Two-agent discrete choice model with random coefficient utility functions for structural analysis on household labor supply

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<u>PRELIMINARY AND INCOMPLETE</u>

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Abstract

This paper discusses a bargaining model on discrete choices of individual household based on two-agent qualitative choice model. The two-agent qualitative choice model describes discrete choices made through bargaining interactions between two agents. This paper presents a bargaining model of discrete choices on labor supply of husband and wife of households.

Contrary to self-employed workers, employees' hours of work tends to be assigned by employers. In the case where hours of work is restricted, the choices the agents make will not be continuous but discrete, i.e., binary choices of whether the agents work or not. This model explicitly demonstrates utility maximizing behavior of two interacting agents under such discrete constraint imposed on hours of work.

As structural equations, an income-leisure preference function of wife and that of husband are introduced in this paper. These functions have random coefficients, which represent taste differences among wives and husbands in population.

This paper focuses on labor supply decisions made by Japanese households that consist only one couple of wife and husband with children under fifteen years old.

Keywords: binary choice, bargaining model, employee opportunity, income-leisure preference function, structural estimation

JEL Classification Numbers: C35, C82, J20

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1 Introduction

In the literature of path breaking analyses on qualitative economic decisions, McFadden (1973;1981), Hausman and Wise (1978) focus on describing decisions of individual agents. Most of the models following these contributions, such as Dubin and McFadden (1984), describe discrete choices based on single-agent qualitative choice model. On the other hand, this paper discusses a bargaining model on discrete choices of individual household based on two-agent qualitative choice model. The two-agent qualitative choice model describes discrete choices made through bargaining interactions between two agents.

This paper presents a bargaining model of discrete choices on labor supply of husband and wife of households. The model gives probabilistic distributions of discrete choices on household labor supply. According to the probabilistic distributions, the model describes binary choice behavior of whether the agents accept the employee job opportunity or not.

Contrary to self-employed workers, employees' hours of work h tends to be assigned by employers as $h = \overline{h}$, where $\overline{h}$ stands for assigned hours. In the case where hours of work is restricted to $\overline{h}$, the choices the agents make will not be continuous but discrete, i.e., binary choices of whether the agents work or not.

This model explicitly demonstrates utility maximizing behavior of two interacting agents under such discrete constraint imposed on hours of work. As structural equations, an income-leisure preference function of wife and that of husband are introduced in this paper. These functions have random coefficients, which represent taste differences among wives and husbands in population. While indirect utility functions are implicitly introduced in most analyses on qualitative economic decisions, direct income-leisure utility functions, as well as income-leisure restrictions are explicitly introduced as structural equations in this paper, and structural parameters of the utility functions are estimated.

Let μ_w denote wife's labor supply probability to her employee job opportunity, and let μ_h denote husband's labor supply probability to his employee job opportunity. Based on the estimated structural parameters, conditional forecasts on μ_w as well as on μ_h are performed, given wage rates and assigned hours of work.

In the literature of quantitative analyses on labor supply, the cross sectional analysis by Douglas (1934) gave a significant evidence that the observed job participation ratios of females are negatively correlated to the observed household income levels[1] . This finding implies that labor supply decision made by household members may not be independent, and thus interacting decision making behavior between household members need to be introduced in theory explicitly.

As for labor supply behavior of females, Mincer (1962) described patterns of female labor supply in a long run by introducing a lifetime hypothesis, and presented females' work hour allocation in their lifetime. More than that, Heck-

[1] See Douglas (1934) pp.279-294.

man (1974) demonstrated that the notion of "reservation wage" gave a way to probabilistic analyses on binomial working decisions made by females.

In the literature of household decision making behavior, Mancer and Brown (1980), Bjorn and Vuong (1984) presented theories explicitly describing bargaining interactions between wife and husband in a household. Instead of describing interactions between household members explicitly, Obi (1969a, 1969b, 1979) introduced an analytical notion of "principal earner" and "non-principal earners" of household. "Principal earner" is analytically defined as a household member whose wage rate is the highest among the wages of household members in each specific household. Given the observed wage differentials between male's labor market and female's labor market, husbands are assigned as principal earners, and wives are assigned as non-principal earners in most cases. Obi described the non-principal earners' labor supply probability to their employee and/or self-employee job opportunities, given the principal earners' job participation, and thus given the principal earners' income level.

This paper focuses on labor supply decisions made by Japanese households that consist only one couple of wife and husband with children under fifteen years old[2] . This paper explicitly describes work decisions of household through interactions between wife and husband in a household, thus both wife's work decision and husband's work decision are described endogenously.

In Section 2, I propose a two-agent discrete choice model of household on labor supply decisions to employee job opportunities. Based on the model in Section 2, Section 3 gives a stochastic model of household labor supply. This model introduces random coefficients to income-leisure preference functions representing taste difference among agents in population. Estimation of structural parameters and simulation is presented in Section 4. Concluding remarks are given in Section 5.

2 A two-agent discrete choice model of household on labor supply

This section presents a model of household labor supply of husband and wife, which is a special case of a two-agent discrete choice model of household on labor supply. The household described here is supposed to consist of only one couple of husband and wife, and their children under 15 years[3] of age, if any.

Household members, not limited to husbands and wives, generally have choices among self-employed work opportunities as well as employee work opportunities. The model in this paper exclusively focuses on the decision-making

[2] Children under fifteen years old are prohibited to work by law in Japan, and thus the agents that make work decisions are limited to wife and husband in such a household.

[3] See footnote 2.

behavior concerning employee work opportunities only[4] . Contrary to self-employed workers, employees' hours of work h tends to be assigned by employers as $h = \overline{h}$, where $\overline{h}$ stands for assigned hours. In this case, the decision making of labor supply has characteristics of discrete choice, because what each agent can choose is, not how long he or she works, but whether he or she works.

2.1 Income-leisure preference function and constraints

Unearned income (in real term) which the ith household gains during a unit period is denoted as I_A^i. The wage rate and assigned hour of work of the employee work opportunity, which the husband of the ith household faces, is denoted by w_h^i and $\overline{h}_h^i$ respectively. Analogously, the wage rate and assigned hour of work, which the wife of the ith household faces, is denoted by w_w^i and $\overline{h}_w^i$ respectively. The wage rates, w_h^i and w_w^i, as well as the unearned income I_A^i, are measured in real term. The total income of the ith household in real term is denoted by X^i. X^i is the sum of the unearned income, I_A^i, and the income actually earned by the husband and the wife of ith household.

Let the leisure of husband and wife be denoted by Λ_h^i and Λ_w^i respectively. The range of Λ_h^i and Λ_w^i should be $0 \leq \Lambda_h^i \leq T$ and $0 \leq \Lambda_w^i \leq T$, where T is the agent's maximum amount of consumable leisure during a unit period.

The total income, X^i is subject to expenditure by the wife as well as by the husband of ith household. On the contrary, the husband's leisure, Λ_h^i, is not subject to consumption by the wife, nor the wife's leisure, Λ_w^i, is not subject to consumption by the husband. In short, it is reasonably assumed that each member of the ith household can exclusively consume his own or her own leisure only. Therefor the following assumption is introduced in this paper.

> **Assumption 1**:Each of the husband and the wife of ith household has the following utility indicator function, ω_h and ω_w respectively.

$$\omega_h^i = \omega_h(X^i, \Lambda_h^i | \boldsymbol{\Gamma_h^i}) \tag{1}$$
$$\omega_w^i = \omega_w(X^i, \Lambda_w^i | \boldsymbol{\Gamma_w^i}) \tag{2}$$

where $\boldsymbol{\Gamma_h^i}$ and $\boldsymbol{\Gamma_w^i}$ are parameter vectors of the utility function, each for the ith household's husband and wife respectively [5] .

[4] See Miyauchi (1992) for a two-agent discrete/continuous choice model of household on labor supply for employee and/or self-employed job opportunity.

[5] Although the case $\dfrac{\partial \omega_h}{\partial \Lambda_w} \neq 0$ or $\dfrac{\partial \omega_w}{\partial \Lambda_h} \neq 0$ cannot be excluded a priori, a functional form such that these values are constantly zero is assumed for simplicity in this paper.

Each husband and wife maximizes his or her own utility indicator function subject to the constraints of;

$$
\begin{aligned}
X^i &= I_A^i + w_h^i h_h^i + w_w^i h_w^i && (3)\\
\Lambda_h^i &= T - h_h^i \qquad (h_h^i = 0 \text{ or } \overline{h}_h^i) && (4)\\
\Lambda_w^i &= T - h_w^i \qquad (h_w^i = 0 \text{ or } \overline{h}_w^i) && (5)
\end{aligned}
$$

where each h_h^i and h_w^i denotes hours of work of husband and wife of ith household respectively.

The husband of ith household maximizes the utility indicator function (1) subject to the constraints (3) and (4). Similarly, the wife of ith household maximizes the utility indicator function (2) subject to the constraints (3) and (5). Inserting these restrictions (3) through (5) into the utility indicator function (1) and (2) yields

$$
\begin{aligned}
\omega_h^i &= \omega_h(I_A^i + w_h^i h_h^i + w_w^i h_w^i, T - h_h^i | \boldsymbol{\Gamma_h^i}) && (6)\\
\omega_w^i &= \omega_w(I_A^i + w_h^i h_h^i + w_w^i h_w^i, T - h_w^i | \boldsymbol{\Gamma_w^i}) && (7)
\end{aligned}
$$

$$
\text{where} \begin{cases} h_h = 0 \quad \text{or} \quad h_h = \overline{h}_h \\ h_w = 0 \quad \text{or} \quad h_w = \overline{h}_w \end{cases}
$$

Note that both h_h^i and h_w^i enter the each of husband's utility function (6), and wife's utility function (7). For this reason, the attainable maximum utility of each husband and wife is not independently determined, but affected by the labor supply choice of the other belonging to the same household.

2.2 Payoffs of labor supply choice in terms of preference

As shown in equations (6) and (7), the utility level of husband and wife, ω_h and ω_w respectively depends on the combination of (h_h, h_w). The combination of (h_h, h_w) is exhaustible within the following four cases according to the labor supply choice made by husband and wife.

(i)	$(h_h, h_w) = (0, 0)$	neither works
(ii)	$(h_h, h_w) = (\overline{h}_h, 0)$	only husband works
(iii)	$(h_h, h_w) = (0, \overline{h}_w)$	only wife works
(iv)	$(h_h, h_w) = (\overline{h}_h, \overline{h}_w)$	both work

where the superscript i is suppressed for simplicity.

Let the above (i) through (iv) be noted as the "combination of labor supply choice." The husband's preference order related to the "combination of labor supply choice" (i) through (iv) is determined by the inequalities among the set of values of ω_h's, which are obtained by inserting the each value of (h_h, h_w) in (i) through (iv) into the right hand side of the husband's utility function (6). In other words, the preference order is determined by the shapes of the husband's indifference curves. The wife's preference order is also determined similarly by inserting the each value of (h_h, h_w) in (i) through (iv) into her utility function (7), which is equivalent to the statement that the order is dependent on the shapes of the wife's indifference curves.

Income-leisure restrictions, (3) through (5), are graphically depicted in Figure 1. The preference field depicted on the right half of Figure 1 is the husband's, and that of wife is depicted on the left half. The vertical axis depicts the total income X, which commonly enters the right hand sides of husband's and wife's utility function, (1) and (2). The horizontal axis depicts the leisure of husband Λ_h or that of wife Λ_w, according to whose field it belongs to. Note that Λ_h and Λ_w are depicted separately in Figure 1, because Λ_h enters exclusively into the husband's utility function (1), and so does Λ_w into the wife's utility function (2). The length of $\overline{\text{or}}$, and that of $\overline{\text{o'r'}}$ shows the husband's and the wife's maximum amount of consumable leisure during a unit period, T, respectively.

The set of the wage rate, w_h, and the assigned hours of work, $\overline{h}_h$, of husband's employee work opportunity is depicted by the portion of the line $\overline{\text{ab}}$ or by $\overline{\text{cd}}$, depending whether his wife works or not. The portion of the line $\overline{\text{ab}}$ and $\overline{\text{cd}}$ are depicted so that $\tan \alpha = w_h$ holds.

On the other hand, the set of the wage rate, w_w, and the assigned hours of work, $\overline{h}_w$, of wife's employee work opportunity is depicted by the portion of the line $\overline{\text{a'b'}}$ or by $\overline{\text{c'd'}}$, depending whether her husband works or not. The portion of the line $\overline{\text{a'b'}}$ and $\overline{\text{c'd'}}$ are depicted so that $\tan \beta = w_w$ holds.

(i) In case neither works $[(h_h, h_w) = (0, 0)]$, the husband is located at point a, and the wife is at point a'. Let the intersection point with the Λ_h axis and a vertical line passing point b and d be denoted by H. The length of the portion of the line $\overline{\text{rH}}$ is the husband's assigned hours of work. Similarly, let the intersection point with the Λ_w axis and a vertical line passing point b' and d' be denoted by H'. The length of the portion of the line $\overline{\text{r'H'}}$ is the wife's assigned hours of work. (ii) In case only husband works $[(h_h, h_w) = (\overline{h}_h, 0)]$, the husband is located at point b, and the wife is at point c'. (iii) In case only wife works $[(h_h, h_w) = (0, \overline{h}_w)]$, the husband is at point c, and the wife at point b'. Lastly (iv) in case both work $[(h_h, h_w) = (\overline{h}_h, \overline{h}_w)]$, the husband is at point d, and the wife at point d'.

Because the total income X, which is common to the husband and the wife, enters the husband's and the wife's utility functions, (1) and (2), simultaneously, the ordinate of the husband's position and that of the wife's position in their preference field are always the same to each other.

Given the income-leisure constraint equations, (3) and (4), the husband faces one of the two alternative choice sets {a, b} or {c, d}, depending upon the wife's decision of whether she works. If the wife works, the husband faces the choice set of {c, d}, otherwise {a, b}. The values of utility index can be assigned to these points a, b, c, and d by inserting the coordinates of these points to the right hand side of the formula (6), and these values are regarded as payoffs of the husband. Similarly, given the income-leisure constraint equations, (3) and (5), the wife faces one of the two alternative choice sets {a', b'} or {c', d'}, Wife's income-leisure preference field

$\overline{h}_w$:Wife's assigned hours of work

w_w:Wife's wage rate in real term

$\tan\beta = w_w$

Husband's income-leisure preference field

$\overline{h}_h$:Husband's assigned hours of work

w_h:Husband's wage rate in real term

$\tan\alpha = w_h$

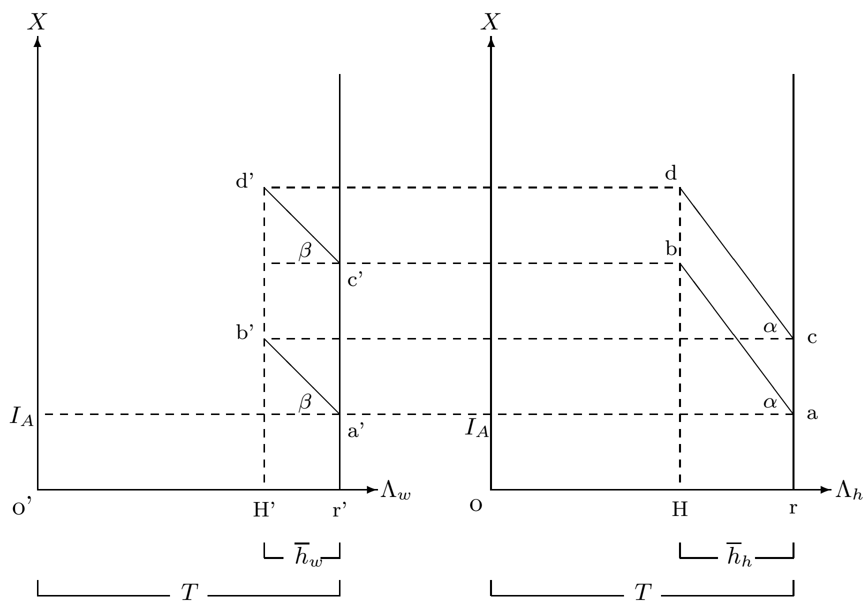


Figure 1: Husband's and wife's income-leisure preference field and their income-leisure constraints

depending upon the husband's decision of whether he works. If the husband works, the wife faces the choice set of {c', d'}, otherwise {a', b'}. The values of utility index at these points a', b', c', and d' are given by the formula (7), and these values are regarded as payoffs of the wife.

The husband's payoffs varies depending upon the shapes of his indifference curves, and so does the wife's payoffs. How many types of payoffs possibly exist as the shapes of indifference curves vary? The permutation of husband's set of four points a, b, c, and d or that of wife's set of a', b', c', and d' is 4! = 24 in all respectively. However, only 6 types of payoffs out of 24 proved to be plausible under the conditions of the positivity of marginal utility and the convexity of indifference curves[6] .

Let the values of husband's preference index at the points a, b, c, and d be denoted by ω_h^{a}, ω_h^{b}, ω_h^{c}, and ω_h^{d} respectively. Similarly let the values of wife's preference index at the points of a', b', c', and d' be denoted by $\omega_w^{\mathrm{a'}}$, $\omega_w^{\mathrm{b'}}$, $\omega_w^{\mathrm{c'}}$, and $\omega_w^{\mathrm{d'}}$ respectively. Thus the payoffs of husband and wife is given as in Table 1 and Table 2 respectively.

Table 1: Husband's payoffs

wife / husband	$h_w = 0$	$h_w = \overline{h}_w$
$h_h = 0$	ω_h^{a}	ω_h^{c}
$h_h = \overline{h}_h$	ω_h^{b}	ω_h^{d}

Table 2: Wife's payoffs

wife / husband	$h_w = 0$	$h_w = \overline{h}_w$
$h_h = 0$	$\omega_w^{\mathrm{a'}}$	$\omega_w^{\mathrm{b'}}$
$h_h = \overline{h}_h$	$\omega_w^{\mathrm{c'}}$	$\omega_w^{\mathrm{d'}}$

Let the husband's payoff matrix of ω_h's in Table 1 be denoted by π_h^k $(k = 1, 2, \cdots, 6)$. Similarly let the wife's payoff matrix of ω_w's in Table 2 be denoted by π_w^ℓ $(\ell = 1, 2, \cdots, 6)$. The superscript k of π_h^k indicates the plausible type of husband's payoffs, and the superscript ℓ of π_w^ℓ indicates the plausible type of wife's payoffs.

The 6 types of husband's payoff matrix are listed in the left column of Table 3, while the 6 types of wife's payoff matrix are in the right column[7] . Since the utility index is ordinal, the payoffs in the matrices π_h^k and π_w^ℓ are indicated ordinal number 1, 2, 3, and 4, where 1 is least preferred and 4 is most preferred.

[6] See Miyauchi(1991).

[7] See Miyauchi(1991) for more precise discussions on the relation between the types of payoff matrix and the shapes of indifferent curves.

Table 3: Six types of payoff matrix

$\pi_h^1 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$	$\pi_w^1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
$\pi_h^2 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$	$\pi_w^2 = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$
$\pi_h^3 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$	$\pi_w^3 = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$
$\pi_h^4 = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$	$\pi_w^4 = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$
$\pi_h^5 = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$	$\pi_w^5 = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$
$\pi_h^6 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$	$\pi_w^6 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

The payoff matrices $\pi_h^k \, (k = 1, 2, \cdots, 6)$ and $\pi_w^\ell \, (\ell = 1, 2, \cdots, 6)$ shown in Table 3 are relevant and exhaustible for all the plausible cases under the basic assumption of utility function. In other words, the payoff matrices in Table 3 cover whole plausible cases. Note that the wife's payoff matrix π_w^6 should be excluded from the plausible set of payoff matrix if and only if $w_h \, \overline{h}_h \geq w_w \, \overline{h}_w$ holds, and the husband's payoff matrix π_h^6 should be excluded from the plausible set of payoff matrix if and only if $w_h \, \overline{h}_h \leq w_w \, \overline{h}_w$ hols[8] .

2.3 Guaranteed income level of husband and wife

Let I_h^0 denote husband's guaranteed income, that is the amount of income available to husband even if he does not work. I_h^0 is the sum of unearned income of the household, I_A, and the earned income by his wife, $w_w h_w$, as

$$I_h^0 = I_A + w_w h_w \tag{8}$$

where $h_w = 0$ in case his wife does not work, and where $h_w = \overline{h}_w$ in case she works.

Similarly, let I_w^0 denote wife's guaranteed income, that is the sum of unearned income of the household, I_A, and the earned income by her husband, $w_h h_h$, as

$$I_w^0 = I_A + w_h h_h \tag{9}$$

where $h_h = 0$ in case her husband does not work, and where $h_h = \overline{h}_h$ in case he works.

[8] See Miyauchi(1991) for more details.

2.4 Nash-equilibrium of the two-person game

Combining the husband's payoff matrices, $\pi_h^k\,(k = 1, 2, \cdots, 6)$, and the wife's payoff matrices, $\pi_w^\ell\,(\ell = 1, 2, \cdots, 6)$, payoff table, $\Pi^{k-\ell}$ $(k, \ell = 1, 2, \cdots, 6)$, is obtained. For example, combining the husband's payoff matrix, π^1, and the wife's payoff matrix, π^3, yields a payoff table Π^{1-3}.

$$\Pi^{1-3} = \begin{bmatrix} (1,1) & (3,2) \\ (2,4) & (4,3) \end{bmatrix}$$

Some payoff tables have Nash-equilibria, and others not. Among the payoff tables that have Nash-equilibria, some tables have unique Nash-equilibria and others have non-unique equilibiria. In the left half of Table 4-1 through Table 4-3, Nash-equilibria corresponding to each payoff table is shown by $\circ$ or $\odot$ if the Nash-equilibria exists. The payoff tables, Π^{2-3} and Π^{3-2} have no Nash-equilibria. The payoff table, Π^{3-3}, has two Nash-equilibira, which means only husband works or only wife works. Other payoff tables have unique Nash-equilibria.

Let the husband's and wife's utility indicator corresponding to a unique Nash-equilibria be denoted by ω_h^* and ω_w^* respectively. Moreover, let the husband's and wife's utility indicator, that they obtain when they change their choice on labor supply corresponding to Nash-equilibria simultaneously, be denoted by ω_h' and ω_w' respectively. Cooperative equilibrium can exist if and only if the two inequalities

$$\omega_h^{1*} < \omega_h^2 \tag{10}$$

$$\omega_w^{1*} < \omega_w^2 \tag{11}$$

hold simultaneously. For the payoff tables, Π^{2-4}, Π^{4-2}, and Π^{4-4}, there exists cooperative equilibria, as shown in the right half of the Table 4-1 through Table 4-3. (Actually, cooperative equilibria exists only for the payoff tables in Table 4-1.)

Table 4-1: Solution of the game (case 1)

$\pi_h^k - \pi_w^\ell$	Nash-equilibria				cooperative equilibria			
	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$
1–1				$\odot$				
1–2				$\odot$				
1–3		$\odot$						
1–4		$\odot$						
1–5		$\odot$						
2–1				$\odot$				
2–2				$\odot$				
2–3	no equilibria							
2–4	$\circ$							$\odot$
2–5	$\odot$							
3–1			$\odot$					
3–2	no equilibria							
3–3		$\circ$	$\circ$					
3–4		$\odot$						
3–5		$\odot$						
4–1			$\odot$					
4–2	$\circ$							$\odot$
4–3			$\odot$					
4–4	$\circ$							$\odot$
4–5	$\odot$							
5–1			$\odot$					
5–2	$\odot$							
5–3			$\odot$					
5–4	$\odot$							
5–5	$\odot$							

Table 4-2: Solution of the game (case 2)

valid only for the case $w_h\overline{h}_h > w_w\overline{h}_w$

$\pi_h^k - \pi_w^\ell$	Nash-equilibria				cooperative equilibria			
	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$
6–1				$\odot$				
6–2				$\odot$				
6–3		$\odot$						
6–4		$\odot$						
6–5		$\odot$						

Table 4-3: Solution of the game (case 3)

valid only for the case $w_h\overline{h}_h < w_w\overline{h}_w$

$\pi_h^k - \pi_w^\ell$	Nash-equilibria				cooperative equilibria			
	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$
1–6				$\odot$				
2–6				$\odot$				
3–6			$\odot$					
4–6			$\odot$					
5–6			$\odot$					

3 A stochastic model of household labor supply of husband and wife

A stochastic model of household labor supply is constructed based on the results presented in Section 2.

3.1 Threshold income of labor supply (TILS)

In this subsection, a concept of "Threshold Income of Labor Supply (TILS)" is defined.

Given the wage rate, w_h, and also given the assigned hours of work, $\overline{h_h}$, the husband's TILS, denoted by I_h^*, is defined as the income level such that the equation

$$\omega_h(I_h^*, T|\boldsymbol{\Gamma_h^i}) = \omega_h(I_h^* + w_h\,\overline{h}_h, T - \overline{h}_h|\boldsymbol{\Gamma_h^i}) \tag{12}$$

holds, where the function, ω_h, is given by the formula (1). Solving the equation (12) regarding to I_h^* yields the formula

$$I_h^* = I_h^*(w_h,\,\overline{h}_h|\boldsymbol{\Gamma_h^i}) \tag{13}$$

indicating that I_h^* is a function of w_h, $\overline{h}_h$, and parameter vector, $\boldsymbol{\Gamma_h^i}$.

Analogously, the wife's TILS, denoted by I_w^*, is defined as the income level such that the equation

$$\omega_w(I_w^*, T|\boldsymbol{\Gamma_w^i}) = \omega_w(I_w^* + w_w\,\overline{h}_w, T - \overline{h}_w|\boldsymbol{\Gamma_w^i}) \tag{14}$$

holds, given the wage rate, w_w, as well as given the assigned hours of work, $\overline{h}_w$. The function, ω_w, is given by the formula (2). Solving the equation (14) regarding to I_w^* yields

$$I_w^* = I_w^*(w_w,\,\overline{h}_w|\boldsymbol{\Gamma_w^i}) \tag{15}$$

indicating that I_w^* is a function of w_w, $\overline{h}_w$, and parameter vector, $\boldsymbol{\Gamma_w^i}$.

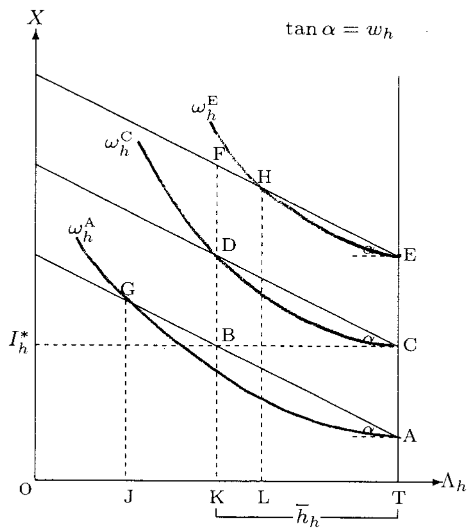


Figure 2: Husband's TILS and MHLS where $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds.

A brief explanation on the relation between labor supply decision and TILS might be necessary. Figure 2 describes the relation between husband's decision on labor supply and his TILS, given the wage rate, w_h, and also given the assigned hours of work, $\overline{h_h}$, of his employee opportunity. Suppose his guaranteed income level is the amount of the length $\overline{\mathrm{TA}}$. In this case, he will accept the employee opportunity, because his utility indicator at point B, where he is if works, is higher than his utility indicator at point A, where he is if does not work. Next, suppose his guaranteed income increases up to the amount of the length $\overline{\mathrm{TE}}$. Contrarily in this case, he will reject the employee opportunity, because his utility indicator at point E, where he is if does not work, is higher than his utility indicator at point F, where he is if he works. Finally, suppose his guaranteed income, I_h^0, is exactly the amount of the length $\overline{\mathrm{TC}}$. He will be at point C if he does not work, and he will be at point D if he does work. In this final case, whether he works or not is exactly indifferent to him, because both point C and D are on the same indifference curve, ω_h^{C}. The guaranteed income level that makes it indifferent whether the husband works or not is the husband's TILS, I_h^*, such as the amount of the length $\overline{\mathrm{TC}}$ in Figure 2. The wife's TILS can be explained analogously.

3.2 Maximum hours of labor supply

In this subsection, a concept of "Maximum Hours of Labor Supply (MHLS)" is defined.

Given the wage rate, w_h, and also given the guaranteed income, I_h^0, husband's MHLS is defined as the hours of work, h_h, such that the equation

$$\omega_h(I_h^0, T|\boldsymbol{\Gamma_h^i}) = \omega_h(I_h^0 + w_h\, h_h, T - h_h|\boldsymbol{\Gamma_h^i}) \tag{16}$$

holds, where the function, ω_h, is given by the formula (1). Note that h_h is a variable and not necessarily equal to $\overline{h_h}$. Solving the equation (16) regarding to h_h yields the formula

$$h_h^x = h_h^x(I_h^0, w_h|\boldsymbol{\Gamma_h^i}) \tag{17}$$

where the husband's MHLS is denoted by h_h^x. The formula (17) indicates that the husband's MHLS is the function of husband's guaranteed income, I_h^0, wage rate, w_h, and parameter vector, $\boldsymbol{\Gamma_h^i}$, of his utility function.

Analogously, wife's MHLS is defined as the hours of work, h_w, such that

$$\omega_w(I_w^0, T|\boldsymbol{\Gamma_w^i}) = \omega_w(I_w^0 + w_w\, h_w, T - h_w|\boldsymbol{\Gamma_w^i}) \tag{18}$$

holds, given the wage rate, w_w, as well as given the guaranteed income, I_w^0. The function, ω_w, is given by the formula (2). Note that h_w is also a variable and not necessarily equal to $\overline{h_w}$. Solving the equation (18) regarding to h_w yields

$$h_w^x = h_w^x(I_w^0, w_w|\boldsymbol{\Gamma_w^i}) \tag{19}$$

where the wife's MHLS is denoted by h_w^x. The formula (19) indicates that the wife's MHLS is the function of wife's guaranteed income, I_w^0, wage rate, w_w, and parameter vector, $\boldsymbol{\Gamma_w^i}$, of her utility function.

Husband's MHSL can be shown in Figure 2. Points L, K, J, are the feet of perpendiculars to Λ_h axis originating points H, D, G, respectively. Suppose husband's guaranteed income, I_h^0, is the amount of the length $\overline{\text{TA}}$. In this case, the husband will accept the employee opportunity of wage rate, w_h, and assigned hours of work, $\overline{h_h}$. Given his guaranteed income, $I_h^0 = \overline{\text{TA}}$, and also given the wage rate, w_h, he will accept any employee opportunity as long as the assigned hours of work is at most the amount of length $\overline{\text{TJ}}$. He will reject the employee opportunity if the assigned hours of work exceeds the length $\overline{\text{TJ}}$. The husband's MHLS is the assigned hours of work that makes it indifferent whether he works or not. The wife's MHLS can be shown analogously.

Note that the husband's MHLS, h_h^x, decreases as his guaranteed income, I_h^0, increases, i.e., $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds in Figure 2. On the contrary, husband's indifference curves depicted in Figure 3 make his MHLS increase as his guaranteed income increases, so that $\dfrac{\partial h_h^x}{\partial I_h^0} > 0$ holds in Figure 3.

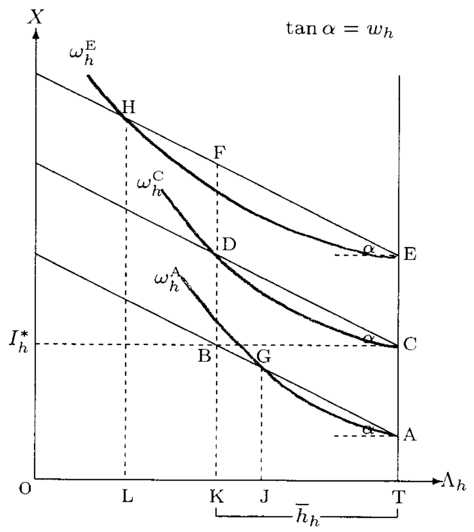


Figure 3: Husband's TILS and MHLS where $\dfrac{\partial h_h^x}{\partial I_h^0} > 0$ holds.

It should be noted that whether the sign of $\dfrac{\partial h_h^x}{\partial I_h^0}$ is positive or negative depends on the characteristics of his indifference curves. Moreover, note that the sign of $\dfrac{\partial h_h^x}{\partial I_h^0}$ determines the relationship between his decision of labor supply and the regions of his Threshold Income of Labor Supply, TILS. Similar statements apply to the case of wife.

3.3 The relation between TILS and decision on labor supply

Let the relation between husband's TILS and his decision be described first. Given the husband's guaranteed income, I_h^0, let the husband's utility index be denoted by ω_h^0 in case he does not accept the employee opportunity of w_h and $\overline{h}_h$, and let it be denoted by ω_h^1 in case he does accept the same employee opportunity. The values of ω_h^0 and ω_h^1 can be given by the formula

$$\begin{aligned}\omega_h^0 &= \omega_h(I_h^0, T|\mathbf{\Gamma}_h^i)\\ \omega_h^1 &= \omega_h(I_h^0 + w_h\overline{h}_h, T - \overline{h}_h|\mathbf{\Gamma}_h^i)\end{aligned}$$

where ω_h is the husband's utility function (1). The relation between the region of his TILS, I_h^*, and the inequalities of ω_h^0 and ω_h^1 is

$$\left.\begin{array}{l} I_h^0 < I_h^* \rightleftharpoons \omega_h^0 < \omega_h^1 \\ I_h^0 > I_h^* \rightleftharpoons \omega_h^0 > \omega_h^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_h^x}{\partial I_h^0} < 0 \tag{20}$$

$$\left.\begin{array}{l} I_h^0 < I_h^* \rightleftharpoons \omega_h^0 > \omega_h^1 \\ I_h^0 > I_h^* \rightleftharpoons \omega_h^0 < \omega_h^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_h^x}{\partial I_h^0} > 0 \tag{21}$$

For the case of wife, let the wife's utility index be denoted by ω_w^0 in case she does not accept the employee opportunity of w_w and $\overline{h_w}$, and let it be denoted by ω_w^1 in case she does accept the same employee opportunity, given her guaranteed income, I_w^0. The values of ω_w^0 and ω_w^1 can be given by the formula

$$\begin{aligned} \omega_w^0 &= \omega_w(I_w^0, T|\mathbf{\Gamma}_{\boldsymbol{w}}^{\boldsymbol{i}}) \\ \omega_w^1 &= \omega_w(I_w^0 + w_w\overline{h}_w, T - \overline{h}_w|\mathbf{\Gamma}_{\boldsymbol{w}}^{\boldsymbol{i}}) \end{aligned}$$

where ω_w is the wife's utility function (2). The relation between the region of her TILS, I_w^*, and the inequalities of ω_w^0 and ω_w^1 is

$$\left.\begin{array}{l} I_w^0 < I_w^* \rightleftharpoons \omega_w^0 < \omega_w^1 \\ I_w^0 > I_w^* \rightleftharpoons \omega_w^0 > \omega_w^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{22}$$

$$\left.\begin{array}{l} I_w^0 < I_w^* \rightleftharpoons \omega_w^0 > \omega_w^1 \\ I_w^0 > I_w^* \rightleftharpoons \omega_w^0 < \omega_w^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{23}$$

The husband's indifference curves depicted in Figure 2 correspond to the case $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$. In Figure 2, the husband's TILS, I_h^*, is shown by the length $\overline{\text{TC}}$. Note that as long as his guaranteed income, I_h^0, is equal to his TILS, I_h^*, his MHLS, h_h^x, is equalized to the assigned hours of work, $\overline{h_h}$, of his employee opportunity ($I_h^0 = I_h^* \rightleftharpoons h_h^x = \overline{h}_h$). Suppose his guaranteed income, I_h^0, falls below his TILS, I_h^* ($I_h^0 < I_h^*$), to the level such as the length $\overline{\text{TA}}$. As long as $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds as in Figure 2, $h_h^x > \overline{h}_h$ holds for the region of $I_h^0 < I_h^*$. This means that as long as $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds as in Figure 2, the point G is necessarily located on the left side of the point B in Figure 2, because $h_h^x > \overline{h}_h$ holds and because the amount of h_h^x is, by definition, the distance between $\overline{\text{TE}}$ and point G, which is the intersection of line AB and the indifference curve on point A, ω_h^{A}. Considering the convexity of the indifference curves to the origin, it is concluded that utility index on point B is necessarily higher than that on point A , because both point A and G is on the same indifference curve and because point B is located between A and G on the line AG. Thus, the relation

$$I_h^0 < I_h^* \rightharpoonup \omega_h^0 < \omega_h^1 \tag{24}$$

is obtained.

Next, on the contrary, suppose husband's guaranteed income, I_h^0, raises above his TILS, I_h^* ($I_h^0 > I_h^*$), to the level such as the length $\overline{\mathrm{TE}}$. In this case $h_h^x < \overline{h}_h$ holds for the region of $I_h^0 > I_h^*$, as long as $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds as in Figure 2. By the similar reasoning above, it is concluded that point H is necessarily located on the right side of point F and that the utility index on point H is necessarily higher than that on point E. Thus, the relation

$$I_h^0 > I_h^* \rightharpoonup \omega_h^0 > \omega_h^1 \tag{25}$$

is obtained.

The relation (24) and (25) yield the relation (20).

In the similar manners as described above, it is shown that the relation (21) holds under the condition $\dfrac{\partial h_h^x}{\partial I_h^0} > 0$, using Figure 3.

The relations (20) and (21) shows that the formulas of inequalities between the guaranteed income and the TILS can be assigned to the payoff matrices.

3.4 Introducing a random coefficient into utility function

In Subsection 3.9, husband's and wife's utility functions with random coefficients are introduced. It is assumed that husbands in population share the same parameters of their utility function except the intercept (constant term) parameter of marginal utility of leisure, which means that the intercept parameter distributes among households in population. Moreover it is assumed that wives in population also share the same parameters of their utility function except the intercept parameter of marginal utility of leisure. This subsection induces the distribution of husband's or wife's Threshold Income of Labor Supply (TILS).

It is assumed that the husband's and wife's parameter vectors of their utility function,

$$\begin{aligned} {}^{\mathbf{t}}\boldsymbol{\Gamma_h} &= (\gamma_{h1}, \gamma_{h2}, \cdots, \gamma_{hm}) \\ {}^{\mathbf{t}}\boldsymbol{\Gamma_w} &= (\gamma_{w1}, \gamma_{w2}, \cdots, \gamma_{wm}) \end{aligned}$$

contains random coefficients, γ_{h4} and γ_{w4}, respectively. γ_{h4} is the intercept parameter of husband's marginal utility of leisure, and γ_{w4} is the random coefficient of wife's marginal utility of leisure. The superscript i of the parameter vector is suppressed for simplicity.

The random coefficients, γ_{h4} and γ_{w4}, are also assumed to follow the joint probability density function

$$f(\gamma_{h4}, \gamma_{w4} \,|\, \boldsymbol{\zeta}\,) \tag{26}$$

where $\boldsymbol{\zeta}$ is the parameter vector of the probability density function f.

The distribution of husband's TILS, I_h^*, and that of wife's HILS, I_w^*, are induced as follows. Solving the formulas (13) and (15) of husband's and wife's TILS, regarding γ_{h4} and γ_{w4} respectively, yields

$$\begin{aligned}\gamma_{h4} &= \gamma_{h4}(I_h^*, w_h, \overline{h}_h | \widetilde{\boldsymbol{\Gamma_h}}) &(27)\\ \gamma_{w4} &= \gamma_{w4}(I_w^*, w_w, \overline{h}_w | \widetilde{\boldsymbol{\Gamma_w}}) &(28)\end{aligned}$$

where $\widetilde{\boldsymbol{\Gamma_h}}$ and $\widetilde{\boldsymbol{\Gamma_w}}$ are the parameter vectors of husband's and wife's including the common parameters among households in population,

$$\begin{aligned}{}^{\mathbf{t}}\widetilde{\boldsymbol{\Gamma_h}} &= (\gamma_{h1}, \gamma_{h2}, \gamma_{h3}, \gamma_{h5}, \cdots, \gamma_{hm})\\ {}^{\mathbf{t}}\widetilde{\boldsymbol{\Gamma_w}} &= (\gamma_{w1}, \gamma_{w2}, \gamma_{w3}, \gamma_{w5}, \cdots, \gamma_{wm})\end{aligned}$$

excluding the random coefficients γ_{h4} and γ_{w4}.

Inserting the formula (27) and (28) into (26) yields

$$f[\gamma_{h4}(I_h^*, w_h, \overline{h}_h | \widetilde{\boldsymbol{\Gamma_h}}), \gamma_{w4}(I_w^*, w_w, \overline{h}_w | \widetilde{\boldsymbol{\Gamma_w}}) \,|\, \boldsymbol{\zeta}\,] \tag{29}$$

The joint probability density function of I_h^* and I_w^* is obtained as a product of the formula (29) and the Jacobean

$$J \equiv \begin{vmatrix} \dfrac{\partial \gamma_{h4}}{\partial I_h^*} & \dfrac{\partial \gamma_{h4}}{\partial I_w^*} \\ \dfrac{\partial \gamma_{w4}}{\partial I_h^*} & \dfrac{\partial \gamma_{w4}}{\partial I_w^*} \end{vmatrix}$$

The Jacobean reduces into the form

$$J = \left| \frac{\partial \gamma_{h4}}{\partial I_h^*} \cdot \frac{\partial \gamma_{w4}}{\partial I_w^*} \right|$$

because $\dfrac{\partial \gamma_{h4}}{\partial I_w^*} = \dfrac{\partial \gamma_{w4}}{\partial I_h^*} = 0$ holds. Multiplying the formula (29) by the Jacobean J yields the joint distribution density function of I_h^* and I_w^*, $g(I_h^*, I_w^*)$, as

$$\begin{aligned} g(I_h^*, I_w^*) &= f[\gamma_{h4}(I_h^*, w_h, \overline{h}_h | \widetilde{\boldsymbol{\Gamma_h}}) \\ &\qquad , \gamma_{w4}(I_w^*, w_w, \overline{h}_w | \widetilde{\boldsymbol{\Gamma_w}}) \,|\, \boldsymbol{\zeta}\,] \cdot \left| \frac{\partial \gamma_{h4}}{\partial I_h^*} \cdot \frac{\partial \gamma_{w4}}{\partial I_w^*} \right| \end{aligned} \tag{30}$$

The formula (30) shows that the shape of the joint distribution density function of I_h^* and I_w^*, $g(I_h^*, I_w^*)$, depends upon the variables w_h, $\overline{h}_h$, w_w, $\overline{h}_w$ and the parameter vectors, $\widetilde{\boldsymbol{\Gamma_h}}$ and $\widetilde{\boldsymbol{\Gamma_w}}$, as well as the parameter vector, $\boldsymbol{\zeta}$, of the joint distribution density function f.

Let the probability density function of each husband's TILS, I_h^*, and wife's TILS, I_w^*, be denoted by $g_h(I_h^*)$ and $g_w(I_w^*)$ respectively. The probability density function $g_h(I_h^*)$ and $g_w(I_w^*)$ are the marginal distributions of the joint distribution, $g(I_h^*, I_w^*)$, given by the formula (30).

3.5 The sign of $\frac{\partial h_w^x}{\partial I_w^0}$ and observation on labor supply

Let the fitted value of wife's labor supply probability be denoted by $\widehat{\mu}_w$. Given the values of w_h, $\overline{h}_h$, w_w, $\overline{h}_w$ and the parameter vectors, $\widetilde{\boldsymbol{\Gamma_h}}$, $\widetilde{\boldsymbol{\Gamma_w}}$ and $\boldsymbol{\zeta}$ in the formula (30), the wife's labor supply probability $\widehat{\mu}_w$ can be calculated by using the probability density function of wife's TILS, $g_w(I_w^*)$. The probability, $\widehat{\mu}_w$, in the population where the wife's guaranteed income is I_w^0 is given by the formula

$$\widehat{\mu}_w = \int_{I_w^0=I_A+I_h^j}^{\infty} g_w(I_w^*)dI_w^* \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{31}$$

or

$$\widehat{\mu}_w = \int_{-\infty}^{I_w^0=I_A+I_h^j} g_w(I_w^*)dI_w^* \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{32}$$

depending upon the sign of $\dfrac{\partial h_w^x}{\partial I_w^0}$. The formula (31) is justified by the relation (22), and the formula (32) is justified by the relation (23). Differentiating $\widehat{\mu}_w$ in respect to wife's guaranteed income, I_w^0, yields

$$\frac{\partial \widehat{\mu}_i}{\partial I_w^0} = -g_w(I_w^0) < 0 \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{33}$$

or

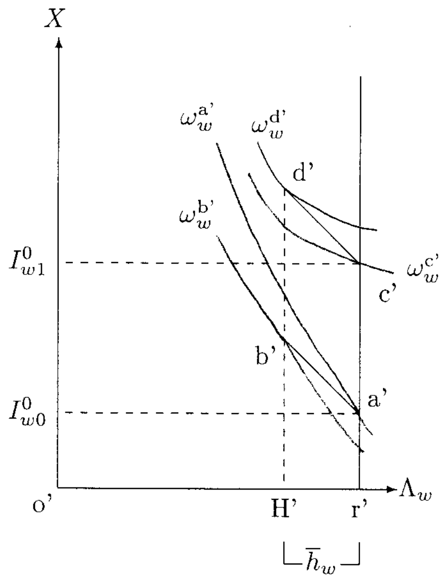
$$\frac{\partial \widehat{\mu}_i}{\partial I_w^0} = g_w(I_w^0) > 0 \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{34}$$

again depending upon the sign of $\dfrac{\partial h_w^x}{\partial I_w^0}$.

Taking the observations of Douglas(1934) into account, the sign of $\dfrac{\partial h_w^x}{\partial I_w^0}$ is necessarily negative because it is observed that household income levels and wife's labor supply ratios are negatively correlated. Thus the equality in formula (34) is inconsistent with observed characteristics of wife's labor supply ratio. This concludes that the inequality $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$ must hold as for the wife's MHLS, h_w^x.

3.6 The consistency between sign of $\frac{\partial h_h^x}{\partial I_h^0}$, $\frac{\partial h_w^x}{\partial I_w^0}$ and payoff matrices

The condition, $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$, which is clarified in Subsection 3.5, is consistent to the wife's payoff matrices listed in Table 3 except π_w^2. This means that the



$$\pi_w^2 = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Figure 4: Wife's indifference curves yielding her payoff matrix π_w^2

whole set of wife's indifference curves that generate her payoff matrix, π_w^2, is not consistent with the condition, $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$.

Indifference curves that generate wife's payoff matrix, π_w^2, is depicted in Figure 4. The wife's guaranteed income when her husband does not work is denoted by I_{w0}^0 in Figure 4. $I_{w0}^0 = I_A$ holds where I_A is the unearned income of the household. Next, in Figure 4, the wife's guaranteed income when her husband does work is denoted by I_{w1}^0, where $I_{w0}^0 = I_A + w_h\overline{h_h}$ holds.

Let the region of wife's TILS, I_w^* of the wife's , be examined if the wife's payoff matrix is π_w^2. Note that wife's indifference curves, which are consistent with her payoff matrix, π_w^2, is depicted in Figure 4.

Let wife's utility index at point a' in Figure 4 be denoted by $\omega_w^{\text{a'}}$, and similarly, let the wife's utility indexes at point b', c', and d' be denoted by $\omega_w^{\text{b'}}$, $\omega_w^{\text{c'}}$, and $\omega_w^{\text{d'}}$ respectively. The relation between the group of ω_w^0, ω_w^1 and the group of $\omega_w^{\text{a'}}$, $\omega_w^{\text{b'}}$, $\omega_w^{\text{c'}}$, $\omega_w^{\text{d'}}$ is

$$\left.\begin{array}{l} \omega_w^0 = \omega_w^{\text{a'}}, \\ \omega_w^1 = \omega_w^{\text{b'}} \end{array}\right\} \quad \text{if} \quad I_w^0 = I_{w0}^0$$
$$\left.\begin{array}{l} \omega_w^0 = \omega_w^{\text{c'}}, \\ \omega_w^1 = \omega_w^{\text{d'}} \end{array}\right\} \quad \text{if} \quad I_w^0 = I_{w1}^0$$

depending whether her husband works or not.

Now suppose wife's indifference curves fulfill the condition $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$. Let us examine the inequality between $\omega_w^{\text{a'}}$ and $\omega_w^{\text{b'}}$, and the inequality between $\omega_w^{\text{c'}}$ and $\omega_w^{\text{d'}}$, according to the relation (22).

1. If $I_w^* < I_{w0}^0 < I_{w1}^0$ hods,
   then $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} > \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_w^* < I_{w0}^0$ does not apply because the inequality $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ holds in Figure 4.

2. If $I_{w0}^0 < I_w^* < I_{w1}^0$ holds,
   then $\omega_w^{\text{a'}} < \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} > \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_{w0}^0 < I_w^* < I_{w1}^0$ does not apply because $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ holds in Figure 4.

3. If $I_{w0}^0 < I_{w1}^0 < I_w^*$ holds,
   then $\omega_w^{\text{a'}} < \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_{w1}^0 < I_w^*$ does not apply because $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ holds in Figure 4.

Following the reasoning 1 through 3 above, the region of wife's TILS, I_w^*, does not exist if her payoff matrix, π_w^2, is assumed. This concludes wife's payoff matrix, π_w^2, is inconsistent with the condition $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$. Thus, the wife's payoff matrix, π_w^2, is excluded from the further discussion.

Two payoff tables, Π^{2-3} and Π^{3-2}, are the two cases where no equilibria exist. (See Table 4-1.) Setting the consistency condition $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$ can preclude the payoff table Π^{3-2}, which gives no equilibria, because wife's payoff matrix, π_w^2, is no longer relevant.

Following the similar reasoning above, it can be concluded that husband's payoff matrix, π_h^2, is inconsistent with the condition $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$, because the region of husband's TILS, I_h^*, does not exist if his payoff matrix, π_h^2, is assumed.

As for the wife's MHLS, h_w^x, the condition, $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$, is required for the consistency with observations. Along with this condition, if the condition, $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$, is assumed for the husband's MHLS, h_h^x, we can exclude the payoff table, Π^{2-3}, where neither Nash-equilibria nor cooperative equilibria exists, because the husband's payoff matrix, π_h^2, becomes no longer relevant. Thus, combining the assumption $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ with the consistency condition $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$ precludes all the cases where no equilibria exist.

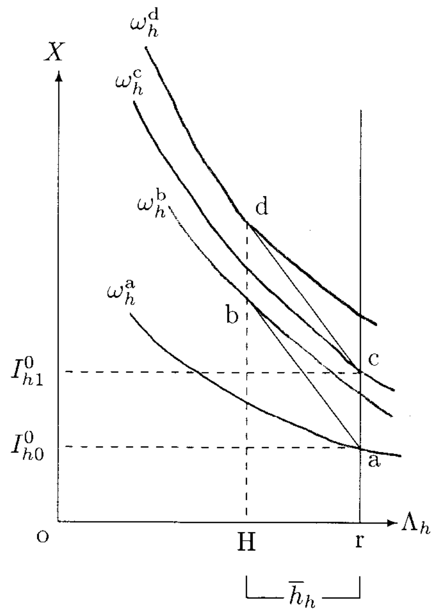
Assumption 2: $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$

3.7 The relation between payoff matrix and the region of TILS

In this subsection, the relations between the payoff matrices, except π_h^2 and π_w^2, listed in Table 3 and the regions of "Threshold Income of Labor Supply (TILS)" is considered.

Firstly, let the region where husband's TILS, I_h^*, exists be considered when his payoff matrix is π_h^1. Husband's indifference curves that generate his payoff matrix π_h^1 is depicted in Figure 5. Let his guaranteed income when his wife does not work be denoted by I_{h0}^0. Now $I_{h0}^0 = I_A$ holds where I_A is the unearned income of the household. Next, let his guaranteed income when his wife does work be denoted by I_{h1}^0, where $I_{h0}^0 = I_A + w_w\overline{h_w}$ holds. I_{h0}^0 and I_{h1}^0 are depicted in Figure 5.

Let husband's utility index at point a in Figure 5 be denoted by ω_h^{a}, and similarly, let the husband's utility indexes at point b, c, and d be denoted by ω_h^{b}, ω_h^{c}, and ω_h^{d} respectively. The relation between the group of ω_h^0, ω_h^1 and the



$$\pi_h^1 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Figure 5: Husband's indifference curves yielding his payoff matrix π_h^1

group of ω_h^{a}, ω_h^{b}, ω_h^{c}, ω_h^{d} is

$$\left.\begin{array}{l}\omega_h^0 = \omega_h^{\mathrm{a}} \\ \omega_h^1 = \omega_h^{\mathrm{b}}\end{array}\right\} \quad \text{if} \quad I_h^0 = I_{h0}^0$$
$$\left.\begin{array}{l}\omega_h^0 = \omega_h^{\mathrm{c}} \\ \omega_h^1 = \omega_h^{\mathrm{d}}\end{array}\right\} \quad \text{if} \quad I_h^0 = I_{h1}^0$$

depending whether his wife works or not.

Let us examine the inequality between ω_h^{a} and ω_h^{b}, and the inequality between ω_h^{c} and ω_h^{d}. The relation (20) gives the relevant inequalities between these utility indexes because $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ is assumed.

1' $\quad I_h^* < I_{h0}^0 < I_{h1}^0 \rightleftharpoons \omega_h^{\mathrm{a}} > \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} > \omega_h^{\mathrm{d}}$ holds.
Nevertheless, $I_h^* < I_{h0}^0$ does not apply because the inequality $\omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ holds in Figure 5.

2' $\quad I_{h0}^0 < I_h^* < I_{h1}^0 \rightleftharpoons \omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} > \omega_h^{\mathrm{d}}$ holds.
Nevertheless, $I_{h0}^0 < I_h^* < I_{h1}^0$ does not apply because the inequality $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ holds in Figure 5.

3' $\quad I_{h0}^0 < I_{h1}^0 < I_h^* \rightleftharpoons \omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ holds.
The inequalities $\omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ comply with the preference curves shown in Figure 5.

The above reasoning 1', 2', and 3' concludes that the region of husband's TILS, $I_h^* = I_h^*(w_h, \overline{h}_h | \boldsymbol{\Gamma_h})$, is $I_{h1}^0 < I_h^*$.

The plausible regions of husband's TILS for the rest of husband's payoff matrices, π_h^3, π_h^4, π_h^5, and π_h^6 are induced following the similar reasoning above. The plausible regions of wife's TILS for wife's payoff matrices, π_w^1, π_w^3, π_w^4, π_w^5, and π_w^6 are also induced similarly. The plausible regions of husband's and wife's TILS are shown in Table 5-1 and 5-2 respectively.

Table 5-1: Regions of husband's TILS corresponding to his payoff matrices

Husband's payoff matrix k's index of π_h^k	Region of husband's TILS, I_h^*
1	$I_{h1}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \boldsymbol{\Gamma_h})$
3	$I_{h0}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \boldsymbol{\Gamma_h}) \quad < I_{h1}^0$
4	$I_{h2}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \boldsymbol{\Gamma_h}) \quad < I_{h0}^0$
5	$I_h^*(w_h, \overline{h}_h \vert \boldsymbol{\Gamma_h}) \quad < I_{h2}^0$
6	$I_{h1}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \boldsymbol{\Gamma_h})$

Note that $I_{h0}^0 = I_A$, $\quad I_{h1}^0 = I_A + w_w\overline{h}_w$.

See formula (37) for the definition of I_{h2}^0.

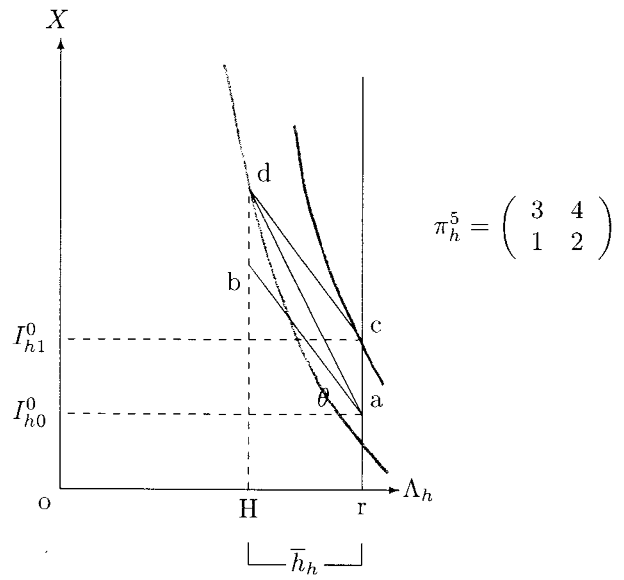


Figure 6: Husband's indifference curves yielding his payoff matrix π^5_w

Table 5-2: Regions of wife's TILS corresponding to her payoff matrixes

Wife's payoff matrix ℓ's index of π^ℓ_w	Region of wife's TILS, I^*_w
1	$I^0_{w1} < \quad I^*_w(w_w, \bar{h}_w \vert \boldsymbol{\Gamma_w})$
3	$I^0_{w0} < \quad I^*_w(w_w, \bar{h}_w \vert \boldsymbol{\Gamma_w}) \quad < I^0_{w1}$
4	$I^0_{w2} < \quad I^*_w(w_w, \bar{h}_w \vert \boldsymbol{\Gamma_w}) \quad < I^0_{w0}$
5	$I^*_w(w_w, \bar{h}_w \vert \boldsymbol{\Gamma_w}) \quad < I^0_{w2}$
6	$I^0_{w1} < \quad I^*_w(w_w, \bar{h}_w \vert \boldsymbol{\Gamma_w})$

Note that $I^0_{w0} = I_A$, $\quad I^0_{w1} = I_A + w_h \bar{h}_h$.
See formula (39) for the definition of I^0_{w2}.

In Table 5-1, a variable I^0_{h2} bounds the region of husband's TILS, which corresponds to his payoff matrix, π_h^5. Similarly in Table 5-2, a variable I_{w2}^0 bounds the region of wife's TILS, which corresponds to her payoff matrix, π_w^5. Some description should be made on these variables, I_{h2}^0 and I_{w2}^0.

Let the variable, I_{h2}^0, be described first. Husband's indifference curves yielding his payoff matrix, π_h^5, is depicted in Figure 6. The coordinates of husband's available leisure, Λ_h, and his available income, X, are indicated by points a, b, c, and d, given the wage rate, w_h, and the assigned hours of work, $\overline{h_h}$, of his employee opportunity. Let an auxiliary line passing point a and d be drawn in Figure 6. The gradient of the line ad to the horizontal axis, θ, is given by the formula

$$\tan\theta = \frac{w_h\overline{h}_h + w_w\overline{h}_w}{\overline{h}_h}$$

Suppose husband's wage rate raises up to

$$W_h \equiv \frac{w_h\overline{h}_h + w_w\overline{h}_w}{\overline{h}_h}$$

and his assigned hours of work unchanged as $\overline{h}_h$. The point d is where husband would be located if he accepts this employee opportunity of W_h and $\overline{h_h}$, in case his wife does not work. Since his utility index at point a is higher than his utility index at d, he does not accept the employee opportunity of W_h and $\overline{h_h}$. Let $\Re_h$ denote the subset consisting of Γ_h's such that inequality $\omega_a > \omega_d$ holds. For the Γ_h's in the subset $\Re_h$,

$$I_h^*(W_h, \overline{h}_h|\boldsymbol{\Gamma_h}) < I_{h0}^0, \quad \forall\, \Gamma_h \in \Re_h \tag{35}$$

holds according to the relation (20). Note that the argument W_h, instead of w_h, enters the husband's TILS in formula (35).

For the Γ_h's in the subset $\Re_h$, what will be the region of husbands' TILS, I_h^*, when the wage rate again reduces to w_h, with the assigned hours of work being constant as $\overline{h_h}$? Suppose the parameter vector, ${}^{\mathbf{t}}\boldsymbol{\Gamma_h}= (\gamma_{h1}, \gamma_{h2}, \cdots, \gamma_{hm})$, of husband's utility function has constant elements over the population except γ_{h4}, which is randomly distributed in the population. Solving the equation

$$I_h^*(W_h, \overline{h}_h|\boldsymbol{\Gamma_h}) = I_{h0}^0$$

in respect to γ_{h4} yields the formula

$$\gamma_{h4} = \gamma_{h4}(I_{h0}^0, W_h, \overline{h}_h|\widetilde{\boldsymbol{\Gamma_h}}) \tag{36}$$

Inserting the formula (36) into the element γ_{h4} in the parameter vector, $\boldsymbol{\Gamma_h}$, of the formula (13) yields

$$\begin{aligned} I_{h2}^0 &= I_h^*\big[w_h, \overline{h}_h\,|\big(\gamma_{h1}, \gamma_{h2}, \gamma_{h3}, \gamma_{h4}(I_{h0}^0, W_h, \overline{h}_h|\widetilde{\boldsymbol{\Gamma_h}}), \cdots, \gamma_{hm}\big)\big] \\ &= I_{h2}^0(I_{h0}^0, w_h, \overline{h}_h, w_w, \overline{h}_w|\widetilde{\boldsymbol{\Gamma_h}}) \end{aligned} \tag{37}$$

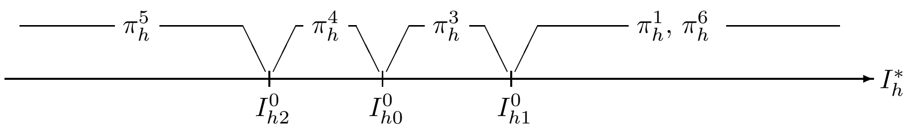


Figure 7: Correspondence between husband's payoff matrixes and the region of his TILS, I_h^*

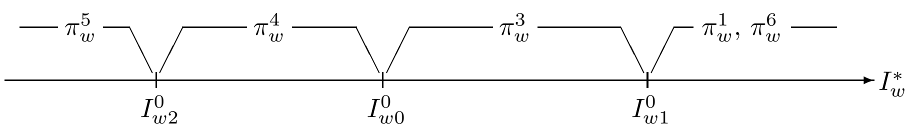


Figure 8: Correspondence between wife's payoff matrixes and the region of her TILS, I_w^*

Using the formula (37), the region of husbands' TILS, whose payoff matrix are π_h^5, can be shown as

$$I_h^*(w_h, \overline{h}_h|\boldsymbol{\Gamma_h}) < I_{h2}^0, \quad \forall\, \Gamma_h \in \Re_h \tag{38}$$

For the variable, I_{w2}^0, bounding the region of wife's TILS, which corresponds to her payoff matrix, π_w^5 in Table 5-2, a formula

$$I_{w2}^0 = I_{w2}^0(I_{w0}^0, w_h, \overline{h}_h, w_w, \overline{h}_w|\widetilde{\boldsymbol{\Gamma_w}}) \tag{39}$$

can be obtained. Using the formula (39), the region of wives' TILS, whose payoff matrix are π_w^5, can be shown as

$$I_w^*(w_w, \overline{h}_w|\boldsymbol{\Gamma_w}) < I_{w2}^0 \tag{40}$$

Regions of TILS corresponding to payoff matrices can be shown graphically in figure 7 and 8 [9] .

[9] For the inequalities between I_{h2}^0 and I_{h0}^0, and for the inequalities between I_{w2}^0 and I_{w0}^0, it can be proved that $\dfrac{\partial h_h^x}{\partial I_h^0} < 0 \rightharpoonup I_{h2}^0 < I_{h0}^0$ and $\dfrac{\partial h_w^x}{\partial I_w^0} < 0 \rightharpoonup I_{w2}^0 < I_{w0}^0$ holds. See Miyauchi(1991) for precise discussion.

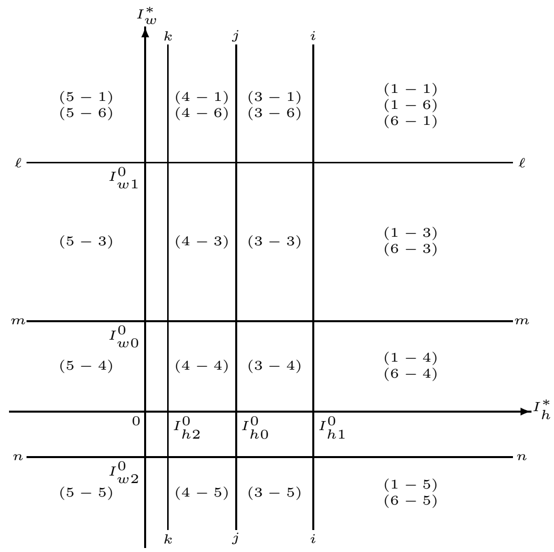


Figure 9: Correspondence between the payoff tables and the region of (I_h^*, I_w^*)

3.8 Joint distribution of TILS and decision on household labor supply

In Subsection 3.7, husband's and wife's TILS, I_h^* and I_w^* respectively, are partitioned into regions corresponding to each of their own payoff matrices. Employing this result, the two dimensional plane of TILS, (I_h^*, I_w^*), can be mapped to household's Nash-equilibria and cooperate equilibria clarified in Section 2.

Combining the I_h^* axis in Figure 7 and the I_w^* axis in right angle in Figure 8 yields two-dimensional coordinate system of (I_h^*, I_w^*), which is shown in Figure 9. Horizontal axis is of I_h^* and vertical axis is of I_w^*. As shown in Figure 9, the (I_h^*, I_w^*) plane is partitioned into regions by orthogonal lines ii, jj, kk passing $I_{h1}^0, I_{h0}^0, I_{h2}^0$ on the I_h^* axis respectively, and also by orthogonal lines $\ell\ell$, mm, nn passing $I_{w1}^0, I_{w0}^0, I_{w2}^0$ on the I_w^* axis. Note that any region on (I_h^*, I_w^*) plane partitioned by lines ii, jj, kk, $\ell\ell$, mm, nn can be assigned one of the payoff tables listed in Table 3. By showing the superscript of payoff tables, Figure 9 assigns the corresponding payoff table to each region on (I_h^*, I_w^*) plane.

Figure 10 assigns the corresponding Nash-equilibria or cooperate equilibria on husband's and wife's labor supply to the region on (I_h^*, I_w^*) plane, based on

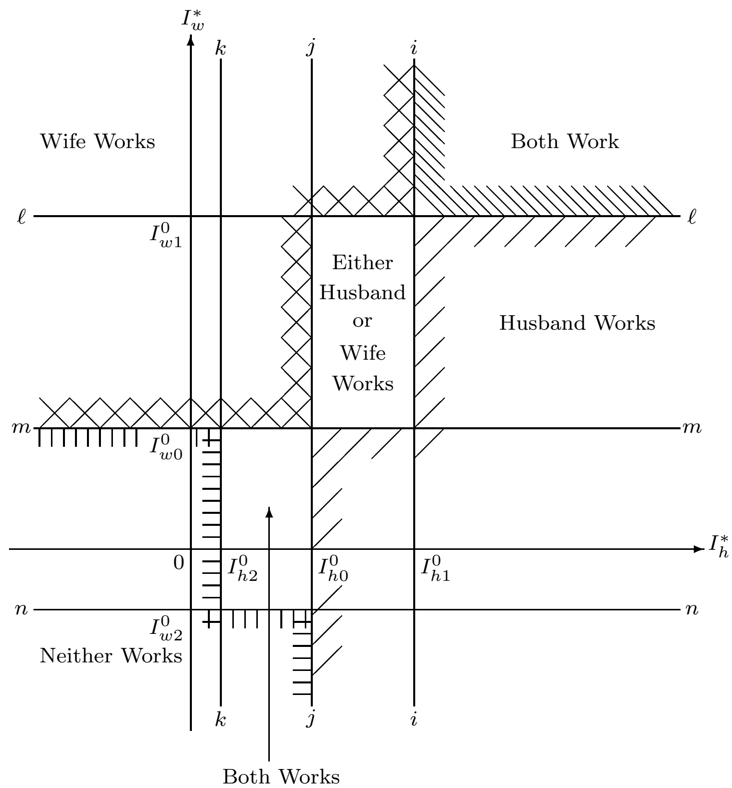


Figure 10: Mapping the region of (I_h^*, I_w^*) to household decisions

Figure 9 and Table 4-1 through 4-3. The region in Figure 10 corresponding to the payoff table Π^{3-3} (i.e., the region indicated by $(3-3)$ in Figure 9) will be of special interest. Note that the households in population where the coordinate of $(I_h^*,\ I_w^*)$ belongs to this region yield non unique Nash-equilibria, so that the theory can only suggest either husband works or wife works.

Integrating the probability density function $g(I_h^*, I_w^*)$ in each region shown in Figure 10 yields probabilities of both work, husband works, wife works, and neither works.

3.9 Utility function in quadratic form

Utility functions of husband is specified in quadratic forms as

$$\omega_h = \frac{1}{2}\gamma_{h1}X^2 + \gamma_{h2}X + \gamma_{h3}X\Lambda_h + \gamma_{h4}\Lambda_h + \frac{1}{2}\gamma_{h5}\Lambda_h^2 \tag{41}$$

where γ_{hj} $(j = 1, \cdots, 5)$ are the parameters of the utility function. γ_{h4} is defined as $\gamma_{h4} \equiv \gamma_{h4}^0 + \overline{\gamma_{h4}} \cdot u_h$, where stochastic variable u_h, is assumed to follow logarithmic normal distribution.

$$\log_e u_h \sim N(m_h, \sigma_h^2) \tag{42}$$

γ_{h4} is the intersection of marginal utility of husband's leisure, which is assumed to be a random coefficient distributed in population. Other parameters, γ_{h1}, γ_{h2}, γ_{h3}, γ_{h4}^0, $\overline{\gamma}_{h4}$, and γ_{h5} are assumed to be common over the population. γ_{h1} is normalized as $\gamma_{h1} \equiv -1$.

Let u_h^* denote the stochastic variable following the standard normal distribution. Standardizing the stochastic variable $\log_e u_h$ in formula (42) yields

$$\frac{\log_e u_h - m_h}{\sigma_h} = u_h^* \rightharpoonup u_h = \exp(m_h) \cdot \exp(\sigma_h \cdot u_h^*)$$

where a constraint, $m_h = -\dfrac{1}{2}\sigma_h^2$, is imposed on m_h, so that $E(u_h) = 1$ follows. Thus a formula

$$u_h = \exp(-\frac{1}{2}\sigma_h^2) \cdot \exp(\sigma_h \cdot u_h^*) \tag{43}$$

is obtained.

Given the formula (41) and (43), the formula of husband's TILS is

$$I_h^* = H_0^h + H_2^h \cdot \exp(\sigma_h \cdot u_h^*) \tag{44}$$

where

$$\begin{aligned} H_0^h &\equiv \frac{\gamma_{h4}^0 - \gamma_{h2}w_h - \gamma_{h3}w_h(T - \overline{h}_h) + \gamma_{h5}(T - \frac{1}{2}\overline{h}_h) - \frac{1}{2}\gamma_{h1}w_h^2\overline{h}_h}{\gamma_{h1}w_h - \gamma_{h3}} \\ H_2^h &\equiv \frac{\overline{\gamma}_{h4}}{\gamma_{h1}w_h - \gamma_{h3}} \cdot \exp(-\frac{1}{2}\sigma_h^2) \end{aligned}$$

Similarly, utility functions of wife is specified as quadratic forms as

$$\omega_w = \frac{1}{2}\gamma_{w1}X^2 + \gamma_{w2}X + \gamma_{w3}X\Lambda_w + \gamma_{w4}\Lambda_w + \frac{1}{2}\gamma_{w5}\Lambda_w^2 \tag{45}$$

where γ_{wj} $(j = 1, \cdots, 5)$ are parameters of the utility function. γ_{w4} is defined as $\gamma_{w4} \equiv \gamma_{w4}^0 + \overline{\gamma}_{w4} \cdot u_w$, where stochastic variable u_w, is assumed to follow logarithmic normal distribution.

$$\log_e u_w \sim N(m_w, \sigma_w^2) \tag{46}$$

where a constraint, $m_w = -\frac{1}{2}\sigma_w^2$, is imposed on m_w, so that $E(u_w) = 1$ follows. γ_{w4} is the intersection of marginal utility of wife's leisure, which is assumed to be a random coefficient distributed in population. Other parameters, γ_{w1}, γ_{w2}, γ_{w3}, γ_{w4}^0, $\overline{\gamma}_{w4}$, and γ_{w5} are assumed to be common over the population. γ_{w1} is normalized as $\gamma_{w1} \equiv -1$.

Given the formula (45) and (46), the formula of wife's TILS

$$I_w^* = H_0^w + H_2^w \cdot \exp(\sigma_w \cdot u_w^*) \tag{47}$$

is obtained where

$$\begin{aligned} H_0^w &\equiv \frac{\gamma_{w4}^0 - \gamma_{w2}w_w - \gamma_{w3}w_w(T - \overline{h}_w) + \gamma_{w5}(T - \frac{1}{2}\overline{h}_w) - \frac{1}{2}\gamma_{w1}w_w^2\overline{h}_w}{\gamma_{w1}w_w - \gamma_{w3}} \\ H_2^w &\equiv \frac{\overline{\gamma}_{w4}}{\gamma_{w1}w_w - \gamma_{w3}} \cdot \exp(-\frac{1}{2}\sigma_w^2) \end{aligned}$$

Let ρ denote the correlation coefficient of two dimensional normal distribution of u_h^* and u_w^*.

3.10 A priori restrictions on structural parameters

A priori restrictions imposed on structural parameters are as follows [10] .

a priori restrictions on parameters of utility function:

1. $\frac{\partial h_h^x}{\partial I_h^0} < 0$, $\quad \frac{\partial h_w^x}{\partial I_w^0} < 0$
2. $\frac{\partial \omega_h}{\partial X} > 0$, $\quad \frac{\partial \omega_w}{\partial X} > 0$
3. $\frac{\partial \omega_h}{\partial \Lambda_h} > 0$, $\quad \frac{\partial \omega_w}{\partial \Lambda_w} > 0$
4. Indifference curves are convex to the origin.
5. $H_0^h > I_h^{0\,max}$ and $H_0^w > I_w^{0\,max}$ must hold, where $I_h^{0\,max}$ and $I_w^{0\,max}$ are the maximum values of observed guaranteed income of husband and wife respectively.
6. $H_2^h < 0$, $\quad H_2^w < 0$

a priori restrictions on distribution parameters of u_h and u_w:

a $\sigma_h > 0$

b $\sigma_w > 0$

c $|\rho| < 1$

[10] See Miyauchi(1991) for detailed discussions on a priori restrictions on structural parameters.

4 Estimation of structural parameters

Applying Japanese data to the model discussed in Section 3, the structural parameters were estimated. The estimation was performed by minimizing the χ^2 for observed cases of household labor supply and these simulated values. Optimal parameter set was searched within a parameter space that is consistent with a priori restrictions discussed in Subsection 3.10.

Observations on household labor supply probability are obtained by "Household Labor Status Survey" for the years of 1971, 1974, 1977, 1979, and 1982. Observations on wage rate and assigned hours of work are obtained by "Wage Census" for the corresponding years. Because appropriate observations on unearned income, I_A, was not available, the value of I_A was assumed to be zero for the first attempt. These observations are stratified by husband's and wife's age classes.

The estimates of the structural parameters are as follows.

$$
\begin{array}{llll}
\gamma_{h2} & = 6540.59 & \gamma_{w2} & = 1150.25 \\
\gamma_{h3} & = 520.02 & \gamma_{w3} & = -21.36 \\
\overline{\gamma}_{h4} & = 816058.3 & \overline{\gamma}_{w4} & = 269220.5 \\
\gamma_{h5} & = -32724.1 & \gamma_{w5} & = -962.0 \\
\gamma_{h4}^{0} & = 79727.4 & \gamma_{w4}^{0} & = 26956.1 \\
\sigma_h & = 2.534 & \sigma_w & = 1.0474 \\
\rho & = 0.602 & &
\end{array}
$$

A comparison between observed and simulated labor supply probabilities is presented graphically in Figure 11. Although the observed and the simulated probabilities are obtained for each stratum of husband's and wife's age classes, let the graphical comparison be presented on the basis of aggregating these strata for the sake of briefness.

5 Concluding remarks

1. The magnitude in probabilities of husband's and wife's labor supply is significantly different by these patterns of household decision, i.e., both work, husband works, wife works, and neither works. The model presented in this paper simulates well the difference in the magnitude of these probabilities.

2. The model simulates well the time trend observed in these probabilities, although systematic biases in simulated probabilities are persistent in each year.

3. The calculated probability in the region corresponding to the payoff table, Π_h^{3-3}, where the model fails to give a unique equilibria on household labor supply, is less than 0.001.

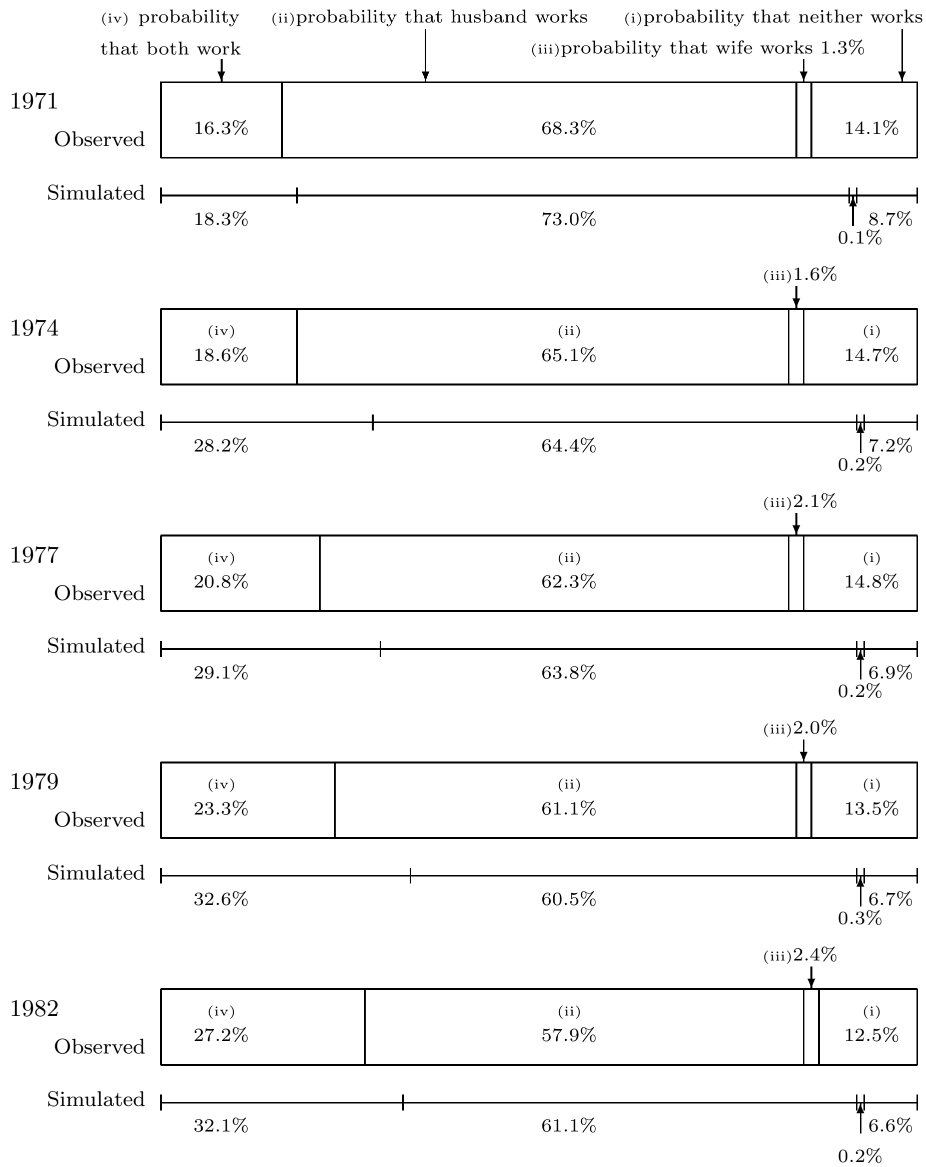


Figure 11: Observed and simulated probabilities of household decision on labor supply

4. The model gives systematic biases in simulated probabilities in each year. The model should be modified so that the systematic biases can be resolved.

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