ECONOMIC POLICY AND GENERAL INTERDEPENDENCE

A Quantitative Theory of Price and Empirical Model Building

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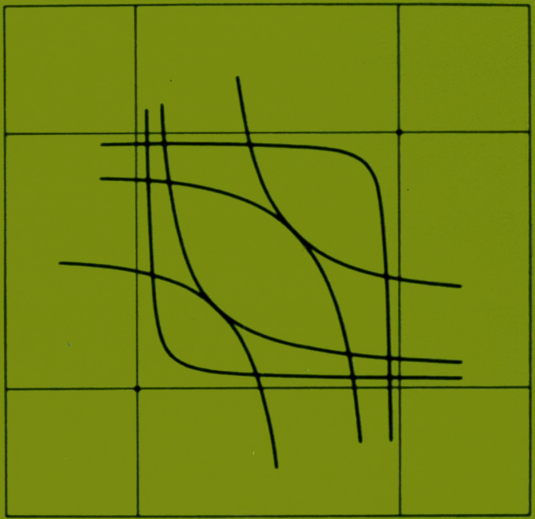

KEIO ECONOMIC OBSERVATORY
KEIO University

Keio Economic Observatory
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Monograph No. 1

Our institute is named the 'Keio Economic Observatory', even though an 'observatory' usually means an astronomical or meterological institute for the observation of natural phenomenon. We call our institute an 'observatory' because we wish to treat economics as an empirical science and thereby intend to analyze economic phenomena objectively, being completely detached from any ideologies, by making use of economic theory as an equivalent to theories of other physical empirical sciences. The K.E.O. monograph series, of which this book is one, is designed to publicly demonstrate this spirit. We hope that this book presents a tangible example of economics as an empirical science.

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Dire libre concurrence n'est pas du tout comme on voit, dire absence de toute intervention de l'Etat. Et, d'abord, cette intervention est nécessaire pour établir et maintenir la libre concurrence là où elle est possible.[1]

Laisser faire ne doit pas signifier ne rien faire, mais laisser agir la libre concurrence. Là donc où la libre concurrence ne peut agir, il y a lieu, pour l'Etat, à intervenir afin de la suppléer ; et là où la libre concurrence peut agir, il y a lieu, pour l'Etat, à intervenir afin de l'organiser, d'assurer les conditions et le milieu de son foncionnement. Or, cette sorte d'intervention donne lieu, en matière de capitalisation, à plusieurs observations importantes.[2]

— *Léon Walras* —

1) "Esquèsse Dúne Doctrine, Economique Et Sociale", 1898.
2) "La Bourse—La Spéculation Et L'Agiotage", *Bibliothèque universelle*, mars et avril 1880. Walras (1874).

Preface

After a score of relatively stable years since the World War II, economies in the world recently seem to have entered a new age of instability, disorder and conflict. One symptom of the new age which has grown conspicuous is the phenomenon of "stagflation" which has prevailed among an increasingly large number of countries since around the beginning of the 1970's. Although it is obvious that this is largely a consequence of accumulated effects of inadequate or unreasonable policy interventions made during the preceding decades, this phenomenon, which appears to be irreconcilable with the existing theories of economics, has given rise to serious doubt concerning not only the workability of the conventional "Keynesian free market system" but also to contemporary economic theory itself.

Disenchanted with the viability of the Post-Keynesian mixed economic system, some economists advocate hastily the need for direct wage-price controls, while others even recommend classical free capitalism. This chaotic situation in economics is aggravated further by the increasingly intensified confrontations and potential conflicts between advanced and developing nations as symbolically represented by serious issues of petroleum. In the face of such conflicts, the question at stake is whether the contemporary economics is capable of providing meaningful suggestions in this arena in attaining a more equitable distribution of resources without sacrificing the efficiency of the world's economic system.

A sufficient amount of criticism and dissatisfaction has been expressed so far concerning the usefulness of the science of economics. A typical example is the recent upsurge of "monetarism" equipped with new theoretical and methodological spears to attack Keynesian economics and its policy implications. However, did these critics really explore the full scope and capacity of economic analysis before they publicized their disappointment with it? To find our own answer to this question, we attempt in this volume to reconstruct conventional theoretical framework of economic analysis encompassing a broad conceptual world which includes domains that have not been treated properly by contemporary economics. The primary feature of this book may be found in our basic position to pursue this goal by means of careful observation of data and sound quantitative empirical analysis rather than mere logical deductions.

This book is an outcome of long and patient efforts concertrated on empirical analysis of economic phenomena. The original motivation for this book can be traced back to the early 1960's when one of the authors, Tsujimura, was inspired and encouraged during his stay at Harvard University by witnessing the sincere dedication of Professors Simon Kuznets and Wassily W. Leontief to empiricism in economic science.

This book is in effect a consolidation of research results accumulated during the last ten to fifteen years by members of Keio Economic Observatory (KEO) of Keio University. Much of the content included in this volume has been published in Japanese in K. Tsujimura, *Keizai Seisaku Ron* (A Treatise on Economic Policy), Tokyo: Chikuma-Shobo, 1977 and K. Tsujimura and M. Kuroda, *Nihon Keizai no Ippan Kinko Bunseki* (A General Equilibrium Analysis of the Japanese Economy), Tokyo: Chikuma-Shobo, 1974. While this book heavily draws upon these two publications in the sense that it translates results reported in them, it integrates their conceptual and empirical contents in a new systematic form.

The Japanese Ministry of Education kindly gave us the Scientific Research Grant for translation and reviewing. Keio University granted us the Fukuzawa Memorial Grant for Publication. The International House of Japan provided us with excellent facilities and accommodations. We are grateful to these organizations for their financial as well as non-monetary assistance.

Finally, we would like to acknowledge considerate and careful help of Mr. Nakaba Kawaguchi of Kogakusha Co., Ltd. and Mr. Yasuo Imai of T&T Co., Ltd. We are also greatly indebted to innumerable people for their valuable assistances at various stages of making this book; translation, typing, reviewing and proof-reading. Although we do not have space to list their names, we would like to express our sincere gratitude to all this help without which this volume could not have been published in this form.

We and KEO would like to express our sincere gratitude to Mr. Jiro Enjoji, former President of Nihon-Keizai Shinbun, and the late Mr. Hideo Shinojima, former President of Mitsubishi-Kasei Corp., for their generous financial support of our academic research.

January 1981,
The Authors

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Part I The Theory of Market Competition and Policy Interventions

Introduction

"Disequilibrium" or "Out of Equilibrium"

During the past decade and a half, disequilibrium theories have been developed as a key to integrate price theory and Keynes' theory. Disequilibrium theories, side by side with theory of rational expectations, had given considerable influences not only on controversies concerning the effectiveness of Keynesian policies but also upon econometric model building. Since a number of intriguing issues directly pertaining to the main theme of this volume have been raised in the process of development of disequilibrium theories, a brief critical review of these issues would probably give a helpful introduction to readers to understand objectives of the book.

In his oft-quoted book Leijonhufvud raised two problems:[1] (1) the stagnation of Keynesian economics at the theoretical level, (2) the incompatibility of micro economics and macro economics. Insisting that Keynes had in mind neo Walrasian disequilibrium theory of ("involuntary") unemployment, he explained that economics of Keynes sought to deal with problems that would arise in general equilibrium systems only if "the auctioneer" was removed.

Prior to the publication of Leijonhufvud's book, Clower took up similar problems in his extremely interesting and stimulating article and developed his "dual decision hypothesis."[2] In a slightly different way from Leijonhufvud, Clower drew readers' attention to the distinction between "planned" and "realized" magnitudes in the theory of household behavior which would possibly lead to states of transactor "disequilibrium." From this point of view he argued that not every household can buy and sell just what it pleases when supply exceeds demand in the economy, and so the other side of involuntary unemployment would have to be involuntary under-consumption. Clower's "dual decision hypothesis" has considerable intuitive appeal to readers. He argued that differences between realized and planned purchases and sales of individual households "might properly be supposed to occur more or less at random." Then he introduced an inequality between realized current income and notional current income and derived modified budget constraint and "constrained" demand functions. Thus, he explained "disequilibrium" and in effect offered an integrated account of price theory and income analysis.

Although the problems handled by Leijonhufvud and Clower are not exactly the same, both of them are quite important. No one would disagree with the view that an instantaneous adjustment under perfect information, as once assumed by Walras, is hardly realizeable in the real market place. However, the question as to whether the time lags which accompany the diffusion of information in the real world are more than negligible can not be answered a priori. This question needs to be judged empirically. In this sense, the concept of disequilibrium put forth by Leijonhufvud may be regarded as being quantitative.

In contrast, Clower's exposition of disequilibrium which utilizes the distinction between "plans" and "realizations" certainly contains quantitative elements especially when we focus on the magnitude of the difference between these two concepts. However, Clower's construct of disequilibrium may be characterized as being primarily qualitative in the sense that theoretical distinction between "plans" and "realizations" is emphasized. In the case where elements of disequilibrium exist as described by Clower, a disequilibrium may take place even though the process of market adjustment proceeds instantaneously. In this sense, the elements of disequilibrium as pointed out by Clower are more fundamental from the viewpoint of price theory, and they deserve careful examination before one proceeds to empirical estimation of time lags. However, Clower's concept of "planned consumption" is not sufficiently clear. Although his example of "champagne appetites" is introspectively appealing, it is not quite articulate enough for a researcher to develop an objective experimental design to conduct empirical analysis. Arrow and Hahn have dealt more rigorously with a problem similar to the one pointed out by Clower.

In their elegant mathematical exposition of general equilibrium theory Arrow and Hahn took up a problem somewhat similar to Clower's.[3] They argued that in a monetary economy intentions to buy become relevant signals only when they are accompanied by willingness and ability to pay in money. Further, they pointed out that the distinction between desired purchase and realized one arises also in a barter economy; for instance, a household may have to acquire some goods before it can exchange them for the desired ones.

In the final chapter of their *General Competitive Analysis*, Arrow and Hahn state, "From what we have learned already we know that we must be able to establish the appropriate continuity properties of the behavioral functions or correspondences that an equilibrium exists (p. 355). Clearly, the actual bankruptcy procedure is at least a matter of law, but it seems plain that the history of the economy may make it impossible to guarantee the continuity properties of the various functions and correspondences and this is bad for existence proofs (p. 354)."

In Chapter 5 of their book they develop a theory of "compensated equilibrium with bankruptcy" with redistribution of income. In this connection they argued, "We can now interpret it as the minimum level that society insists on providing for every household, even for those that cannot achieve this level in the marketplace. Of course, the budgetary deficits of the bankrupts must be balanced by budgetary surpluses of others; they may be interpreted to be paid to maintain the minimum guaranteed utility level (p. 120)."

Now we can see that the problems put forth by Clower and Arrow-Hahn resemble with each other quite well when we replace Clower's concept of "planned" or "desired" consumption by "social minimum" formulated by Arrow and Hahn, and also Clower's concept of "gap between realized income and notional income" by Arrow-Hahn's "bankruptcy." The concept of "the minimum level" coined by Arrow-Hahn is somewhat more neat and precise than Clower's "planned consumption." This is because the magnitude of "social minimum" may be interpreted to be common across different households while "planned consumption," as it is explained by Clower, can differ at random from household to household.

When Clower's case of "disequilibrium" and Arrow-Hahn's case of "out of equilibrium" are re-examined in Keynes' theory, these issues both appear to be, to the eyes of the present authors, old issues that were already considered in some depth by Adam Smith. As well known, Smith put forth the concept of "minimum subsistence level." In contrast to Arrow-Hahn's "social minimum," this concept may be characterized as "natural minimum." When explaining the meaning of bankruptcy, Arrow-Hahn posit a case in which there exist debtor and creditor households. It seems to us that the problem of Arrow-Hahn will become identical to that of Smith if we replace "Nature" for a "creditor household." The fact that a human being cannot survive unless he consumes each period the minimum amount of various necessities may be interpreted to mean that he is destined to pay back to "Nature" more than a certain amount of debt each period. Smith observed, "Wages depend on contract between masters and workmen. The masters have the advantage. ...But masters cannot reduce wages below a certain rate, namely, subsistence for a man and something over for a family... Wages may be considerably above this rate, when there is an increasing demand for labourers (pp. 67-68)."[4] This means that wages tend to be pegged at the subsistence level if there is no excess demand for labor in the market.

When we regard "necessities of life" as the same thing as "debt from Nature," it is possible to interpret the situation in which household's earnings fall below subsistence as being equivalent to the situation of "bankruptcy" or "out of equilibrium" described by Arrow-Hahn. Incidentally, when Smith qualifies "subsistene wage" he clearly admits that "the masters have the advantage." This implies that Smith did not acknowledge such a condition in the market as being "normal" performance of market competition in the sense that bargaining positions between masters and workmen are equal.

When we interpret the possibility of getting "out of equilibrium" triggered by an Arrow-Hahn type "bankruptcy" to mean the same thing as having a "debt from Nature" which essentially implies the existence of "necessities of life," we are reminded of the fact that Jevons once emphasized that the marginal utility of necessities such as grain increases infinitely large, and so does its price as well, as its amount available for consumption approaches some minimum critical level.[5] Marshall, too, has called attention to the peculiar property of labor market by pointing to the fact that bargaining positions between buyers and sellers in the labor market tend to be lopsided on the ground that the marginal utility of wage income for poor workers can be extremely high.[6]

In this respect, we would like to call readers' attention to the fact that in his *Theory of Value* (1959), a celebrated monumental work of neo-Walrasian theory, Debreu stipulated "the free disposal assumption" as well as "the assumption of non-satiation" to ensure "the convexity of consumption set" which are crucial for proofs of fundamental theorems of economic equilibrium.[7] By "free disposal" he means the exclusion of possibilities that a consumer fails to secure adequate amounts of necessities of life which are necessary for his survival. Thus, it seems clear that the possibility of Jevons-Marshall type "infinite marginal utility" in real economic life may well lead to states of "out of equilibrium."

Adam Smith maintained that workmen in the labor market are at a disadvantage since they have to earn a living every day. In Jevons' or Marshall's terms, the very high marginal utility of necessities drives workmen to work every day even though wages they earn are less than "desired" levels. When the marginal utility grows infinitely large, the existence of an equilibrium may be endangered by the discontinuity as pointed out by neo-Walrasians. When we introduce this concept of infinite marginal utility, we will find that Smith's "subsistence income" may be defined more clearly as a price theoretic concept than as Clower's "notional income."

In the context of affluent life in contemporary advanced nations, Smith's subsistence wage or Jevons' infinitely large marginal utility of food may be relevant only in their history of long ago. However, in underdeveloped nations, the problems pointed out by Smith or Jevons are real even today. Moreover, we suspect that the oil crisis in the 1970s has made people in advanced nations keenly aware of the concept of necessities of life. Indeed, the dangers are getting increasingly apparent that markets cease to operate properly when people merely "expect" a shortage of supplies to take place even before it becomes real.

Economic Theory and Economic Policy

Theories of disequilibrium, which intend to analyze the possibility of non-existence of equilibrium in neo-Walrasian terms, whether depending on Clower's or Leijonhufvud's conceptualization, have been developed with reference to Keynes' concpet of "involuntary unemployment." However, we have to keep in mind that the theoretical possibility of disequilibrium discovered through modifications of neo-Walrasian abstract price theory will not necessarily explain by itself the existence of unemployment in actual economies. Attempts by some econometricians to build quantitative models by which to describe actual economies with unemployment by making use of disequilibrium theories appear to us only hasty short circuits. This is because advanced economies after the world war II or earlier ones observed by Lord Keynes had incorporated within themselves effects of a variety of institutional and policy interventions and consequently these economies were under quite different conditions from what would be supposed by neo-Walrasian pure theory.

It is probably Keynes himself who should be blamed for the fact that these short circuits take place easily. When criticizing Pigou's view that trade unions' nonacceptance of money wage cuts is the cause of unemployment, Keynes pointed to the fact that in the United States in 1932 unemployment increased while money wages declined. However, Keynes did not really explain why money wages were rigid in Britain and not in the United States. Hicks provides a clear explanation on this point in his *The Theory of Wages*. Hicks discusses the rigidity of money wages in conjunction with the system of unemployment insurance as follows:

> Further, throughout the post-war period, all Governments have undoubtedly been strengthening the hands of the Unions, by the system of Unemployment Insurance. If it had not been for Unemployment Insurance, there can be little doubt that many of the national agreements would long ago have broken down, or been rendered much more flexible. It is not so much that the Unions, if they had had to look after their own unemployed, would have been financially weakened, and thus less able to resist wage-cuts, although this may be of some importance. The significance lies rather in that clause, which has run through all the multitude of Insurance Acts, decreeing that employment "at a rate of wages lower, or on conditions less favourable, than those generally

> observed in that district by agreement betwen associations of employers and employees" shall not be regarded as suitable employment, refusal of which disqualifies for benefit. If it had not been for this clause, it is impossible to believe that it would have been possible to enforce agreements in the face of large and persistent percentages of unemployed in regular trades.[8]

In the stream of postwar economics, this remark of Hicks has rarely been refered to. Two reasons for this are conceivable. One is the overwhelming popularity of Keynes' *General Theory* in postwar economics. The other, and perhaps more important, is the fact that to question the reasons for existence of unions and unemployment insurance systems themselves would not have gained social sympathy at the time when the major objective of societies was perceived to be what may be called "welfare state" by which people pursue such goals as "freedom from want" or "freedom from idleness" as stated in the *"Atlantic Charter"* and *the Beveridge Report*. However, it would be timely and worthwhile to re-examine implications of the remark of Hicks today.

If Hicks' explanation of the rigidity of money wages is correct—and we think it is correct—then the validity of Keynesian policies should be qualified in connection with this point. He describes the essential nature of unions as follows:

> About the origin of such combination it is unnecessary to say much; where it is possible for men to snatch gains, real or apparent, permanent or temporary, from the abandonment of separate individual action, it would be surprising if they did not sometimes attempt it. Monopolistic combination is common enough in all parts of the economic system; very much the same motives which drive business men to form rings and cartels drive their employees to form unions. The one, as much as the other, is a natural product of a gregarious animal.[9]

He acknowledges that an attempt of monopolistic combination of workers is a natural product. However, one would be curious to know why a monopolistic combination of workers is permitted while that of business men has been regarded since the time of Adam Smith as the one which should be eliminated. This question, to our knowledge, has never been tackled seriously. Even a book which presents comprehensive discussions of eight prominent scholars on various topics in this area, Wright, D.M. ed. *The Impact of the Unions*, does not provide an answer to this question.[10] Even Milton Friedman, one of the eight contributors, purports simply that trade unions do not have as strong an influence upon the economy as is usually considered.

In order to consider this point properly, therefore, we need to go back again to the writing of Adam Smith. Although Smith's *Wealth of Nations* is undoubtedly a book which accuses monopoly, Smith takes a generous attitude toward workmen's combinations as we have seen earlier. This is probably because he recognized that there exist situations in which bargaining positions between masters and workmen are not equal, which can be seen in his oft-appearing expressions such as "the masters have the advantage." He recognized well that while masters' combinations are offensive, workers' combinations are defensive. From the standpoint of Smith, an unemployment insurance system would be well accepted as a device to counteract workmen's disadvantage unless it does yield undesirable effects as in the case of the Poor Law at the time.

However, we should note that "workmen's disadvantage" as conceived by Smith is the one which would disappear when demand for labor exceeds supply—as in the case of the North American labor market at the time of Smith—and consequently the labor market works as a normal competitive market where bargaining positions between masters and workmen become equal. When normal competitive conditions are restored, therefore, monopolistic combination of unions would not be allowed. We shall define, in a later chapter, such concepts as "polypoly" and "polyopsony" as disturbances of competition which are distinguished from familier concepts as monopoly and monopsony. To rephrase using this new terminology, we may say that the reason why Smith recognized the necessity of workmen's combination was because it can ease the conditions of "out of equilibrium" in the labor market where demand does not exceed supply and as a result masters enjoy positions of "polyopsony" and workmen suffer from positions of "negative-polypoly."

Viewed in this way, the possibility of wage-cost push inflations which took place in some of the advanced industrialized countries in the postwar period would not have been anticipated in the conceptualization of Smith. Hicks had reasons for expressing in his *The Theory of Wages*, a negative view of unions and unemployment insurance systems, since he apparently meant to point out the excessive labor protection policies in British labor market in those years. Yet, his argument would have been much more persuasive had he accused an excessive dose of policy interventions after acknowledging in general theoretical terms the necessity of labor protection policies within a broader perspective as did Smith. For example, the Japanese labor market in those days was quite different from the British labor market as observed by Hicks. It was perhaps closer to the British labor market in the age of Smith. This implies that government interventions were insufficient. What is important, therefore, is not the question of whether policy interventions are necessary or not but rather is the question to know exactly how much of what kind of policy interentions are necessary to ensure the existence of a normal competitive equlibrium.

Keynes and Keynesians alike insisted upon the necessity of aggregate demand controls without paying attention to the warning of Hicks. However, in the actual world, effects of Keynesian policies obviously relate to already existing institutional systems which regulate markets. As Keynes contended, an increase in aggregate demand would sooner or later give rise to excess demand in the labor market which in turn would prepare conditions in which a normal competitive market operates as envisioned by Smith. However, if labor protective arrangements were already existing, then excess demand would emerge earlier than otherwise. This implies that if the effect of an increase in aggregate demand and the effect of labor protective regulations reinforce with each other too strongly, employers rather than workers would suffer from malfunctioning of labor market in the sense, for example, of having to pay excessively high wages.

Neo-Walrasian economists, who are interested only in mathematical rigor, seldom talk about the validity of policy interventions. In contrast, Leon Walras himself emphasized that what he meant by "laissez faire," to prepare adequate conditions in order to make the actual market operate in such a way as described by his pure theoretical model. Comparing the concepts of "laissez faire" of Smith, Ricardo and Walras in a broad context, we may infer that the "laissez faire" of Smith was perhaps closer to that of Walras than to that of Ricardo.

As is well known, Ricardo's writings are interpreted to mean that everything should be put to the hands of the competitive market without relying on policy interventions. For example, he aptly criticizes the Poor Law by stating that:

> These then are the laws by which wages are regulated, and by which the happiness of far the greatest part of every community is governed. Like all other contracts, wages should be left to the fair and free competition of the market, and should never be controlled by the interference of the legislature.[11]

This kind of statement, however, has been interpreted by his followers as a declaration of overall denial of policy interventions. A review of Smith's observations on labor market problems will reveal that his view is quite different from that of Ricardo. Smith pointed out the irrationality of the government which permits masters' combinations on the one hand while oppresses workmen's combinations on the other. He criticized the government because he knew that the bargaining positions of masters and workmen were unequal. Ignoring this important element, Ricardo, in contrast, declared that the labor market, too, will take care of itself. The complete denial of a desirable allocative function of the market by Marx, who applied the idea of Ricardian labor value theory in his own theorizing, may well be regarded as the other side of the coin of Recardo's complete reliance on the competitive market mechanism, except for sociological ingredients in Marxist theory.

The example of Arrow-Hahn's "compensated equilibrium" as quoted earlier suggests that there is a possibility by which an equilibrium may still be attained by some proper policy interventions even in cases in which no equilibrium may be attained otherwise. The Arrow-Hahn's concept of "compensated equilibrium," in this sense, will remind one of the concept of "laissez faire" of Leon Walras.

It seems that most theoreticians who develop and advocate disequilibrium theories do not distinguish explicitly between a hypothetical situation of general equilibrium in the absence of policy interventions and the actual situation of an economy. It must be born in mind that the actual performance of economies is governed by effects of a variety of policy and institutional interventions such as aggregate demand control policies, collective bargaining by unions, unemployment insurance systems, price regulations of agricultural products and petroleum etc.

Theory and Empirical Model Building

Discussions presented so far suggests that it is useful and meaningful to develop a general theoretical framework of the market, taking into account fully Adam Smith's image of a market mechanism which allows for the possibility of "out of equilibrium". And it is to this objective that Part I of this volume is devoted. In Part I, we will first develop a theoretical framework of what we call 'Generalized Edgeworth's Box Diagram" or "***GEBD*** in short. Making use of "***GEBD***," we will then examine in price theoretic terms economic rationales of not only familiar Keynesian demand control policies but also various institutional and policy interventions of the government which have been actually introduced ever since the 19th century.

Our theoretical examinations will suggest that there are certain cases in which governmental interventions are necessary to assure an equilibrium in a "fair" competitive market which has been conceptualized since the time of Adam Smith. We may interpret an equilibrium in the actual economy which is attained with adequate government interventions as being analogous to what Arrow and Hahn call "compensated equilibrium." However, various governmental interventions other than Keynesian demand control policies have been introduced into actual economies either as a result of pursuit of a "welfare state" or as a result of political compromise between various interest groups, and their economic rationales have rarely been clarified in the framework of price theory. It is not suprising, therefore, that some of the policies and institutional regulations, which are necessary conditions to attain a "fair competitive market," are made either excessively or deficiently when they are actually introduced into the economy, and that some of the policies are even undesirable when judged from the viewpoint of price theory.

In Part II of this book, we present a proto-type of econometric model building by which to evaluate effects of various institutional regulations and policies which are built into actual economies.

In recent years, various appreciable achievements have been made in the field of econometric model building by incorporating new theoretical elements such as "disequilibrium theories" and "theories of rational expectations." For example, in a comprehensive model he developed, Ray C. Fair describes carefully decisions of behavioral units on the basis of sound microeconomic foundations, incorporates elements of disequilibrium theories taken out of theories of Patinkin, Clower, Leijonhufvud and subsequent developments, attempts to endogenize the determination of prices and wages, and sheds special lights upon the functions of the monetary market.[12]

In contrast, our basic objective is to develop a model by which we can evaluate effects and even examine reasons for existence of various policies and regulations; such as the familier Keynesian demand control policies, labor standard laws, unemployment insurance systems, price support of agricultural products, various industrial policies and so on. From the viewpoint of our objective such as this, we need to have a measurable model which is constructed more closely along the lines of Neo-Classical general equilibrium theory. This is because the model needs to be capable of following closely the cross-market effects of disequilibrium by means of stated described general interdependence of different sectors of an economy described in the form of a disaggregated multi-sectoral model, and also capable of describing quantitatively the maximization behavior of households and firms in terms of micro economic theory.

In Fair's model, it is postulated that prices and wages are determined by firms according to a kind of full cost principle. In contrast, our model adopts the approach by which labor demand functions, investment functions and short-run commodity supply functions are derived explicitly from production functions of firms, and system of demand functions for major consumption items are derived consistently from explicitly specified preference functions of households. The model then describes the mechanism of simultaneous determination of quantities and prices of various commodities and factor inputs through their demand and supply relationships by making use of a Leontief matrix of intermediate inputs. For each commodity market, demand and supply schedules are described in a Marshallian way for each period. The model describes in detail the way in which quantities and prices for all commodity sectors of an economy are determined through interactions of these sectoral demand and supply schedules with different shapes and positions. Having described the interdependent structure of an economy in this way, the model is now capable of following closely the effects of various policies upon relative prices of different commodities and sectors as well as upon income distribution.

The other major objective of our model is to specify the structure of the model in such a flexible way that the number of sectors can be increased with improved data availability, and also new theoretical components can be added in accordance with modifications in analytical purposes and methodology. For example, candidates for such new theoretical components which may be incorporated in the more developed version of our model are labor supply functions as well as saving functions of households and production and labor demand functions with heterogeneous labor inputs distinguished at least in terms of sex.

We feel that many of the econometric models developed elsewhere have lost the theoretical clarity found in basic textbooks largely due to the fact that they contain too many proxies which are introduced inevitably in the complex process of repeated theoretical deductions between the stage of basic theoretical construction and the stage of empirical application. It is our contention that for us to be able to evaluate and determine the validity of policies making use of a measured model, the model has to have a theoretical clarity as one may find in basic economic textbooks.

Notes to Introduction

1) Leijonhufvud (1966).
2) Clower (1965).
3) Arrow and Hahn (1971).
4) Smith (1776).
5) Jevons (1871).
6) Marshall (1920).
7) Debreu (1959).
8) Hicks (1932), p.177.
9) Hicks, *ibid.*, p.137.
10) Wright (1951).
11) Ricard (1821), Chapter V, p.82.
12) Fair (1974).

Chapter 1
The Limit to the Market Mechanism and Inevitability of Governmental Interventions

1.1 Adam Smith's Concept of Competition

In modern economics it is conventionally believed that a free competitive market is always workable except for the case in which a monopolistic element exists. This belief is said to be based on the Neo-Classical theory of general equilibrium. It must be noted, however, that Leon Walras himself, the original founder of the general equilibrium theory, warned that his theory was not always valid in explaining the performance of an actual market, especially the labor market.[1]

He wrote that the economist should not be duped by those abstractions developed in his own pure theory, and warned that the tendency of employers to extend indefinitely the hours of the working day must be arrested by governmental intervention. Alfred Marshall, another great Neo-Classist, also pointed out that there exists an important exception to the theory of market equilibrium which is found in the labor market.[2]

The main stream of modern economics, however, has paid little attention to these warnings and has been quite optimistic about the workability of a free market system. Some critiques of this stream of thought express their skepticism about the workability of the free market system on the ground that the present situation of market competition in highly industrialized countries has already become highly oligopolistic where the condition of atomistic competition is scarcely fulfilled.[3]

It is curious, however, that these mutually incompatible judgements are made in spite of the fact that both of them are based primarily on the same Neo-Classical theory of competitive market.[4] This apparent contradiction leads us to suspect that their basic theory, namely the contemporary Neo-Classical price theory, contains some serious defects. In particular we wonder whether the theory is merely a theoretical artifact instead of one of empirical science formulated on the basis of empirical evidence. We, then, are led further to ask whether the contemporary price theory is rather a degenerated version of the insightful classical theories instead of a more developed outcome if evaluated from the view point of empirical science. If this is indeed the case, then it would not be surprising that the contemporary theory misleads people in analyzing and explaining the complex economic phenomena of the real world today.

Motivated by these questions, let us first go back to the perspective of Adam Smith, the acknowledged founder of modern economics, and see how insightful his original thoughts about the market mechanism were when used as building blocks of a more viable empirical theory of the market which we are going to develop.[5]

In Chapter VII of *The Wealth of Nations* Smith analyzed the determination of prices in three cases: the case of excess demand, the case of excess supply, and the case where supply is equal to demand.[6] He suggested as follows:

1) In the case of short supply, competition will immediately begin among buyers, and the market price will rise as the greatness of deficiency animates the eagerness of competition. Among competitors the same deficiency will induce more eager competition when the acquisition of the commodity is more important to them. Hence the necessities of life will have exorbitant prices during a famine or a blockade of a town.
2) In the case of excess supply the market price will fall as competition among sellers becomes more intense with an increase in excess supply. The excess supply in perishable goods will make the competition much more intense than in the case of durable commodities.
3) In the case where supply equals demand, the market and natural prices coincide and suppliers are obliged to accept this price being forced by the competition among them.

The concept of competition in the third case seems to correspond to that of the "price taker" in Neo-Classical theory and leads to the law of indifference of W. S. Jevons.[7] It is doubtful, however, whether the Neo-Classical theory has fully explained cases (1) and (2). In cases (1) and (2), Smith mentioned the relationship between the importance of acquiring or getting rid of the commodity as well as the relationship between the greatness of excess demand or excess supply and the intensity of the competition among buyers or sellers. The importance of acquiring or selling of commodities determines the relative bargaining positions between sellers and buyers. Smith took the labor market as a typical example. He stated as follows:

"What are the common wages of labor, depends everywhere upon the contract usually made between those two parties, whose interests are by no means the same. ... In all such disputes the masters can hold out much longer. A landload, a farmer, a master manufacturer, or a merchant, though they did not employ a single workman, could generally live a year or two upon the stocks which they have already acquired. Many workmen could not subsist a week, few could subsist a month, and scarce any a year without employment." [8]

This was the reason why the level of wages tended to remain at the subsistence minimum in Smith's day. The urgency of acquiring wages or daily necessities for a workman to maintain the life of himself and his family makes his bargaining position very weak relative to that of the masters.

It should be added quickly that there also exists a critical minimum amount of leisure time, since human beings can not keep working 24 hours a day as pointed out by Walras.

The concept of the necessities of life was interpreted by Jevons as being the case in which the marginal utility of a commodity approaches infinity when its quantity available falls short of a certain critical level.[9] Marshall referred to the nearly infinite marginal utility of the meager income of a wage earner when he pointed out the exceptional property of the labor market.[10] In this connection, we should like to call readers' attention especially to Notes in the Mathematical Appendix VI, VII, and VIII on Marshall's *Principles*.

Edgeworth and his successors, however, did not take the possible case of the necessities of life into account when they developed a theory of the competitive market. In Edgeworth's Box in its usual display, it is not specified whether or not the origin of the indifference maps of the two groups (sellers and buyers) coincide with the origin of the quantities of two commodities. When the two commodities which are to be exchanged with each other in the market are the necessities of life, such as wages and disposable hours in the labor market, the marginal utility of each commodity would be infinite at the minimum critical amount of the necessity.[11]

Taking this concept of minimum critical amount $X_{\text{min.}}$ into account, the marginal utility of a necessity will grow infinitely large approaching asymptotically the vertical line at the level of critical amount, as shown in Figure 1.1. If the quantity falls below the level $X_{\text{min.}}$, then a man would not be able to survive. In the case of non-necessities, on the other hand, the marginal utility may have finite values at the point of zero quantity, as shown by Figure 1.2, since a man can survive without them.

Both daily necessities and leisure hours are classified in the category of marginal utility curve which may be expressed as in Figure 1.1.[12]

If the daily necessities Y and leisure time X have their own minimum critical amounts necessary for survival, $Y_{\text{min.}}$ and $X_{\text{min.}}$ respectively then what

Figure 1.1
THE MARGINAL UTILITY CURVE FOR NECESSITIES

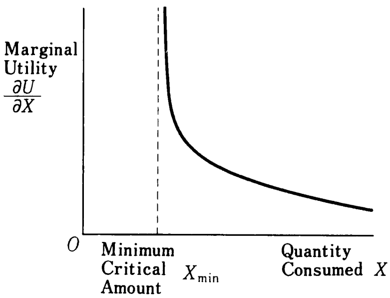
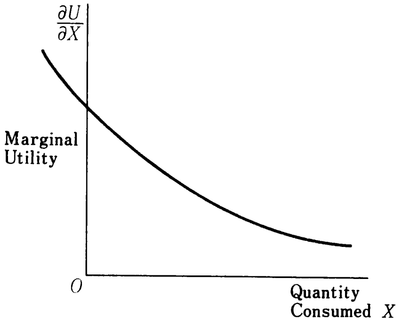


Figure 1.2
THE MARGINAL UTILITY CURVE FOR NON-NECESSITIES



would the indifference curve between them look like?

In textbooks, indifference curves are drawn in the shape of downward sloping and convex to the origin illustrating simply that the stability condition for consumer's equilibrium is satisfied.

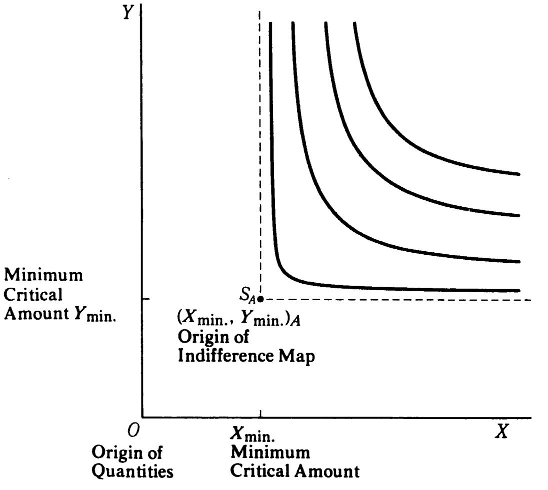
In his epoch making book, *Value and Capital*, John R. Hicks did not stress the possible disparity between the origin of the quantities and the origin of the indifference map. The influence of the book has been so overwhelming that the relationship between the two origins has rarely been discussed explicitly.[13]

It is important to note, however, that if the goods to be exchanged have the minimum critical amounts for survival as shown by Figure 1.1, then the origin S of the indifference map and the origin O of quantities of the goods would not be in the same position. The origin S would be located above and to the right of the origin O, at $(X_{min.}, Y_{min.})$.

It must be emphasized that Vilfredo Pareto, the very founder of indifference curve theory, was well aware of this possibility. In his *Manuel* he took an example of the case of bread X and water Y and argued "Without bread he dies of hunger... without water he dies of thirst." He pointed out that if the quantity of either bread or water at his disposal were reduced less than the smallest quantity he needs in order not to die the total utility would be equal to zero and the marginal utility of either good would be infinite. Then he drew a figure of an indifference map like Figure 1.3 below (Figure 33 in *Manuel*).[14]

The fact that the marginal utility curve takes the shape described by Figure 1.1 implies that the marginal utility can not be defined in the region where the quantity is less than the minimum critical amount. Consequently, the marginal rate of substitution against any other commodity can not be defined either in this region. If the quantity of either of the two goods is reduced less than the critical minimum amount, therefore, the indifference curve cannot be drawn. In the neighborhood of the asymptotic line where marginal utility grows infinitely large, the indiference curve becomes either almost vertical or almost horizontal since the slope of the indifference curve is determined by the ratio of marginal utilities of the two goods (the marginal rate of substitution). In other words, the indifference map will take the shape shown in Figure 1.3. There exists, as shown by the Figure, a blank peripheral region between the zero quantity axes of the two goods and the axes of the indifference map.

Figure 1.3 THE DISPARITY BETWEEN THE ORIGIN OF QUANTITIES AND THE ORIGIN OF INDIFFERENCE MAP

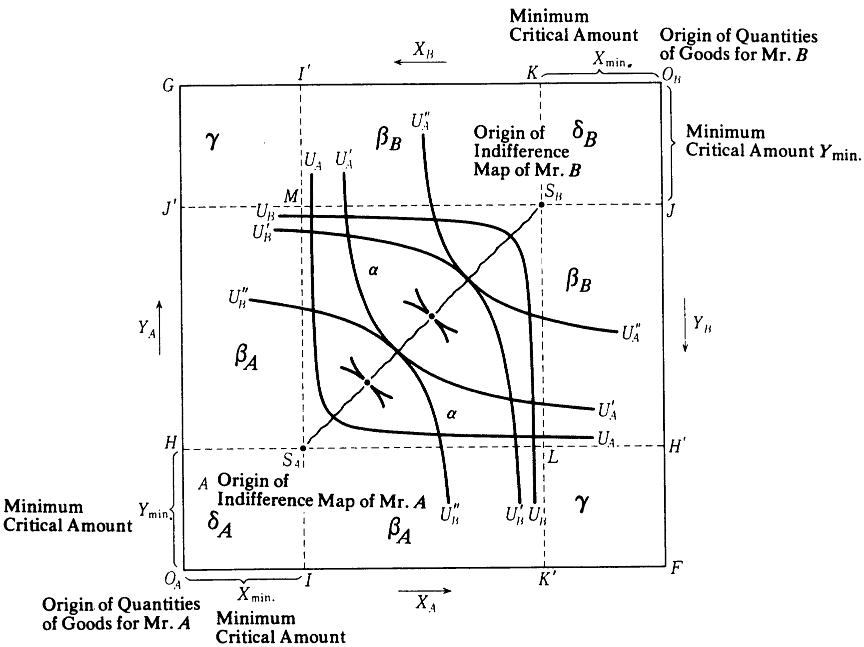


In the case where both of goods X and Y are not necessities and have marginal utilities as depicted by Figure 1.2, the origin of the indifference map between them and the origin of their quantities are the same.

1.2 Generalization of Edgeworth's Box-Diagram

The relationship between the origin of the indifference map and the origin of the quantities of goods is not explicitly qualified usually when the problem of exchange is explained using Edgeworth's Box-Diagram. However, if one considers the market mechanism bearing the question of "survival" in mind, as Smith did, then one has to consider the relationship between the two origins explicitly. Suppose that both of goods X and Y are necessities for Mr. A and Mr. B. The indifference maps of both Mr. A and Mr. B, then, will be of a shape like that in Figure 1.3. Taking this fact into account, the Edgeworth's Box-Diagram between A and B will be reconstructed as in Figure 1.4. To describe it in short, the Box is now surrounded by a frame. The length of the outer horizontal edge of the frame represents the total quantity of $\overline{X}$, or $\overline{X} = \overline{X_A} + \overline{X_B}$, and length of the outer vertical edge represents the total quantity of $\overline{Y}$, or $\overline{Y} = \overline{Y_A} + \overline{Y_B}$.

Figure 1.4 GENERALIZED EDGEWORTH'S BOX-DIAGRAM (*GEBD*)



The indifference map of Mr. A develops right and upward from the origin S_A $(X_{\text{min.}}, Y_{\text{min.}})_A$, and the indifference map of Mr. B develops left and downward from the origin S_B $(X_{\text{min.}}, Y_{\text{min.}})_B$. Note that the indifference curves of Mr. A can not be defined in the area below line HS_AH' and left of line IS_AI', and likewise the indifference curves of Mr. B do not exist in the area above line JS_BJ' and right of line KS_BK'.

It is, therefore, only in the α zone or S_ALS_BM that the indifference maps of Mr. A and Mr. B co-exist. Neither of the indifference curves of Mr. A nor Mr. B are drawn within the two quadrates $FH'LK'$ and $GJ'MI'$, two γ

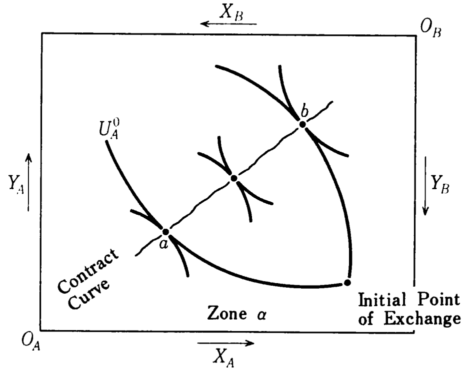
zones. Only the indifference curves of Mr. *B* are drawn in zone β_A and zone δ_A, and only those of Mr. *A* are drawn in zone β_B and zone δ_B. Usually, expositions of Edgeworth's Box-Diagram deal only with zone α.

When Edgeworth attempted to explain the theory of market competition using his Box-Diagram he quoted, as an example of exchange, a transaction between Robinson Crusoe (Mr. *B*) who pays wages in return for the labor services of Friday (Mr. *A*).[14] He said that if Robinson Crusoe offered to have an employment contract at any point below and to the left of point a on the contract curve of Figure 1.5, namely the point at which Friday's indifference curve which passes through the initial point intersects the contract curve, then Friday would stop bargaining and decide to work on his own. This implies that bargaining between Mr. *A* and Mr. *B* is based on the prerequisite that the indifference curves of both parties pass through the initial point which represents their initial shares. In other words, Edgeworth's example, as explained by Figure 1.5, corresponds to zone α of ***GEBD*** in terms of Figure 1.4.

It is interesting that Edgeworth happens to choose an employment contract as an example. In discussing the employment contract, he assumed that the worker can always work on his own if he is not satisfied with the terms of contract offered by the employer. In other words, the worker can choose freely between being employed by someone else and employing himself.

Adam Smith, in contrast, assumed that the workman can earn a living only by being employed by some master, and that the option of self-employment is not available for the worker. As a reason for this assumption he pointed to factors such as the occupancy of land and accumulation of

Figure 1.5 EDGEWORTH'S IMAGE OF CONTRACT



capital and goods. At any rate, in the view of Smith, the worker is obliged to be employed in order to earn a living quite unlike the case of Friday as quoted by Edgeworth.

Although there are exceptions like Walras and Pareto, Neo-Classical economics generally has an optimistic connotation. There were times when such an elegent example as a choice between a silk hat and a pair of kid gloves was preferred in explaining the theory of consumer's choice through the notion of indifference curve. The basic problems of economics with which Smith was concerned were, however, the issue of survival and not merely of choice between comparative luxuries. In view of this serious concern of the Classical economists with the question of "life or death," one would be led to suspect the adequacy of a scheme such as Edgeworth's which is concerned only with the "safe" area, in dealing with the whole question of market competition.

Needless to say, workers are not always deprived of alternatives to being employed by employers. In contemporary Japan, for example, workers of households which are partially engaged in agriculture or wives and children of urban households of which the principal earner's income more or less meets the family needs can enjoy alternative options, namely either work as an employee or not working. Nevertheless, we can not ignore elements of a forced exchange with which Smith was deeply concerned. The basic question here is why those economic historians who were interested in the relationship between the industrial revolution and the enclosure movement, focused their attention on the hypothesis that the enclosure movement had given rise to ample and cheap labor supply by limiting opportunities for self-employment.

The perspective of Edgeworth's theory on market competition was confined within the α zone of Figure 1.4. Although there remains the problem of Walras's reservation on the initial asset distribution even within the α zone, insofar as the initial point is within zone α and there exist a plurality of competitors on both sides, the merit of market competition will surely be realized in the sense that resources are reallocated more efficiently through competitive exchanges. However, what would happen if the initial point was in zones γ or δ of Figure 1.4.

Let us begin our examination with zone δ_A at the south-west corner of Figure 1.4. There are indifference curves of Mr. B in this zone. The level of Mr. B's utility rises higher as the point of exchange moves left and downward. In contrast, the indifference curves of Mr. A do not exist in this zone. Both goods X and Y are at levels below the critical minimum amount for Mr.

A within zone δ_A. If the initial point was in zone δ_A, it would mean that Mr. A would not be able to survive even if he consumes all of his initial holdings. In this situation, Mr. A could not afford to give away any of his initial goods to exchange with another. In other words, the δ_A zone is the area in which Mr. A can not afford to exchange at all. The same is true for Mr. B in the δ_B zone.

Those members of society who can not survive by themselves, or equivalently those members whose holdings (X, Y) are within the δ zone, would have to be taken care of by the society in the sphere other than production and exchange. The fact that the initial point falls within zone δ implies that there exists a situation in which market functions do not operate. It is in this situation where social security policies are called for.

Smith was well aware of this situation. He pointed out that:

> "... every individual who is able to work, is more or less employed in useful labor, and endeavors to provide, as well as he can, the necessaries and conveniencies of life, for himself, or such of his family or tribe as are either too old, or too young, or too infirm to go a hunting and fishing."[16]

Let us now look at zone γ at the southeast corner of Figure 1.4. The fact that the initial point is in the γ zone means for Mr. A that his initial holding of Y, or Y_A, is less than the critical minimum amount $Y_{\text{min.}}$ and for Mr. B that his initial holding of X, or X_B, is less than the critical minimum amount $X_{\text{min.}}$ and Mr. A's initial holding X, X_A, is more than the critical level and Mr. B's initial holding of Y, Y_B, is more than the critical level. Nevertheless, indifference curves of neither Mr. A nor Mr. B exist in the γ zone since the marginal rate of substitution between the two goods can not be defined here. This implies that neither A nor B can maintain his livelihood within the γ zone. The situation in the γ zone differs from the δ zone in that Mr. A can afford to sell good X and Mr. B can sell good Y.

Mr. A exchanges X for Y and Mr. B exchanges Y for X. Through this exchange, the point of exchange of X and Y is pushed above the line H—H' by Mr. A and to the left of the line K—K' by Mr. B. That is, the point of exchange contract would eventually have to be within the α zone. If for example in Figure 1.6, point c is reached then both A and B can survive. Once point c is reached, the situaiton would no longer be different from the situation where the point of exchange has initially been within the α zone. The indifference curves U_A of A and U_B of B, which pass through the point c, intersect the contract curve at the end points a and b respectively. And if there is a sufficiently large number of competitors on the side of both A and B, then the contract point will finally be determined at a point of competitive equilibrium E which is between a and b.

However, the process by which A and B reach a point within the α zone from the initial point is quite different. Since no indifference curves exist in the γ zone, neither contract relations nor supplementary contract relations between A and B can be defined. In such a situation, regardless of the number of competitors, the competitive mechanism as postulated by Edgeworth would not operate.

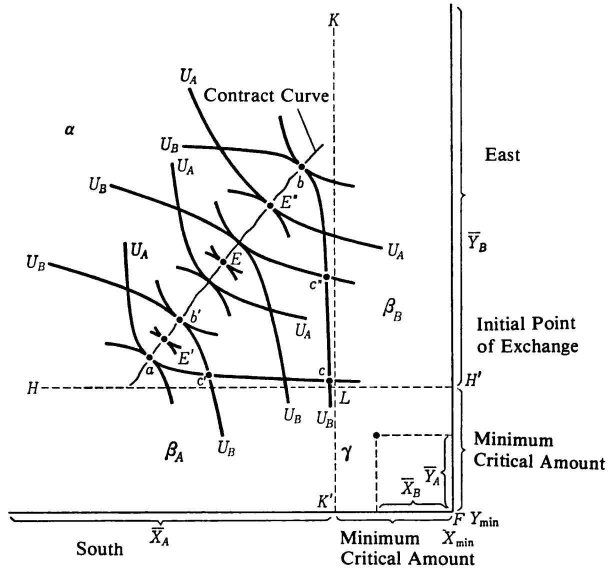
Consequently it is impossible to predict how a point at the edge of the α zone will be reached. Both Mr. A and B must be very anxious to reach the edge of the α zone to make a contract since both of them could not survive as long as they stay in the γ zone. Under such circumstances, both of them would try to reach a point on the edge of the α zone no matter where it might be.

Suppose that a contract in the first round happened to be made at point c at the southeast corner of the α zone through this kind of unpredictable process. In the neighborhood of c, the indifference curve U_A representing the lowest utility level of Mr. A and the indifference curve U_B representing the lowest utility level of Mr. B intersect. These indifference curves representing the lowest utilities intersect with the contract curve nearly at S_A in the southwest corner and S_B in the northeast corner of the α zone respectively. Therefore, the end points a and b of the effective contract curve will be located in the neighborhood of the two corners of the α zone, S_A and S_B, and consequently the entire span of the contract curve will be effective. If the quasi-initial point c is located in this way, the point of competitive equilibrium E will be determined at a position somewhere between and also more or less equidistant from the two corners of the α zone.

However, the consequence would be quite different if the first contract happened to be made at a point such as c' or c'', as shown in Figure 1.6. The indifference curve of Mr. B which passes through c', for example, is ranked high in terms of Mr. B's utility, and thus its intersection with the contract curve b' will necessarily be located not far from a, unlike point b discussed earlier. In this situation, the point of competitive equilibrium E' would have to be located in a position close to a, since E' has to be below and to the left of point b'. Similarly, if the first contract happened to be made at point c'', then the point of final contract E'' would be located close to point b.

As suggested from the above discussion, the fact that the location of the point of the first contract (the quasi-initial point) on the edge of the α zone is indeterminate implies that the point of perfectly competitive equilibrium E is also indeterminate no matter how perfect the subsequent competition may be in Edgeworth's sense after this quasi-initial point is reached. Therefore, this case is eventually indistinguishable from Edgeworth's precompetitive case in

Figure 1.6 THE CASE IN WHICH THE INITIAL POINT IS IN ZONE γ



which the initial point of contract between one person of *A* and one person of *B* was located on the contract curve. In other words, the indeterminate region of contract as broad as from *a* to *b* will not be compressed by competitive functions of the market. Even though the competitive mechanism may operate after the quasi-initial point is reached with the presence of many competitors, this case will not differ in practice from a case in which the competitive mechanism does not operate at all, since the determination of the quasi-initial point itself is indeterminate.

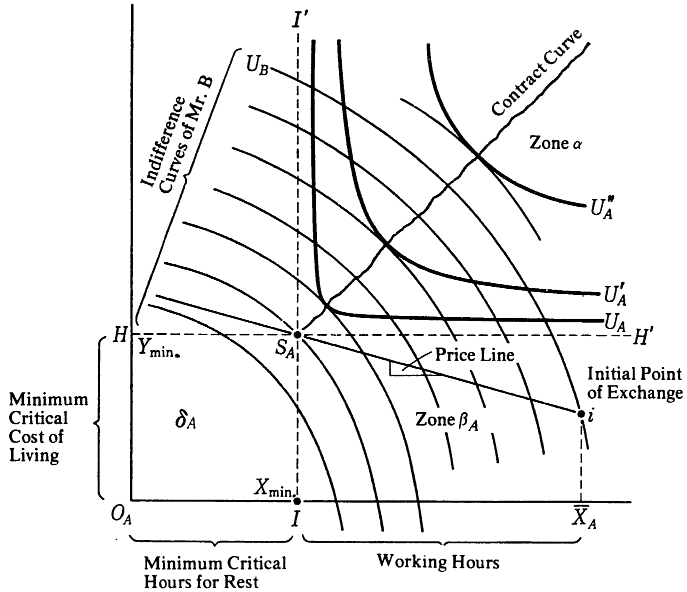
1.3 The Properties of the β Zone

Let us consider the case in which the initial point of Mr. *A* and Mr. *B* is in the β_A zone. In the β_A zone, either good *X* or *Y* falls short of the critical minimum amount for Mr. *A* while Mr. *B* has more than the critical amount of both goods. Therefore, it is an urgent necessity for Mr. *A* to exchange with Mr. *B* but it would not be fatal for Mr. *B* even if he does not exchange with Mr. *A*. Although the use of the expression "fatal" may sound too strongly, problems in the labor market often were really fatal in the days when Smith observed them.

Suppose that good X is leisure time. All workers have 24 hours of time at their disposal everyday as long as they are alive. The hours which remain after subtracting from this the minimum hours $X_{min.}$, necessary for life maintenance functions such as sleeping and eating, are the hours he can offer to the employer as hours for work. Let good Y be a daily necessity. Let us suppose further that the worker A has less than the critical minimum amount $Y_{min.}$ of good Y. The initial point in this situation may be located in a position such as i in Figure 1.7.

While there exists no indifference curve of Mr. A which passes through the initial point i, the indifference curve U_B of Mr. B does pass through point i. Therefore, if Mr. B wishes to make a contract with Mr. A, the point of contract should be to the southwest of the curve U_B. The further the location of the point of contract is to the southwest, the higher the level of Mr. B's utility will be. On the other hand, it is imperative for Mr. A to reach the edge of zone α. Because it means nothing for him to make a contract at a point below line H—H' or to the left of line I—I'. Once Mr. A reaches the α zone he is assured of survival. However, Mr. A will have no freedom of choice as to the position of the point of contract within the α zone. This is because the employer Mr. B enjoys a stronger bargaining position in the sense that he can maintain his livelihood without making a contract with the worker.

Figure 1.7 THE CASE IN WHICH THE INITIAL POINT IS IN ZONE β



In this situation, Mr. *B* would naturally choose a location in the neighborhood of S_A as the most advantageous point for him to have a contract. This is because the point S_A is the point at which Mr. *B* has the highest utility among all the points in the α zone, or equivalently within the points acceptable for Mr. *A*.

After the first contract has been made in the neighborhood of S_A, is there a possibility that the equilibrium point will move toward the central part of the contract curve through the competitive mechanism as depicted by Edgeworth if, for example, there are two competitors on sides *A* and *B*? The answer is no. This is because there exists no indifference curve of Mr. *A* which intersects with the price line *i*—S_A although there are indifference curves of Mr. *B* which intersect with the price line. The Edgeworth-type competition begins to operate only if one of the two *B* persons recontracts with the two *A* persons at the price given by the contract. There is obviously no incentive by which this kind of competition is triggered since the price line does not intersect indifference curves of Mr. *A* with higher utility.

Thus, in a case where the initial point *i* is in the β_A zone, S_A would have to be the final point of contract. The fact that goods *X* and *Y* are exchanged at the point S_A implies that worker *A* receives from his employer *B* the critical minimum amount $Y_{\min}$ of necessities (wages) in return for his offer of working hours ($X_A - X_{\min}$) to his employer. The remaining hours for leisure are the minimum critical amount $X_{\min.}$ necessary for his survival. In other words, the exchange at S_A means that the worker *A* receives the minimum wage for subsistence in exchange for the longest possible hours of work for his employer *B*. This is exactly the type of employment contract which Smith believed to be the likely case and Marx believed to be the inevitable. Walras, as cited above, was looking probably at a similar situation when he noted the fact that workers cannot freely choose the length of working hours in the actual labor market.

When the initial point *i* is in the β_A zone, the point of contract will be determined at the point S_A regardless of the number of competitors. Since the Edgeworth-type competition would not work at all no matter how many competitors there may be, the employment market in such a situation is not at all a competitive market. It is not even a monopsony market in the usual "industrial organization" sense. It may perhaps be more accurate to describe this situation as an absence of a market.

Let us recall the case in which the initial point was in the γ zone. The competitive equilibrium was indeterminate in that case. However, since Mr. *A* and Mr. *B* were more or less equal in the sense that both of them were quite weak, it was not impossible to have a competitive equilibrium in the central part of the contract curve although it was largely dependent on coincidence. In contrast, in the case of the β zone, one side of the bargainers has complete dominance over the other. The weaker side is in a position where the realm in which he can not survive. There remains no room for improving the situation by means of Edgeworth-type competition no matter how great the number of competitors on either side.

One could perhaps maintain, as Ricardo and his followers did, that the contract at S_A is also a consequence of the market competition. If this were so, then it would be more sensible to assert, as Marx did, that it is better not to have such a competition. In fact, it is incorrect to say that the contract at S_A is a result of competition. Strictly speaking, a market, in its normal sense, hardly exists under such circumstances.

A helpless worker, who has no means to employ himself, no savings, no connections with mutual associations, no public unemployment benefits, would be obliged to make a contract at a point like S_A if he has to make a contract under the threat of starvation. The contract is disadvantageous for the worker not only in terms of wages but also in terms of hours. As Walras pointed out, he would be obliged to work much longer than he would be willing to choose to unless external regulations are imposed on working hours. Since the long hours will increase the amount of labor supply measured in terms of man-hours, it gives rise to a tendency for excess supply of labor in the market. With this tendency, the position of workers which is already disadvantageous relative to that of employers, will deteriorate further and the vicious circle of deterioration of workers' positions will be established. The governmental regulation of working hours, the need of which was stressed by Walras, is therefore necessary to counteract this vicious circle and to restore the market functions.

The possibility that the initial position falls in the β zone exists also for markets other than the labor market. This possibility exists, for example, in the case of a transaction between a financially weak small firm A and a strong large firm B. Suppose firm A is a sub-contractor of firm B receiving money Y in return for providing goods X. If A did not get orders from B for a long time, A would face serious financial difficulties or shortage of Y. In this situation, the initial position may be said to be in the β_A zone. The price of good X, or slope y/x in such a situation can not be steeper than the slope of the price line i—S_A. This is because the Edgeworth-type competitive mechanism does not operate between A and B even though there exists a plurality of firms on the sides of both A and B.

Similarly, this kind of possibility applies to the case in which a small firm A purchases raw materials Y from a large firm B and pays money X. If A

could not continue its business activities because of termination of the supply of Y, their contract position would inevitably be at S_A. This means that A is obliged to purchase Y at the highest possible price x/y which is the reciprocal of the slope of the price line $i—S_A$. The highest price is the price above which A cannot afford to secure with its money the critical minimum amount of Y necessary to continue its business. It is important to note that the price of Y will be driven to its highest possible rate even though B is not a monopolist. In other words, this kind of consequence will take place, contrary to the common supposition, even though there are competitors on both sides of A and B. The same thing can be expected when the small firm A, suffering from the lack of money Y, borrows from a bank B. The interest rate x/y in this case will necessarily be the maximum for the same reason as discussed above.

The analoguous situation may take place in the international market. A country whose domestic oil production does not satisfy the critical minimum amount will fall in the position of A when it bargains with an oil producing country. Another example is that a monocultural copper exporting country is prone to fall in the position of A in securing the critical minimum amount of foreign exchange when the international copper price is depressed.

Generally speaking, in the case in which the initial position prior to exchange is in the β_A zone, A is obliged to make a highly disadvantageous contract with B by which A can barely survive because of the failure of the competitive function of the market. The important point to be noted is that this is the result of economically rational actions of B and not necessarily the result of monopoly of B or collusion between B's.

This result is clearly different from the case in which an equilibrium position may be reached through competitive exchanges from the initial position within the α zone. It would be a serious mistake, therefore, to preach the gospel of *laissez faire* on the assumption that market competition always functions irrespective of a possibility of false competition in the β zone.

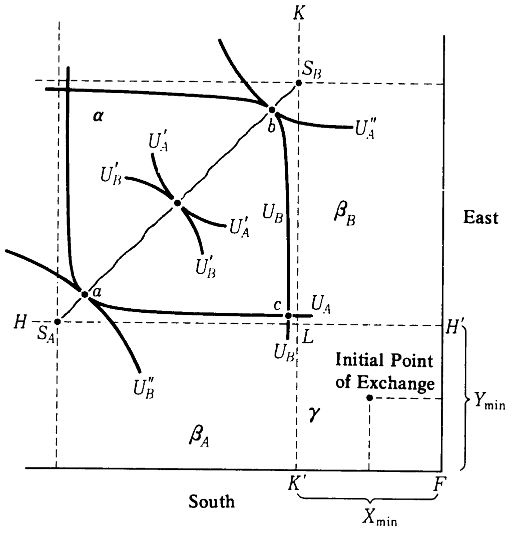
1.4 The Requisites for Market Competition

Market competition would not be assured if the initial position prior to exchanges fell within the β zone when no interventions were made. There are, however, two cases in which the competitive functions of the market may be restored.One is the case in which excess demand emerges for goods X supplied by A, as observed by Adam Smith in his example of the labor market in North America.[17] The other is the case in which the initial position is moved to the α zone by forces external to the market itself, such as governmental interventions in the form of legal labor standards or income transfers. Since we will discuss the former case in detail later in connection with the Keynesian demand control policies, let us consider the latter case here.

It is important to consider first the case of the γ zone. As we have illustrated using Figure 1.6, both A and B have to make a contract promptly somewhere on the border of the α zone when the initial position is in the γ zone. However, there is no way of knowing at which point on the border the contract may be made. The determination of a particular position of the contract will be totally random. However it is easy to see that the consequence will be completely different depending on whether the contract is made on the β_A side or β_B side of the border of the α zone. That is to say, it would be advantageous for B if the quasi-initial position were to reach the β_A side of the α zone, and advantageous for A if it were to reach the β_B side of the α zone.

A criterion for policy interventions should be that the interventions do not result in a position too much in favor of one side relative to the other. To realize the spirit of this criterion in this case, the government should order A to supply slightly more than the critical minimum amount $X_{min.}$ of goods X to B, and order B to supply slightly more than the critical minimum amount $Y_{min.}$ of Y to A. This policy is desirable since both A and B can now be assure of survival with this externally forced exchange and free market competition will occur thereafter.

Figure 1.8 THE CASE IN WHICH THE INITIAL POINT IS IN ZONE γ



With this policy intervention, the quasi-initial position will be located in terms of Figure 1.8, at point c in the Southeast corner of the α zone. The point c is the point where the indifference curve U_A of A's lowest utility and the indifference curve U_B with B's lowest utility intersect. The effective width of the contract curve defined by the distance between the end points a and b will take the maximum possible breadth in this case since a and b are, respectively, in the neighborhood of corners S_A and S_B of the α zone. The equilibrium point reached through competition in this situation will be determined depending on the shapes of indifference curves of A and B, and in this sense the point of contract will not be unduly biased.

As long as necessities exist, the initial position would have to be located either in the γ zone or in the β zone in the world where division of labor is perfectly established. For the purpose of making the competitive function of the market operate normally, it is therefore imperative that the government intervenes in the market to bring the quasi-initial position of exchange at a point within the α zone. This point has an important implication for consideration of problems associated with international division of labor too. That is to say, the standpoint of "unconditional *laissez faire*" should be abandoned if one wishes to take advantage of the merits of greater efficiency by means of division of labor or specialization in production while at the same time maintaining the allocative function of free competitive markets.

1.5 The Inevitability of Institutional and Policy Interventions

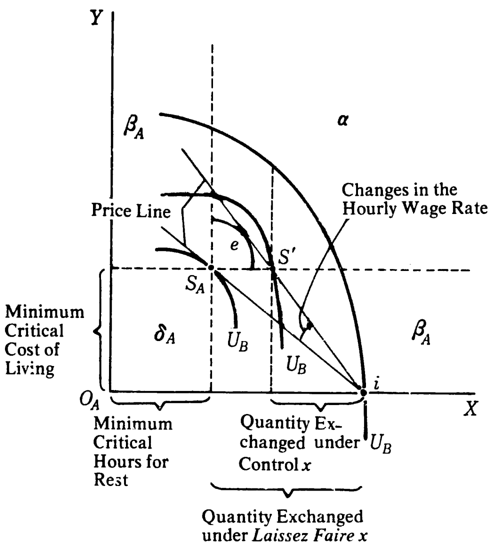
In the γ zone, the bargaining positions of the bargainers are equal in the sense that both of them are in the region in which they cannot survive. In the β zone, the bargaining positions are seriously unbalanced since either one of the bargainers is in the non-survival region while the other is in the survival region. In other words, the former desperately needs to make a contract with the latter while the latter does not have to make a contract with the former. This unbalance in bargaining positions gives rise to, using the classical example of the employment contract, subsistence wages and the longest possible working hours. This situation differs from the concept of competitive market equilibrium, which was embraced vaguely by the Classical school and defined clearly by the Neo-Classical school, in the sense that this is not exchange within the α zone.

The "principle of free contract" upheld by the liberal economics implies freedom to either making or not making a contract. In the case where the initial position is in the β zone, however, one of the bargainers cannot afford not to make a contract since he would not be able to survive without doing so. Therefore, the principle of free contract should be totally irrelevant to exchanges within the β zone.

The fact that the principle of free contract is inapplicable implies that external intervention in the market is inevitable. In other words, governmental interventions are unavoidable. In the labor market, governmental regulations such as factory legislation had been attempted even before the necessity for such interventions was pointed to by Gossen, Marx and Walras. The 19th century Factory Laws, which resemble the contemporary Labor Standard Laws in its essential nature, regulate basically the amount of labor services supplied to the labor market. It restricted legally the amount of labor services used by employers by means of limiting working hours permitted for a day, prohibiting infant labor and restricting night work by female workers. The effect of this kind of quantitative restriction on the form of working hours and types of labor force may be illustrated by Figure 1.9.

The point of contract in the absence of regulations would be at S_A where the worker offers the longest possible working hours x in order to receive subsistence wage y from the employer. Once a greater number of working hours than x' is prohibited by the government, the point of contract would shift to S'. The employer B has no incentive to shift the point of contract from S_A to S' unless forced by outside regulations since his utility at S_A is

Figure 1.9 EFFECTS OF FACTORY LAW UPON LABOR MARKET



higher than at S'. However, since his utility at S' is still higher than at the initial point i, in other words, that it is still more advantageous to make a contrast at S' than to stay at the initial position even under the legal regulations of work hours, employer B will make an employment contract at S' following the regulation. The wage rate per hour increases in this case from y/x to y/x' even though the daily wage rate remains the same. From the viewpoint of worker A, he will enjoy somewhat higher utility at S' than at S_A since the time for rest will be greater than its critical minimum level although his earnings may not increase. In other words, thanks to this governmental regulation the level of worker's utility will be higher and hence the working conditions may be said to be improved accordingly. As seen in Figure 1.9, a similar effect may be expected from regulation of the minimum wage. This is because the contract point can shift from S_A to S' if the minimum hourly rate were regulated at y/x' or at the slope of line i—S'. The exchange will be made in this case at the point of contact of the extended line of the price line i—S' and B'_S indifference curve higher than U_B. This point e is advantageous for the employer as well. In the case of the aforementioned quantity regulation, on the other hand, the contract point will be at S' since a contract made at any point to the left of S' will be illegal. It seems that the price (wage) regulation has a greater favorable effect since A's utility is higher at e than at S' and also the market function operates partially in the sence of adjusting between demand and supply.

However, the implementation of minimum wage regulations involves technical difficulties since the minimum wage rate has to be specified for each of the different types of labor. Since there are various types of labor even within each category of the labor force, such as adult male workers or young female workers, regulations based on broad categories may even confuse the situation. It is not easy, on the other hand, to specify adequate minimum rates for detailed labor classifications. Quantity regulation is advantageous in the sense that objective criteria can be specified more easily since the regulations of hours, sex and ages can be commonly applied without regard to very fine categories of the labor force.

At any rate, governmental regulations, whether in terms of quantity or price, certainly have an impact in modifying the relative bargaining positions in favor of the disadvantaged side of the bargainers.

This kind of governmental regulation will have similar effects in cases other than the classical example of the employment contract. Let us consider, for example, the case of a transaction between a small businessman A and a money lender B. A has to borrow money y from B to run his business. The contract will be made at S_A in this case if no external regulations are made on the terms of contract. If A were to borrow money y from B at the beginning of the period and return the sum of principal and interest at the end of the period, then the rate of interest would have to be $(x/y)-1$. If the government intervened here and fixed the maximum rate of interest at $(x'/y)-1$, then the contract would be made at S'. It is also possible, as we have discussed earlier, that the contract will be made at e. Since x is all of his corporate income, or all of his income remaining after subtracting from turnover minimum subsistence expenses, Mr. A would have nothing in hands at the end of each period if he were to appropriate the minimum amount of money necessary to run his business at S_A. The same situation will recur period after period unless external regulations are made on the rate of interest. If a certain ceiling were imposed on the interest rate by policy interventions, as described above, A could secure a reserve of $x-x'$ after paying the debt back. This will make the amount that A will have to borrow for the next period smaller, and consequently his business will be able to enjoy a gradual accumulation of its own capital.

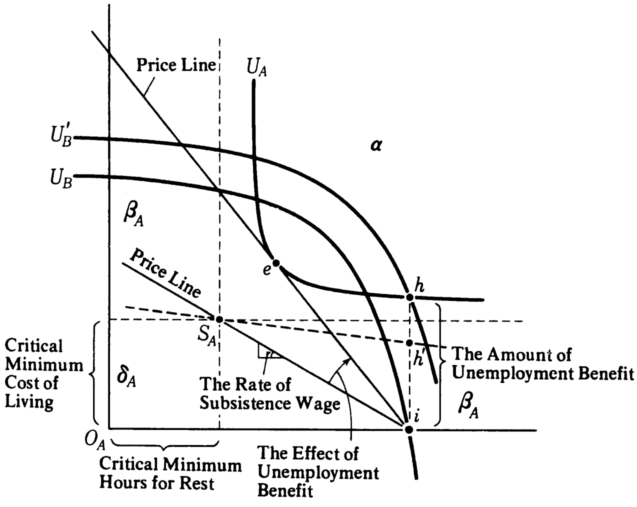
Examples of unbalanced bargaining positions such as this may be found in history in a contract between landlords and tenants, or between whole sale dealers and household manufacturers. This kind of unbalance in bargaining positions may also implicitly or explicitly exist in various cases in contemporary societies. For those cases, governmental regulation of quantities or prices is necessarily called for and also must be effective in restoring competitive market functions.

Returning to the familiar example of the employment contract, the effect of the quantity regulation may be found not only in the shift of the point of contract from S_A to S' but also in bringing the market more nearly to the situation of excess demand. The latter effect takes place since the employer can no longer enjoy the plentiful labor hour supplies as he used to prior to the implementation of the regulation, even for the same wage bill and consequently the tendency for excess supply in the labor market is curtailed.

There is also an indirect intervention where income security is provided in the case where the initial position i is in the β zone. If, for example, unemployment benefit is given to an unemployed worker in excess of his minimum subsistence under a well developed unemployment insurance system, then the initial position i would shift from the β to the α zone. This case is nearly equivalent to the case in which an employed person tries to change his job in the sense the initial position is already in the α zone. In this situation, he does not have to make a new contract if the new contract has inferior working conditions. As shown by Figure 1.10, if unemployment benefit greater than the minimum subsistence is provided to an unemployed worker, then the initial position would shift from i to h.

Since h is within the α zone, there exists an indifference curve U_A of

Figure 1.10 EFFECTS OF UNEMPLOYMENT INSURANCE UPON LABOR MARKET



worker A. This means that worker A will not be obliged to make a contract outside (or to the S_A side) of his indifference curve U_A.

It should be noted, however, that the initial point of exchange in this case should be at i instead of h, unlike the case in which the initial position of exchange has originally been at h, since the unemployment benefit will be terminated whenever A is employed. Accordingly, the price line should be i—e instead of h—e. It is obvious that A will not take a job at a wage rate lower than the slope of price line i—e. From the viewpoint of employer B, on the other hand it is advantageous for employer B to make a contract at a point as close as possible to the indifference curve U_A which is the worst position acceptable for the worker A. Then, the final contract will be determined by the relationship between indifference curves U_A and U_B depending on the labor market demand-supply balance as a whole. The competitive process in this case should be somewhat different from that described by Edgeworth's theory of supplementary contract because the price line i—e is drawn from outside the indifference curve U_A. We shall explain this kind of competitive process in detail in Chapter 3. Without a shortage of labor the contract position will be determined at point e.

Nevertheless, the contract position e backed by policy interventions is more advantageous to worker A than position S_A which would have been reached in the absence of policy interventions since the former brings about longer hours for rest and higher wages. When the unemployement benefit is greater, then the price line i—e would shift further to the right since the level of utility of the indifference curve U_A which passes through h is higher. This means that A does enjoy still higher wages and more time for rest. Needless to say, however, the meaningful amount of the unemployment benefit is not limitless. It should be noted that the contract of employment between A and B can be made only when the B's indifference curve U_B which passes the initial position i intersects A's indifference curve U_A which passes point h. It is easy to see the higher the position of h which is determined by the amount of unemployment benefit the less the possibility of U_B intersects U_A will be. When U_B does not intersect U_A no employment contract will be made. That is to say, the excessively large amount of unemployment benefit relative to the vertical length $\overline{Y}$ of ***GEBD*** will prevent the restoration of full employment. This is the case of which J.R. Hicks warnd in his *The Theory of Wages.*

On the other hand, an insufficient amount of unemployment benefit is also harmful since the contract point will remain at S_A if the benefit is not large enought to pay for the subsistence minimum as illustrated by point h' in Figure 1.10. This example is reminiscent of the effect of the Poor Law. The Poor Law was intended to supplement incomes of workers when their incomes were less than the minimum level to support life by means of transferring incomes from sources other than employers. In this case, employer B can employ labor services x at the wage rate of only h'—S_A as illustrated in Figure 1.10. This eventually results in no improvement in conditions of living or work of worker A even though part of labor costs which should be born by employer B is paid by the public fund. Under this system the result would not be different even if the fund were appropriated from B'_S sources. Ricardo was quite right in pointing to the undesirable results of the system.[18]

The Poor Law was a bad law, however, simply because the way it was used as a policy intervention was inadequate and not because such kind of intervention was unnecesary. Gossen aptly pointed to the necessity of such policy interventions.[19] And also Marx and Walras had good reasons to recognize their inevitability.

The necessity and inevitability of policy interventions as illustrated by Figure 1.10 is not limited to the case of the labor market. A small and financially weak firm A may be obliged to sell at a loss at a position like S_A if the initial position of exchange was in the β zone because its poor financial capacity was insufficient to maintain its inventories for a certain period. If firm A, in this case, could obtain loans at a low interest rate from public financial intermediaries for the purpose of maintaining inventories, then A would be able to bargain in a normal market or the α zone. Generally speaking, policy interventions as illustrated above are required in cases where the initial position is within the β zone or the γ zone for the very purpose of restoring the competitive functions of the free market system.

Notes to Chapter 1

1. Leon Walras states in his paper (1897).
   "L'économique pure suppose, à son point de départ, des propriétaires de facultés personnelles dont chacun offre une quantité de journées de travail

   $$o_p = f_p(p_t, p_p, p_k \cdots p_b, p_c, p_d \cdots)$$

   susceptible de varier avec tous les prix de tous les services et produits et se déterminant, après détermination de ces prix, par la raison du maximum d'utilité effective. Mais l'économiste ne doit pas être la dupe de ses abstractions. En fait, cette hypothèse peut se réaliser dans certains cas, comme peut-être celui des professions libérales; mais, dans beaucoup d'autres cas, elle est irréalisable. Dans la grande et moyenne industrie, la quantité fournie et employée de travail journalier est nécessairement la même pour tous les travailleurs, non seulement, pour des raisons techniques, dans une même entreprise, mais encore, pour des raisons économiques, dans toutes les entreprises d'une même industrie. Et qui fixe alors cette quantité? Sous le régime du *laisser faire,* c'est une concurrence d'entrepreneurs qui, visant au bon marché par la répartition des intérêts nets du capital sur une quantité plus considérable de produits, tend à allonger indéfiniment la journée de travail. Il faut pourtant que cette tendance soit arrêtée. Le travailleur ne peut pas travailler vingt-quatre heures par jour. La fixation d'un maximum s'impose. Et dès lors, quoi de plus naturel que de la confier à l'Etat qui l'effectuera d'après ses desiderata de moralité, d'hygiène, etc."
2. Marshall (1920) pp. 279-280.
3. For instance, see Galbraith (1967).
4. The Neo-Classical theory of market competition is often illustrated by a convenient theoretical device of "Edgeworth's Box-Diagram."
5. Adam Smith obviously had in mind a concept of market competition which was far broader and more comprehensive than the theoretical formulations of his decendants. The classical "Edgeworth's Box-Diagram" as well as the recent highly sophisticated theory of the "core" do not seem to reconstruct the full implication of Smith's conception.
6. Smith (1776).
7. Jevons (1871).
8. Smith (1776), pp. 56-57.
9. Jevons *op.cit.,* p. 107.
10. Marshall *op.cit.* Marshall uses the concept of "bargaining power" here, which was succeeded by Pigou and Slichter later. While the concept of bargaining power plays an important role in contemporary labor economics, the concept has been largely ignored in the mainstream of the Neo-Classical price theory.
11. See notes to Chapter 2.
12. Empirically measured marginal utility curves of consumers will be presented in Chapter 10 of this volume.

13. Hicks (1939).
14. Pareto (1927), English edition (1971) pp. 201-202.
15. Edgeworth (1881), Augustus-Kelley Reprint, especially pp. 28-29.
16. Smith *op. cit.*
17. Smith *op. cit.*, p. IVIII:
18. Ricardo (1821).
19. Gossen (1854). pp.166-167

Chapter 2
A Neo-Classical Interpretation of Smith's Free Market

We have made it clear in the previous chapter that indifference curves of contractors do not necessarily pass through the initial point of exchange once we consider the case of necessities. In other words, we have stressed that the importance of considering the Generalized Edgeworth's Box Diagram (*GEBD*) which consists more than just the α zone as Edgeworth himself probably thought. This does not imply at all, however, that the merit of free market competition, which has been advocated ever since the days of Adam Smith, is denied. Rather, we would like to emphasize the efficacy of the competitive market, even more strongly than the conventional concept of "atomistic competition" would imply, so long as the initial position of exchange is located within the α zone with a given appropriate distribution of assets among agents.

2.1 Production and Utility maximization—A Case of "Beaver and Deer"

Using Smith's example of beavers and deer, let us examine how a hunter allocates his labor for different purposes. In that "early and rude state of society", we assume that every hunter can use any tools on any part of land he likes to chase and capture game. Suppose there exists a hunter who has a certain taste in choosing between beavers (good X) and deer (good Y). Let his preference between goods X and Y be expressed specifically by an utility indicator such as

$$(2.1) \qquad U = \alpha_x \log(a_x + X) + \alpha_y \log(a_y + Y) \ ,$$

where X represents the number of beavers, Y the number of deer, and α and a are preference parameters. Then, marginal utility of beaver and deer is expressed respectively as

$$(2.2)\qquad \frac{\partial U}{\partial X} = \frac{\alpha_x}{a_x + X} \quad , \quad \frac{\partial U}{\partial Y} = \frac{\alpha_y}{a_y + Y} \quad .$$

This specific functional form represented by equations (2.1) and (2.2), which indicate that the marginal utility of a good diminishes asymtotically with the quantity of good consumed, has been found in a number of empirical analyses to have a high degree of approximation of actual consumers' behavior in contemporary society.[1] Therefore, it would not be unreasonable to presume that the hunter in this example behaves also according to the marginal utility curves as represented by this equation (2.2).

His preference about consumption would be realized without any distortions if beavers and deer could be obtained without any "toil and trouble" or without sacrificing at all "his ease, liberty and happiness". However, if it takes "toil and trouble" to obtain them, then how many animals he can obtain will depend upon how he allocates his labor between the two kinds of animals.

Let us suppose that the maximum number of days he can go out for hunting during a year is H, the number of days he has to spend to hunt one beaver is denoted by h_x and one deer by h_y, and that all of his labor is spent for hunting of these animals. In other words, the total workable days H will be divided into hunting days for beavers and deer, that is, h_xX days for beavers and h_yY days for deer. This may be expressed in equation

$$(2.3)\qquad h_xX + h_yY = H.$$

Equation (2.3) represents in effect a kind of budget constraint. The necessary hunting days h_x and h_y in this equation are equivalent to what Smith meant by "real price." Smith meant by "real price" of any good the amount of labor a man has to spend to obtain it. The problem of allocating the workable days for this barbarian between beavers and deer is in effect equivalent for an individual in contemporary society to allocate his income between different consumption items. Therefore, it is required in the former case of a hunter that the marginal utilities of the two animals divided by the respective hunting days are equal as a necessary condition of constrained utility maximization under the budgetary constraint (2.3). This conditon may be expressed as

$$(2.4)\quad \left(\frac{\alpha_x}{a_x + X}\right)/h_x = \left(\frac{\alpha_y}{a_y + Y}\right)/h_y \quad \text{or} \quad \left(\frac{\alpha_x}{a_x + X}\right)/\left(\frac{\alpha_y}{a_y + Y}\right) = \frac{h_x}{h_y} \quad .$$

Solving the simultaneous system of equalities of marginal utilities (the equality between marginal rate of substitution and realtive prices) and the budget constraint for optimal number of beavers X and deer Y, we get

$$(2.5)\quad X = \frac{\alpha_x \frac{H}{h_x} + \alpha_x a_y \frac{h_y}{h_x} - \alpha_y a_x}{\alpha_x + \alpha_y}, \quad Y = \frac{\alpha_y \frac{H}{h_y} + \alpha_y a_x \frac{h_x}{h_y} - \alpha_x a_y}{\alpha_x + \alpha_y}$$

The necessary hunting days to capture one beaver h_x and one deer h_y are a kind of labor input coëfficient in Leontief's sense. In other words, they represent simple production functions. Equations (2.5) therefore represent the scheme of an individual to maximize utility under the given utility indicator U and technological conditions of production (h_x and h_y).

These equations imply that the equilibrium quantities of captured animals will differ between hunters A and B if they differ in their hunting technology even if their preferences are exactly the same. This is because equations (2.5) contain not only preference paramenters α and a but also technology parameters h_x and h_y.

Let us suppose now that the common preference among barbarians is expressed by

$$(2.1)'\quad U = \underset{\alpha_x}{0.4} \log(\underset{a_x}{-20} + X) + \underset{\alpha_y}{0.6} \log(\underset{a_y}{-20} + Y).$$

Let us suppose further that barbarians of type A take on average slightly more than a day ($h_x = 1.017$) to capture a beaver and more than seven days and a half ($h_y = 7.627$) to hunt one deer.

If the barbarian spends his annual total hunting days, 300 days, for hunting only beavers, he will get 295 beavers, while he will get 39.3 deer if he only hunts deer. According to the first and second term of the right-hand side of equation (2.1)′ the total utility indicator of 295 beavers will be 976 while that of 39.3 deer will be 771 respectively. Thus, it would be more advantageous for the barbarian to hunt only beavers if he had to hunt only one kind of games. However, he will probably hunt both beavers and deer in such a way as to satisfy the equilibrium conditions of equation (2.4).

Given the preference function (2.1)′, the optimal numbers of beavers and deer will be determined respectively by

$$(2.5)'\quad \begin{aligned} X &= \frac{120}{h_x} - 8\frac{h_y}{h_x} + 12, &\quad \text{Beavers} \\ Y &= \frac{180}{h_y} - 12\frac{h_x}{h_y} + 8. &\quad \text{Deer} \end{aligned}$$

The optimal numbers under the following technological constraint of A-type barbarian

$$(2.3)'_A \qquad \underset{h_y}{1.017}X + \underset{h_y}{7.627}Y = \underset{H}{300} \quad ,$$

therefore will be given by substituting values of h_x and h_y into equation (2.5)′. They are:

$$\text{Beavers } X_A = 70, \qquad \text{Deer } Y_A = 30.$$

Substituting these numbers into equation (2.1)′, the annual total utility indicator of A-type hunter U_A will be 1280. Since this is greater than the utility indicator which would be achieved when he hunts only beavers or deer, it is implied that he could enjoy greater satisfaction by combining two kinds of animals.

Let us suppose that a B-type hunter has the same consumption preference as his A-type counterpart as expressed commonly by equation (2.1)′ but has different technological conditions of hunting. The B-type barbarian takes nearly 6 days ($h_x = 5.882$) to hunt a beaver while less than two days to hunt a deer ($h_y = 1.756$). In other words, it may be said that an A-type barbarian is good at swimming while a B-type barbarian is good at running. The technological condition for B-type hunters is given by

$$(2.3)'_B \qquad \underset{h_x}{5.882}X + \underset{h_y}{1.765}Y = \underset{H}{300} \quad .$$

Substituting these values of h_x and h_y into equation (2.5)′ we will get equilibrium numbers of game, namely

$$\text{Beavers } X_B = 30, \qquad \text{Deer } Y_B = 70.$$

The annual amount of satisfaction U_B which the B-type hunter will get in this case will be 1419 in terms of the utility indicator, which is greater than that for an A-type hunter.

The number of days necessary to hunt one beaver and one deer is given by $1.017(h_x) + 7.627\ (h_y) = 8.644$ for the A-type and by $5.882\ (h_x) + 1.765\ (h_y) = 7.647$ for the B-type. This means that a B-type hunter is superior than the A-type in overall hunting skill. Moreover, as equation (2.1)′ shows that the utility obtained from one deer is relatively greater than that from one beaver. Therefore, the B-type hunter enjoys utility indicator of 1419 which is greater than that of 1280 for the A-type.

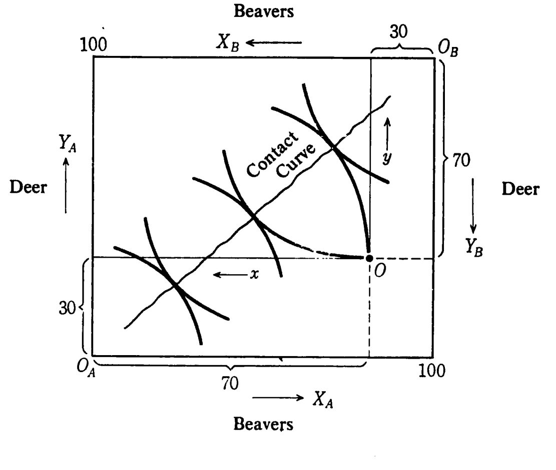
The "primitive" situation as depicted by Smith (in chapters V and VI of *Wealth of Nations*) in which everyone produces and consumes freely is thus well described by the micro optimization theory developed by the Neo-Classical school. As is indicated by this example, individuals with the same preference, so long as they are mutually independent, can enjoy different amounts of optimal production and consumption if they have different technological conditions. They would have even more different consumption patterns should they have different consumption preferences in addition to different technological conditions in production.

2.2 Merits of Free Trade

Thus far, we have discussed cases in which hunters were mutually independent and isolated. Let us now consider cases in which A-type and B-type hunters get acquainted with each other and make transactions.

As an initial condition, let us suppose that A-type hunter hunts during a year 70 beavers and 30 deer while B-type hunter hunts 30 beavers and 70 deer, as indicated above. Since the A-type hunter hunts more beavers and the B-type hunts more deer, A would probably like to exchange his beavers for B's deer. Needless to say, both A and B will try to exchange their games in such a way as to maximize their own utility. These exchanges may be expressed using Edgeworth's Box Diagram as shown by Figure 2.1.

Figure 2.1 A Theoretical and Diagramatical Illustration of Exchange: The Edgeworth's Box Diagram



Let us assume that A and B have the same preference functions which can be expressed as

$$U = \alpha_x \log(a_x + X) + \alpha_y \log(a_y + Y). \tag{2.6}$$

Let us denote the number of beavers exchanged from A to B by x and the number of deer from B to A by y. Although the amounts exchanged, x and y, are the same for both A and B, the signs are opposite between A and B. Denoting the initial stock of A by $(\overline{X}_A, \overline{Y}_A)$ and that of B by $(\overline{X}_B, \overline{Y}_B)$, the consumable amounts, X and Y, for A and B after exchanges are expressed as

(2.7)

	Type-A	Type-B
Beavers	$X_A = \overline{X}_A - x$	$X_B = \overline{X}_B + x$
Deer	$Y_A = \overline{Y}_A + y$	$Y_B = \overline{Y}_B - y$

Substituting equation (2.7) into (2.6), we get utility indicators for A and B which can be written as

$$\begin{aligned} U_A &= \alpha_x \log(a_x + \overline{X}_A - x) + \alpha_y \log(a_y + \overline{Y}_A + y) \quad \text{Type-A,} \\ U_B &= \alpha_x \log(a_x + \overline{X}_B + x) + \alpha_y \log(a_y + \overline{Y}_B - y) \quad \text{Type-B.} \end{aligned} \tag{2.8}$$

U_A is distinguished from U_B, even though they have common parameters α and a, because initial stock $(\overline{X}_A, \overline{Y}_A)$ differs from $(\overline{X}_B, \overline{Y}_B)$. In this situation, variables are the amounts exchanged, x and y, or consumable quantities after exchanges, X_A, Y_A, X_B, and Y_B.

The conditions for exchange shown by Edgeworth are that transactors exchange beavers, x, and deer, y, without causing losses on either side. This means, for example from the viewpoint of A, that A makes an exchange contract with B by which to exchange x of beavers with y of deer in order to maximize A's utility without reducing B's utility. This problem of constrained maximization may be expressed in the form of maximizing objective function φ in the following equation by choosing appropriate x and y

$$\varphi = U_A(x, y) - \lambda\,[U_B(x, y) - \overline{\overline{U}}_B]\ , \tag{2.9}$$

where $\bar{\bar{U}}_B$ is B's utility which is given, and λ is Lagrangian multiplier. Setting $\partial\varphi/\partial x = 0$, $\partial\varphi/\partial y = 0$, we get from equation (2.9) the following relations $\partial U_A/\partial x = \lambda \cdot \partial U_B/\partial x$, $\partial U_A/\partial x = \lambda \cdot \partial U_B/\partial y$. Therefore, equilibrium condition is written by

$$\text{(2.10)} \qquad \frac{\partial U_A}{\partial x} / \frac{\partial U_A}{\partial y} = \frac{\partial U_B}{\partial x} / \frac{\partial U_B}{\partial y}$$

Marginal utilities of X (beavers) and Y (deer) for A and B are given specifically by equation (2.8) as

Marginal utilities of beavers for A and B are;

$$\frac{\partial U_A}{\partial x} = \frac{-\alpha_x}{a_x + \bar{X}_A - x} \quad \text{and} \quad \frac{\partial U_B}{\partial x} = \frac{\alpha_x}{a_x + \bar{X}_B + x}.$$

Marginal utilities of deer for A and B are;

$$\frac{\partial U_A}{\partial y} = \frac{\alpha_y}{a_y + \bar{Y}_A + y} \quad \text{and} \quad \frac{\partial U_B}{\partial y} = \frac{-\alpha_y}{a_y + \bar{Y}_B - y}.$$

Thus, we may write

$$\frac{\partial U_A}{\partial x} / \frac{\partial U_A}{\partial y} = \frac{-\alpha_x}{\alpha_y} \cdot \frac{a_y + \bar{Y}_A + y}{a_x + \bar{X}_A - x}, \quad \frac{\partial U_B}{\partial x} / \frac{\partial U_B}{\partial y} = \frac{\alpha_x}{-\alpha_y} \cdot \frac{a_y + \bar{Y}_B - y}{a_x + \bar{X}_B + x}.$$

Therefore, the specific form of equilibrium condition for exchange is given by

$$\text{(2.10)}' \qquad \frac{a_y + \bar{Y}_A + y}{a_x + \bar{X}_A - x} = \frac{a_y + \bar{Y}_B - y}{a_x + \bar{X}_B + x},$$

which is the equality between ratio of marginal utilities of beavers and deer for A and B respectively at the time of exchange. This equation represents the locus of tangency points between indifference curves of A and B, namely, the equation for the Edgeworth's contract curve.

Rearranging equation (2.10)′, we may get a contract equation of the form

$$y = \frac{(\bar{X}_A \bar{Y}_B - \bar{X}_B \bar{Y}_A) - a_x(\bar{Y}_A - \bar{Y}_B) + a_y(\bar{X}_A - \bar{X}_B) - (2a_y + \bar{Y}_A + \bar{Y}_B)x}{2a_x + \bar{X}_A + \bar{X}_B}.$$

Since hunters A and B have the common preference parameters

$$\alpha_x = 0.4, \qquad \alpha_y = 0.6, \qquad a_x = -20, \qquad a_y = -20$$

and initial stocks of beavers, X, and deer, Y, for A and B are given by

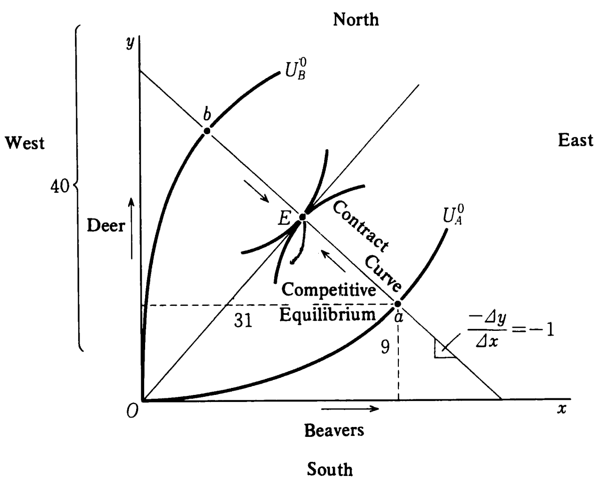
$$\begin{array}{lll} \text{Beavers} & \left.\begin{array}{l} \bar{X}_A = 70 \\ \bar{Y}_A = 30 \end{array}\right\} \text{for A,} & \left.\begin{array}{l} \bar{X}_B = 30 \\ \bar{Y}_B = 70 \end{array}\right\} \text{for B,} \\ \text{Deer} & & \end{array}$$

we can get, by substituting these values into equation (2.11), the specific form of the contract curve which is given by

$$(2.12) \qquad\qquad y = 40 - x.$$

This relationship may be illustrated by Figure 2.2.

Figure 2.2 AN ILLUSTRATION OF A SPECIFIC CONTRACT CURVE



Taking the anti-log of utility indicators for A and B (2.8) will not alter the form of marginal rate of substitution expressed by equation (2.10)′. We may therefore compute utility indicators by the following formula

$$U_A = (50 - x)^{0.4}(10 + y)^{0.6} \qquad \text{for A,}$$

$$U_B = (10 + x)^{0.4}(50 - y)^{0.6} \qquad \text{for B.}$$

According to these equations, the utility indicator for A before exchange, namely $x=0$ and $y=0$, is $U_A^0=19.03$, and for B is $U_B^0=26.25$. In Figure 2.2, U_A^0 and U_B^0 represent respectively indifference curves of A and B which pass through the origin 0 of transaction. Let the southeast point where U_A^0 intersects the contract curve be a and the northwest point where U_B^0 intersects the contract curve be b. The distance between a and b is the region in which exchanges can be made. According to our numerical example, the coordinates of southeast point a are $x_a=31$ and $y_a=9$. This implies that A would reject exchanges by which he has to give more than 31 beavers in return to less than 9 deer since such exchanges will reduce his level of satisfaction to a lower level than his initial situation. Similarly, the coordinates of northwest point b are $x_b=16.3$ and $y_b=23.7$ which prescribe B's initial situation before exchange.

In the case of exchange between barbarians of types A and B, it would be more advantageous for A if the contract point is closer to point b and more advantageous for B if the contract point is closer to a. However, as stated by Edgeworth, in the case of one-to-one bargaining it is impossible to determine specifically at which point between a and b the contract is settled. It is known, however, that the region of indeterminateness would be compressed somewhere between a and b with a help of supplementary contract if one more of A and B join to form a two-to-two bargaining. Edgeworth has proved that in the case of n-persons-to-n-persons bargaining the greater the number n the narrower the region of indeterminateness on the contract curve, and eventually when the number n becomes infinitly large the region of indeterminateness will disappear leaving only one point E between a and b, which is the point of perfectly competitive equilibrium.

Since utility U is constant on each indifference curve, the total differentiation of $U(x,y)$ with respect to x and y gives 0. To put it formally

$$(2.14) \qquad \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0 \qquad \text{therefore } -\frac{\partial U}{\partial x} \Big/ \frac{\partial U}{\partial y} = \frac{dy}{dx} .$$

Since dy/dx indicates the slope of tangency of the indifference curve at point (x,y), equation (2.14) implies the conformity between the slope of tangency and the marginal rate of substitution of X and Y at a certain point of indifference curve. Since a specific form of the ratio of marginal utilities, for example for A, may be given by

$$(2.15) \qquad -\frac{\partial U_A}{\partial x} \Big/ \frac{\partial U_A}{\partial y} = \frac{\alpha_x}{\alpha_y} \cdot \frac{a_y + \bar{Y}_A + y}{a_x + \bar{X}_A - x} = \frac{0.4}{0.6} \cdot \frac{-20+30+y}{-20+70-x}$$

$$= \frac{2}{3} \cdot \frac{10+y}{50-x} = \frac{20+2y}{150-3x} ,$$

equation (2.14) will be written in this case as

$$(2.14)' \qquad \frac{dy}{dx} = \frac{20 + 2y}{150 - 3x} ,$$

which expresses the slope of tangency of A's indifference curve evaluated at point (x,y). As is well known since the time of Edgeworth, at perfectly competitive equilibrium point E the slope of price line or exchange rate y/x conforms with the slope of tangency dy/dx. Therefore, according to equation (2.14)′ we have at point E

$$(2.16) \qquad \frac{y}{x} = \frac{20 + 2y}{150 - 3x} .$$

Since perfectly competitive equilibrium point E is on the contract curve expressed by equation (2.12), we can determine the co-ordinates of point E which satisfy the conditions described above by means of solving simultaneously equations (2.16) and (2.12). The co-ordinates of point E thus determined are now given by

$$x^* = 24 \quad \text{and} \quad y^* = 16.$$

In other words, at perfectly competitive equilibrium point E, hunters A and B exchange 24 beavers for 16 deer. After this exchange Mr.A will enjoy utility $U_A^* = 26$ and Mr.B will enjoy utility $U_B^* = 34$, which are higher than the utility indicators at the initial point, $U_A^0 = 19$ and $U_B^0 = 26$, implying that the degree of satisfaction increased for both bargainers through this exchange.

2.3 Competitive Market and the Division of Labor

So far, we have examined the case in which barbarians, who had been living independently with each other, happened to begin exchanges of their captured animals. Since hunters of type A are good at capturing beavers while the type B are good at hunting deer, the optimal numbers of beavers and deer they hunt independently are different between A and B even though their consumption preferences are the same. Mr. A hunts 70 beavers and 30 deer during a year while Mr.B hunts 30 beavers and 70 deer for the same period. We have discussed the case in which Messrs A and B exchange beavers and deer at this initial situation. Let us call this case as "Case I."

Assuming that the utility indicator is of the form

$$(2.13)' \qquad U = (-20 + X)^{0.4} (-20 + Y)^{0.6} ,$$

we have shown earlier that utility indicators prior to exchange are $U_A = 19$ for Mr. A and $U_B = 26$ for Mr. B. In a competitive market in which many barbarians of types A and B exchange with each other, a A-type barbarian gives 24 beavers to a B-type in return for 16 deer. After the exchange, Mr. A can consume 46 beavers and 46 deer, and accordingly his utility indicator increases from 19 to 26. Similarly, Mr.B can now consume 54 beavers and 54 deer, and his utility indicator increased from 26 to 34. In this case, the sum of consumable amounts of A and B equals the total number of animals captured by them, namely 100 beavers and 100 deer. Let us classify this case as "Case II."

In the case of living alone without making exchangs with other people, it would be rather disadvantageous to chase only beavers even though he is good at capturing beavers or only deer even though he is good at deer hunting. However, the situation may well change if he goes out for hunting with an anticipation that he will exchange his animals with those of others after the hunting.

If A specializes in capturing only beavers he would be able to get 295 beavers during 300 working days of a year since he needs in average only 1.017 days to get one beaver. Similarly, B could get 170 deer during 300 days if he tried to hunt only deer since it takes only 1.765 days for him to hunt one deer. In other words, if A and B specialize, respectively, in hunting one type of animal at which he is good, they can hunt 295 beavers and 170 deer in total. Since these amounts are much greater than 100 beavers and 100 deer of the previous case, the degree of satisfaction would certainly be greater than the previous case as long as these animals are properly distributed between A and B.

Let us suppose that both A and B have to hunt for some reason at least some minimum amounts of both kinds of animals, say, 21 deer for A and 21 beavers for B. Using the rest of hunting days, A can capture 137 beavers and B can hunt 100 deer. Thus the total number of beavers will be 158, and of deer will be 121. Even in this case, the numbers of captured animals are greater than those in Case II. However, the utility indicator of A with 137 beavers and 21 deer is only 6.7, and that of B with 21 beavers and 100 deer is only 13.9. These levels of utilities are much lower than $U_A = 19$ and $U_B = 26$, which were attained when both A and B independently pursued optimal hunting. This is an eloquent example of the disparity between the quantity and the degree of satisfaction obtained from it.

Substituting A's initial stocks ($\bar{X}_A = 137$, $\bar{Y}_A = 21$) and B's initial stocks ($\bar{X}_B = 21$, $\bar{Y}_B = 100$) into equation (2.8), we can specify equation (2.11) of the contract curve as

$$(2.12)' \qquad y = 79.3 - 0.686x \text{ and consequently } \frac{y}{x} = \frac{79.3 - 0.686x}{x} \; .$$

Moreover, substituting the above numbers into the ratio of marginal utilities, we may obtain for example for the case of Mr. A

$$(2.15)' \qquad \frac{-\partial U_A}{\partial x} \Big/ \frac{\partial U_A}{\partial y} = \frac{a_y + \bar{Y}_A + y}{a_x + \bar{X}_A - x} = \frac{0.4}{0.6} \cdot \frac{-20 + 21 + y}{-20 + 137 - x}$$

$$= \frac{2}{3} \cdot \frac{1 + y}{117 - x} = \frac{2 + 2y}{351 - 3x} \; .$$

As we have seen earlier, since this marginal rate of substitution conforms with the slope of tangency of indifference curves, we have the relation

$$(2.14)'' \qquad \frac{dy}{dx} = \frac{2 + 2y}{351 - 3x} \; .$$

At perfectly competitive equilibrium point E, the price line becomes the common tangency line to indifference curves of both A and B, and thus the exchange rate equals the slope of tangency. That is,

$$\frac{dy}{dx} = \frac{y}{x} \; .$$

Substituting equations (2.12)′ and (2.14)″ into this equality, we can determine co-ordinates of the perfectly competitive equilibrium point on the new contract curve (2.12)′, namely,

$$x^* = 69 \quad \text{and} \quad y^* = 32.$$

In other words, at the point of perfectly competitive equilibrium in this new market, A gives 69 beavers to B in return for 32 deer. Consequently, consumable amounts after exchange for A are 68 beavers and 53 deer which provide $U_A^* = 38$, and for B are 90 beavers and 68 deer which give $U_B^* = 56$. Let us classify this case as "Case III."

We may remind ourselves of the consequences of competitive exchange without division of labor (Case II): consumable amounts for A are $X_A=46$ and $Y_A=54$ and his utility indicator is $U_A^*=26$, and consumable amounts for B are $X_B=54$ and $Y_B=54$ and his utility indicator $U_B^*=34$. In comparison with these results, the results of exchange with division of labor (Case III), namely, $X_A=68$, $Y_A=53$, $U_A^*=38$ and $X_B=90$, $Y_B=68$, $U_B^*=56$ are clearly much more advantageous for both A and B. This comparison eloquently illustrates the point that the merits of division of labor *a la* Smith are undeniable when examined by Neo-Classical analysis.

Even though individuals are engaged with production independently from each other, exchanges in a competitive market will surely increase their economic welfare. The level of economic welfare would be increased much more greatly if from the beginning production were carried out with specialization and division of labor anticipating subsequent exchanges in the market and also the products were distributed properly through the working of competitive markets.

If complete specialization of work were adopted and A chases only beavers and captures 295 of them and B runs after only deer and hunts 170 of them, then the total number of captured animals would be the greatest. Let us suppose that a great chieftain of a large tribe which consists of a large number of barbarians of types A and B, being aware of the merits of division of labor, decided to order the A-type to chase only beavers and the B-type to hunt only deer (Case IV).

However, we have to note that if A captures only beavers and B hunts only deer the initial position prior to exchange ($\bar{X}_A=295$, $\bar{Y}_A=0$; $\bar{X}_B=0$, $\bar{Y}_B=170$) would fall into the γ zone of ***GEBD*** of which we have discussed in Chapter I. Since we assume that the specific form of marginal utility is given by

$$\frac{\partial U_A}{\partial y} = \frac{\alpha_y}{a_y + \bar{Y}_A + y}, \quad \frac{\partial U_B}{\partial x} = \frac{\alpha_x}{a_x + \bar{X}_B + x},$$

where $\alpha_x=0.4$, $\alpha_y=0.6$, $a_x=-20$, $a_y=-20$, the marginal utility of deer for A and that of beavers for B at the initial point ($x=0$, $y=0$) of Case IV will be negative, which is theoretically meaningless. In other words, there exist no indifference curves for both A and B which pass through the initial point of exchange. This is what we mean by saying that the initial position of Case IV falls in the γ zone, which implies that there exists no market in which exchanges can take place meaningfully.

Let us then suppose that the chieftain transfers 21 of 295 beavers captured by A to B and in return transfers 21 of 170 deer hunt by B to A. The initial position in this case is given by

$$\bar{X}_A = 274, \quad \bar{Y}_A = 21; \quad \bar{X}_B = 21, \quad \bar{Y}_B = 149.$$

The initial position in this case falls in the neigborhood of the southeast corner of the α zone of ***GEBD***, where indifference curves of A and B corresponding respectively to one of the lowest levels of their utilities intersect.

The equation of the contract curve in this case is given by

$$y = 129 - 0.51x,$$

and the equation of common tangency line to indifference curves of both A and B at the point of perfectly competitive equilibrium E is written as

$$\frac{dy}{dx} = -\frac{\partial U_A}{\partial x} / \frac{\partial U_A}{\partial y} = \frac{2 + 2y}{762 - 3x}$$

Solving these equations simultaneously, we can determine the amounts exchanged at a perfectly competitive equilibrium, that is

$$x^* = 151 \quad \text{and} \quad y^* = 52.$$

This means that each of A gives 151 beavers to each of B in return for 52 deer. Therefore, the consumable amounts and utilities in this case are:

$$X_A^* = 123, \quad Y_A^* = 73, \quad U_A^* = 69; \quad X_B^* = 172, \quad Y_B^* = 97, \quad U_B^* = 101.$$

It is interesting to compare these results with results obtained in Case III. Case III is the case of quasi division of labor in which individuals A and B specialize in their relatively more productive activities after cautiously securing 21 beavers or deer, an amount slightly more than the minimum critical level. The results obtained in Case IV indicate that the degree of satisfaction is greater for both A and B than the results obtained in Case III. This example clearly implies that the level of social and economic welfare will be greater in the case in which complete specialization or division of labor in production and free exchanges in distribution are combined under the assumption that the government guarantees to provide at least the critical minimum amount of beavers or deer than in the case of quasi division of labor in which each individual tries to secure the critical minimum amounts of necessitites on his own.

Case IV may be said to be a typical example of "compensated equilibrium" which we defined earlier.

2.4 The Number of Persons Taking Part in Market Competition and the Convergence Toward a Competitive Equilibrium within the α Zone

The concept of atomistic competition is based on a mathematical proof that a perfectly competitive equilibrium will hold when the number of each of the two groups of persons who participate to market exchanges approaches infinity. Edgeworth has shown, that the locus of tangency points of indifference curves of Mr. A and Mr. B constitutes the contract curve, and proved that a contract made at any point on the contract curve will converge to the perfectly competitive contract point through repetition of recontracts so long as there are plural numbers of Messrs. A and B.

Let us assume for simplicity that the utility functions of Mr. A and Mr. B are identical and express it by U. Denoting Mr. A's marginal utility with respect to good X by $U_{X,A}$ and Mr. B's marginal utility with respect to good Y by $U_{Y,B}$, etc., the equation for the contract curve can be expressed as

$$(2.17) \qquad \frac{U_{X,A}(x,y)}{U_{Y,A}(x,y)} = \frac{U_{X,B}(x,y)}{U_{Y,B}(x,y)} \qquad \text{Contract Relation.}$$

where x and y are the quantities to be exchanged.

In cases where exchanges are made between plural number of contractors such as between two of A and B, three of A and three of B, ... or generally between n persons of A and n persons of B, and also where the first contract happened to be made in favor of Mr. B, Edgeworth has pointed to the fact that it would be even more advantageous for $(n-1)$ persons of Mr. B to make recontracts with n persons of Mr. A eliminating one of Messrs. B. The equilibrium condition for recontract is shown by

$$(2.18) \qquad \frac{U_{X,B}(x',y')}{U_{Y,B}(x',y')} = \frac{U_{X,A}\left(\frac{n-1}{n}x', \frac{n-1}{n}y'\right)}{U_{Y,A}\left(\frac{n-1}{n}x', \frac{n-1}{n}y'\right)} \qquad \text{Supplementary Contract Relation.}$$

Since the utility of the Mr. B who was eliminated would otherwise be lowered to the level at the initial point, he would propose to make a new contract with one of Messrs. A at a point on the contract curve where the relative price ratio y/x is more favorable to Mr. A than was the previous point of contract.

Edgeworth has demonstrated that the contract point will approach a perfectly competitive market equilibrium through the repetition of contract → recontract → new contract → recontract ... and so forth, and that the convergence will be accomplished when the number of contractors n approaches infinity. Since it is proved generally that the process of convergence will be terminated before a convergence is reached at the perfectly competitive equilibrium point in cases where n is a small number such as two or three, it appears as though the condition $n \to \infty$ is indispensable for a competitive equilibrium to hold. It is this that gives a peculiar image of "atomistic competition." However, one will get quite a different impression when the utility indicator is specified with an empirically meaningful mathematical form.

Allowing for the notion, put forth initially by Jevons, that the marginal utility of a necessity grows infinitely large as its quantity approaches the minimum critical amount, and for the requirement that the function fits the actually observed data reasonably well, let us specify the following functional form.

(2.19) $$U_X = \frac{\alpha_x}{a_x + X}, \qquad U_Y = \frac{\alpha_y}{a_y + Y}.$$

As a numerical example for the purpose of illustration, let us consider the case in which Mr. A's initial holding is ($\bar{X}_A = 70$, $\bar{Y}_A = 30$) and Mr. B's initial holding is ($\bar{X}_B = 30$, $\bar{Y}_B = 70$).

We may write a utility indicator corresponding to (2.19) also as

(2.20) $$U = (a_x + X)^{\alpha_x}(a_y + Y)^{\alpha_y}.$$

Needless to say, any monotonously increasing function of U can be a utility indicator. Suppose now that the utility indicator is expressed specifically as

(2.20)′ $$U = (-20 + X)^{0.4}(-20 + Y)^{0.6}.$$

Since the total quantity may be decomposed into the initial holding and the exchanged quantity as ($X_A = \bar{X}_A - x$, $Y_A = \bar{Y}_A + y$) and ($X_B = \bar{X}_B + x$, $Y_B = \bar{Y}_B - y$),

we have

(2.21) $$U_A = (50 - x)^{0.4}(10 + y)^{0.6} \qquad \text{for Mr. } A\text{, and}$$

$$U_B = (10 + x)^{0.4}(50 - y)^{0.6} \qquad \text{for Mr. } B.$$

When $x = 0$, $y = 0$, $U^0{}_A = 19$ and $U^0{}_B = 26.3$ hold.

Substituting numerical forms $U_{X,A}$, ... $U_{Y,B}$ derived from this into the contract relation (2.17) and the supplementary contract-relation (2.18) presented earlier, we will obtain

(2.22) $$y = 40 - x, \qquad \text{the Contract Relation}$$

(2.23) $$y' = \frac{240n - (6n - 5)x'}{6n - 1} \qquad \text{the Supplementary Contract Relation.}$$

Since the quantities to be exchanged at a contract (x, y) and at the succeeding recontract (x', y') are related by the condition that the relative prices must be the same between the two cases, i.e. $y/x = y'/x'$, we have

$$(2.24) \qquad x' = \frac{60nx}{60n - 10 - x}, \qquad y' = \frac{60n(40 - x)}{60n - 10 - x}.$$

At the point of perfectly competitive equilibrium where $n \rightarrow \infty$, we have $x = x'$ any $y = y'$, and therefore, there would remain no room for further recontracts. This condition is filfiled at point E ($x^* = 24, y^* = 16$).

Suppose that the initial contract was made at the end point a ($x=31$, $y=9$) on the contract curve at the southeast corner of the Box-Diagram in Figure 2.2. This is the position most disadvantageous for Mr. A ($U_A = 19$) and most advantageous for Mr. B ($U_B = 41$). At this point, there remains room for $(n-1)$ persons of Mr. B to propose to n persons of Mr. A to have recontracts. For such a proposal to be profitable for each of Mr. B, the following condition has to hold: $U_B(x, y) < U_B(x', y')$. In view of this condition, we can see, in the case of $n=2$, that the contract point can move as far as $x = 26$, $y = 4$ through the repetitive process of contract $\rightarrow$ recontract $\rightarrow$ new contract and so forth.

Since the position of the perfectly competitive equilibrium point E is ($x^* = 24$, $y^* = 16$), the fact that the contract point shifts from the position of the initial contract ($x=31$, $y=9$) to the neigborhood of point ($x=26$, $y=14$) implies that the contract point shifts as much as five sevenths of the distance from the initial point a to the point of perfectly competitive equilibrium E. This differs sharply from the conventional image of atomistic competition when it is interpreted symbolically in abstract terms and $n \rightarrow \infty$. The conventional image of atomistic competition connotes that what could be attained by a 2-persons-to-2-persons competition would be far from the perfectly competitive equilibrium. However, it should be born in mind that the above numerical example indicates that even a competition with $n=2$ could compress the range of indeterminateness along the contract curve down to two seventh of the original extent.

Figure 2.3 and Table 2.1 show changes in the competitive region in this numerical example with increases in the number of contractors n from 2 to 3, 3 to 4, and so on.

The utility indicator $U^0{}_A=19$ indicates the degree of satisfaction of A prior to exchange and $U^0{}_B=26.3$ indicates that of B. In other words, these utility indicators indicate respectively the degrees of satisfaction of A and B when they are mutually isolated and independent. In contrast, their levels of utilities when a perfectly competitive market is established with many A's

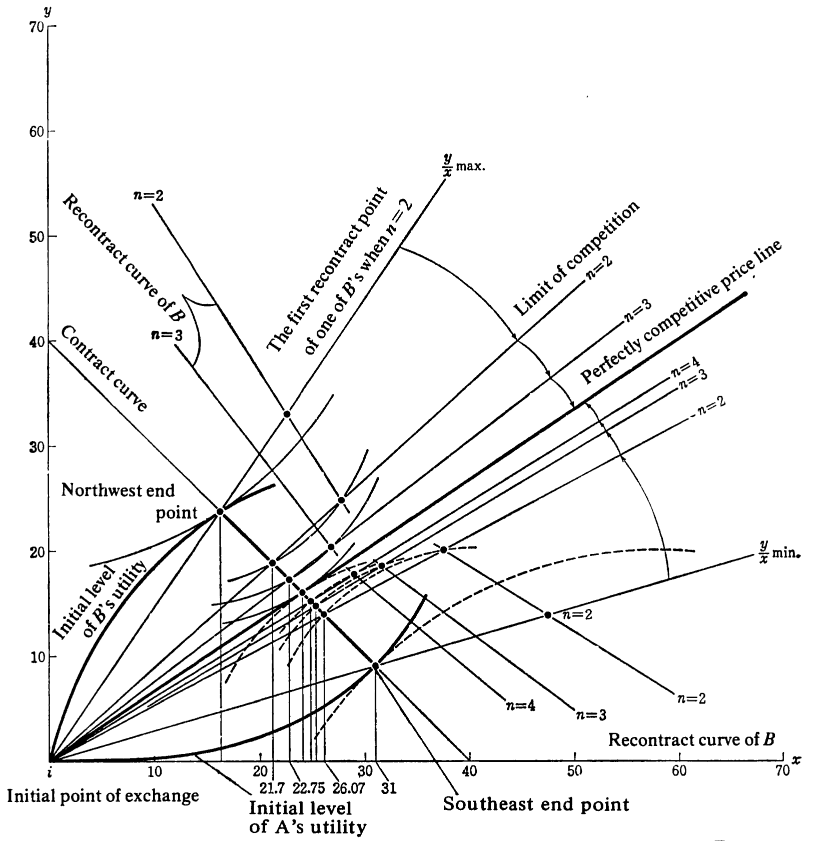


Figure 2.3 AN ILLUSTRATION OF CONVERGENCE TO A PERFECTLY COMPETITIVE EQUELIBRIUM WITH AN INCREASE IN THE NUMBER OF COMPETITORS

and B's taking part in competition are $U_A^* = 26$ for A and $U_B^* = 34$ for B. This implies that the degrees of satisfaction of both sides have been improved by exchanges in a competitive market. This is a concrete illustration of the merit of a competitive market advocated by Adam Smith that economic welfare of each person will be improved by exchanges in a free competitive market.

It is suggested from Edgeworth's theorizing that the greater the number of competitors in the market the better the result of market competition would be. However, the relationship between increases in the number of competitors and the extent of indeterminateness in the contract will remain unknown so long as the indifference maps of the competitors are characterized only in such general terms as being "downward sloping" and "convex to the origin." Under this highly general notion, one might think that the market could perhaps be fairly competitive if there exist, say, 5 or 10 competitors on each side. But at the same time he might also be skeptical as to how competitive the market is when he is told that perfectly competitive equilibrium can hold only when there is an infinite number of competitors on each side.

Table 2.1 indicates that the distance between the end point *a* and the point of perfectly competitive equilibrium *E* is 7 units of *X* and 7 units of *Y*. Since the position of the equilibrium point e_v is ($x=24.7$, $y=15.3$) when $n=5$, the distance between this equilibrium point and the perfectly competitive equilibrium point has already been compressed to 0.7 units of *X* or 0.7 units of *Y*. In other words, the range of indeterminateness of contract about the perfectly competitive equilibrium point *E* has been compressed to 1/10 of the original range at the southeast side and to 1/11 at the northwest side when the number of competitors has increased to 5 persons-to-5 persons. In either side the original range of indeterminateness which was more than 7 units has now been compressed down to less than one unit. Also, the utility indicator of *A*, which was originally $U^0{}_A=19$ at the end point *a* at the southeast corner, has now increased to $U^V{}_A=25.3$ at point e_v, which is not much different from the level $U_A^* = 26$ at point *E*. Similarly, *B*'s utility indicator at point e_v in the northwest side is not much different from the level at point *E*.

Table 2.1 INCREASES IN THE NUMBER OF CONTRACTORS AND COMPRESSION OF THE RANGE OF INDETERMINATENESS OF CONTRACT

Number of Contractors		South-east Side of E: Limits of Range of Indeterminateness of Contract $x>$	$y<$	Degree of Utility Indicator A U_A	B U_B		North-west Side of E: Limits of Range of Indeterminateness of Contract $x<$	$y>$	Degree of Utility Indicator A U_A	B U_B
End Point *a*	1	31	9	19	41	*b*	16.3	23.7	33.7	26.3
	2	26.1	13.9	23.9	36.1		21.7	18.3	28.3	31.7
	3	25.3	14.7	24.7	35.3		22.8	17.2	27.2	32.8
	4	24.8	15.2	25.2	34.8		23.0	17.0	27.0	33.0
	5	24.7	15.3	25.3	34.7		23.3	16.7	26.7	33.3
	10	24.19	15.81	25.81	34.19		23.65	16.35	26.35	33.65
	20	24.16	15.84	25.84	34.16		23.81	16.19	26.19	33.81
	100	24.08	15.92	25.92	34.08		23.98	16.02	26.02	33.98
Perfectly Competitive Equilibrium Point *E*	∞	24	16	26	34		24	16	26	34

In short, the above numerical example demonstrates that the number of competitors does not have to be very large for a market to be competitive both from the viewpoint of the range of indeterminateness of contract settlement and of the degree of satisfaction of contractors. Table 2.1 and Figure 2.3 suggest that a competitive market in its reasonable sense of the word will be established if there exist, say, about 10 competitors on each side. It would not be necessary for a market to have 100 or 1000 competitors for it to be really competitive in its practical sense.

Although the expression that $n \to \infty$ appears to suggest that the existence of innumerable atomistic competitors is indispensable for a market to be competitive, the market can be quite competitive even though the competitors are not literally "atomistic." It is the distinction whether there exist a single or a plural number of competitors on each side that is important in making the market incompetitive or competitive. The above example suggests that the market can be sufficiently competitive so long as a plural number of competitors compete with each other even though the number itself may be small. Implying this point, Adam Smith remarks, "If this capital is divided between two different grocers, their competition will tend to make both of them sell cheaper, than if it were in the hands of one only; and if it were divided among twenty, their competition would be just so much the greater, ..."[2]

It is true, as Smith has pointed out, that the monopolization of the market by collusion is more likely to take place as the number of competitors n gets smaller. Nevertheless, the Galbraithian view that the competitiveness of the market is lost simply for the reason that n is small seems to interpret the word "atomistic" too stubbornly. The naive belief in competitive markets embraced by economists since the days of the Classical School is not necessarily misled so long as the initial point of contract is within the α zone. It is not merely the number of competitors in the market but also the conditions by which the initial point of contract can be maintained within the α zone that economists have to worry about.[3]

Notes to Chapter 2

1. See Tsujimura and Sato (1964), Tsujimura and Kutsukake (1966), and Tsujimura (1968). See discussions in Chapter 10 of this volume.
2. Smith (1977), p. 342.
3. The simple model of two types of contractors who exchange two kinds of goods suffices for our immediate purpose here of explaining the basic set-up of Edgeworth's theory of competition. Readers, however, may be interested in more general cases. The Edgeworth's theory has been generalized appreciably by mathematical economists from the latter half of the 1940's to the 1960's. These more general and refined theoretical analyses indicate that the conclusion

obtained from the simple model will apply basically to more general cases with arbitrary numbers of contractors and types of goods. See for example Aumann (1961), Aumann and Peleg (1960), Debreu (1962) and (1963), Debreu and Scarf (1963), Gilies (1953), Neumann and Morgenstern (1947), Scarf (1962), Shapley (1955) and (1958), Shapley and Shubik (1961) and Shubik (1959).

Chapter 3
Keynesian Policies and the Price Theory

Demand control policies and the market mechanism are theoretically treated separately not only by Keynes himself but also by Keynesians and even monetarists.[1]. Keynes developed his theory of effective demand taking the degree of imperfection in market competition as given. Galbraith warns that the actual market has become much more imperfect than what Keynes might have suspected[2]. The market imperfection which Galbraith emphasizes is the long-term tendency for concentration in the sense of industrial organization and is not necessarily related to governmental demand control policies. Monetarists seem to be concerned with the fact that labor unions make excessive wage demands being encouraged by the government's permissive full employment policies. However, they have not clarified theoretically what this fact means in terms of their cherished theory of competitive market.

All three agree in that they regard the level of effective demand and the competitive mechanism of the market as being mutually independent. Was the original insight of Keynes about the effectiveness of the demand control policies, however, really totally unrelated to the competitive mechanism of the market? To consider this question in depth we need to get back to the perspective embraced by Adam Smith.

3.1 Demand-Supply Balance and Competition

It is necessary to pay special attention to the concept of unemployment when we try to reinterpret the implications of Keynesian theory in the perspective of Adam Smith's economics. It is questionable whether Adam Smith had in mind the concept of "unemployment" in its strict sense. This is because he maintains "Many workmen could not subsist a week, few could subsist a month, and scarce any a year without employment."[3]

How can a person remain unemployed who can not survive even for a week without a job? Unemployment was probably not much different from death from the viewpoint of not only Smith but also of Ricardo and Marx. In other words, workmen in those days could not afford to be unemployed. For the theories of Smith and Ricardo, therefore, the concept of unemployment was probably meaningless.

In view of this, it is irreconcilable that economics after Pigou and Keynes takes the possibility of unemployment for granted. Consideration of the issue of unemployment would probably have been unavoidable since there existed a mass of unemployed workers in the real world. However, has the question of how unemployment as such came into existence ever been seriously examined as a subject of economic theory?

The fact that a mass of unemployed workers actually existed implies that they were able to survive for more than a week. These workers in the twentieth century must have had either a stock of wealth to live on, or access to public aid. For employed members of an union to be able to offer mutual aid to unemployed fellow members, or for friends and relatives of unemployed workers to do the same thing, their earnings had to be well above the minimum subsistence level. The employed workers could not simply afford to do so if their wages were at the level of minimum subsistence as in the days of Smith and Ricardo.

The emergence of unemployment was only possible on the condition that some kind of unemployment compensation was available under protective labor legislations developed in spite of the indifferent attitude of the Classical and Neo-Classical economics, and that wage levels were generally high enough to maintain some savings. Since these conditions were absent in the days of Adam Smith, workmen in those days were unable to remain unemployed and were obliged to work at the level of subsistence wage.

And as Ricardo and Marx pointed out, wages often were unconsciously reduced even below the subsistence level without being aware of and hours were extended to cut into the minimum necessary time for rest.[4] Under these circumstances, many worker's families could not increase their population. The modern economics of development elegantly refers to this as the "Multhusian trap."[5] The Multhusian trap exists in underdeveloped economies in the modern era but not in advanced economies.

Viewed in this way, one may say that both Keynes focussing on the problem of unemployment and Smith on the subsistence wage were looking at essentially the same economic issue. Adam Smith pointed out, in Chapter VIII of his *Wealth of Nations*, that the wage level tends to be depressed to the subsistence level because of the unbalance in bargaining positions between the employers and the workmen. However, he also mentioned cases in which wage levels can be higher than the subsistence. It has to be borne in mind that an increase in the market wage above the subsistence level implied for Smith almost the same thing that the elimination of unemployment implied to Keynes.

The market wage level can increase above the subsistence level when the revenues of employers increase to a level more than sufficient to feed their own families and to pay for costs of raw materials for production and they can thus afford to employ more workers than before using the surplus revenue. Smith argues:

> "When in any country the demand for those who live by wages; laborers, journeymen, servants of every kind, is continually increasing; when every year furnishes employment for a greater number than had been employed the year before, the workmen have no occasion to combine in order to raise their wages. The scarcity of hands occasions a competition among masters, who bid against one another, in order to get workmen, and thus voluntarily break through the natural combination of masters not to raise wages."
>
> "The demand for those who live by wages, it is evident, cannot increase but in proportion to increase of the funds which are destined for the payment of wages. These funds are of two kinds; first, the revenue which is over and above what is necessary for the maintenance; and secondly, the stock which is over and above what is necessary for the employment of their masters."[6]

Adam Smith makes two points here: (1) an increase in the wage funds increases demand for labor, and (2) an increase in demand for labor leads to an increase in wages by altering the relative competitive positions of sellers and buyers in the labor market.

An increasing in the funds, in Smith's terminology, may be rephrased as an increase in the effective demand in terms of Keynesian theory. It should be noted here that Smith specially emphasizes that an increase in demand has the effect of modifying the balance of bargaining positions between the masters and the workmen in favor of the latter by altering the relative competitive conditions in the labor market.

Adam Smith was aware not only of the fact that sellers compete with each other in the face of excess supplies and buyers compete with each other in the face of excess demand but also of the fact that the intensity of the competition depends on the size of the demand-supply discrepancy. That is, the greater the excess supply the more intense the competition among suppliers and the greater the excess demand the more intense the competition among buyers. To view the same thing from a different angle, when there is a great amount of excess supply buyers do not have to compete with each other and when there is a great amount of excess demand suppliers do not need to compete with each other. Smith's remark that workmen did not need to combine for the purpose of raising wages in the phase of increasing demand implies that workmen, as sellers of labor services, do not have to compete with each other in the face of excess demand for them.

Smith noted, quoting an example of imported oranges, that competition among sellers becomes much more intense when perishable goods are in excess supply. Since labor as a kind of commodity is an even more perishable commodity than oranges in the sense that the labor unsold today can not be sold tomorrow, the competition among sellers of labor can be fierce when labor is in the condition of excess supply. Smith mentioned at various places in his book that a combination among workmen is called for as a means to counteract this tendency and that this kind of device does not easily succeed. The fact that workmen do not need to combine when demand for labor increases, therefore, implies that the strong pressure for competition among sellers can wither away due to excess demand.

In terms of the Neo-Classical economics, when effective demand in the Keynesian sense increases the demand curve for any commodity will shift up and to the right due to an increase in the purchasing power. Accordingly the point of intersection between the demand and supply curves will also shift up and to the right, and thereby the quantity and price of transaction will increase. In the Neo-Classical economics, however, the changes in the relative conditions of competition which accompany the shift in the point of equilibrium are not necessarily implied. In the case of Adam Smith, on the other hand, it is explained that an increase in demand for labor due to an increase in national wealth or funds will raise the level of wages above the level of minimum subsistence by altering the intensities of competition in the market.

3.2 The Situation of North America Viewed by Smith

Smith emphasized that the level of wages depends not on the volume of labor demand but rather on the rate of its increase. This point of his assertion may be seen when he writes, for example, that the demand for workers increases with an increase in the national wealth, that high wages are realized not by the size of the nation's wealth but rather by its growth, that North America is more prosperous than England, or that wages cannot be high if the nation is stagnant no matter how wealthy the stock of the nation may be.

If what the stagnation of wages at the level of minimum subsistence implies to Smith or Ricardo is more or less equivalent to what an increase in unemployment implies to Keynes, then an increase in the fund in the eyes of Smith corresponds to an increase in the effective demand in the view of Keynes. The increase in effective demand by means of fiscal policy suggested by Keynes simply implies an expansion of the frontiers of an economy in this way, as aptly explained by early Keynesian economists.

In America in the late 18th century, as seen by Adam Smith, a man who had just completed his apprenticeship was able to possess his own land. A hunter was able to get plenty of game easily in the forests. Under these circumstances it was not difficult for a man to set up on his own. Until the time when the frontiers were exhausted in America, it is not too unrealistic to say that people were able to enjoy conditions of life which were inconceivable by European standards: "In that original state of things, which precedes both the appropriation of land and the accumulation of stock, the whole produce of labour belongs to the labourer. He has neither landlord nor master to share with him."[7]

In this situation, people still had a freedom of choice between employing themselves or being employed by others and were not in the desperate situation in which they had to work for others at the lowest working conditions simply to earn a living. Rephrasing this in term of the Generalized Edgeworth's Box-Diagram (*GEBD*), while Scottish tenants or English workmen in their homeland were in the β zone those who emigrated to America were able to live in the α zone. Viewed in this way, Edgeworth's specification, focusing within the confines of the α zone, may be said to be suitable for the case of North America in its pioneering days.

After the Civil War and after the Western Movement had reached its end, the conditions in America had to change gradually away from what was described by Adam Smith as "the original state of things". It is nevertheless not surprising if the contemporary Americans still have the memories of the pioneering age in their minds and therefore a conviction that the market should be competitive in Edgeworth's sense.

At any rate, it must be clear from the foregoing discussion that so long as the initial position is located within the α zone of *GEBD*, the level of wages can be determined at a level higher than that of minimum subsistence through the Edgeworth-type market competition without necessarily the presence of excess-demand. The high wages in North America as observed by Smith must have been realized by strong excess-demand in addition to the fact that the initial position was already in the α zone.

With the stable climate, rich and extensive land endowed with ample underground resources, and constantly expanding frontiers, it is easy to see that the physical marginal productivity curve of labor in America was much higher than its European counterpart. Under these circumstances, owners of farm land, ranches, and mountains, independent workmen such as weavers or shoemakers, and merchants alike were able to secure revenues more than sufficient to pay of their own living costs and those of raw materials. Smith described the situation as:

> "When an independent workman ... has got more stock than what is sufficient to purchase the materials of his own work, and to maintain himself till he can despose of it, he naturally employes one or more journeymen with the surplus, in order to make a profit by their work. Increase this surplus, and he will naturally increase the number of his journeymen."[8]

Although it is easy to see, as Smith concluded, that the greater the wealth of the employer the greater the number of workers he is likely to employ, how can this reasonable conclusion be expressed in terms of the theory of market competition?

Edgeworth employed the example of a one-to-one transaction between Robinson Crusoe and Friday to make his explanation simple. He demonstrated in his analysis how market competition proceeds in a market where there is n number of such bargaining pairs. How then is the case analyzed in terms of Edgeworth's Box-Diagram where the contact is made between one employer and a plural number of workers rather than a one-to-one contract.

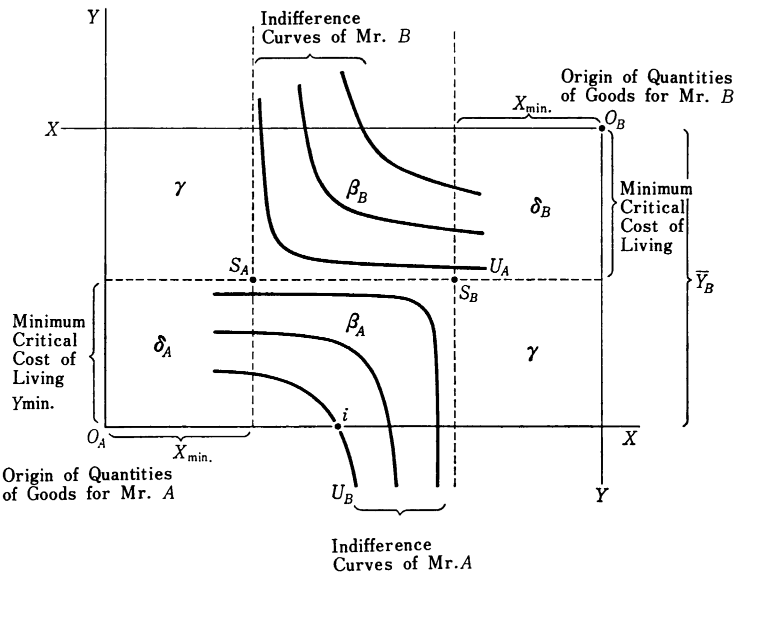
3.3 The Size of the Fund and the Market Mechanism

We have already seen in Chapter 1 that the contracted wage has to conform with the minimum subsistence level at the southwest corner of the α zone S_A if the initial position in the labor market is in the β zone on the A side in *GEBD*. Strictly speaking, however, there are cases in which even this kind of contract cannot hold. That is to say, since an indepndent workman can be a master only when he has accumulated some amount of wealth, with which he can afford to employ workers, he can not employ workers even at the lowest wage level if he does not have the stock to do so. This case may be analysed by the extreme example illustrated by Figure 3.1.

This figure resembles Figure 1.7 in that worker A's holding of daily necessities Y is zero and the initial position i is on the X axis of A. However, it differs from Figure 1.7 in that the master B's holding of Y is just twice as much as the critical minimum amount $Y_{\text{min.}}$ or, in other words, the height of the Edgeworth's Box is just 2 $Y_{\text{min.}}$. Since in this case the distance from the X axis of B to the X axis of A is 2 $Y_{\text{min.}}$, the $Y_{\text{min.}}$ line of worker A and the $Y_{\text{min.}}$ line of employer B both fall on the same position to join a single line S_A—S_B. In this situation, the indifference map of worker A is located above line S_A—S_B, and that of employer B is below line S_A—S_B. In other words, since the indifference maps of A and B do not intersect, there exists no α zone in this Box-Diagram.

In this situation, since even B's indifference curve with the lowest utility does not pass through S_A, the indifference curve U_B of higher utility which passes through the initial point i should not reach S_A. This implies that it is better off for employer B not to employ worker A at all since his utility would be lower even though he employed worker A at the lowest wage level equivalent to the minimum subsistence $Y_{\text{min.}}$. Worker A would have to starve to death in this situation unless B gives half of his holding of Y to A for

Figure 3.1 THE CASE IN WHICH THE HOLDING OF THE EMPLOYER (MR. B) IS ONLY AS MUCH AS THE MINIMUM SUBSISTENCE OF TWO PERSONS



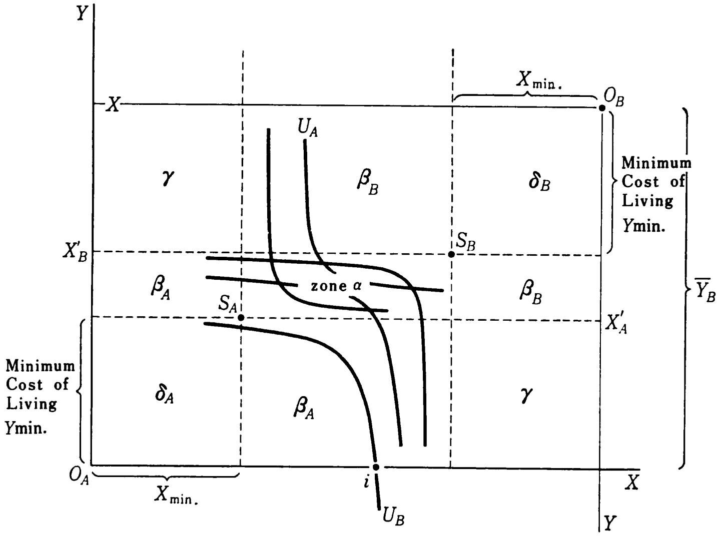
humanitarian reasons quite apart from economic rationality. At any rate, the market as such cannot exist in this situation.

In the case where B's holding of Y is greater than 2 $Y_{min.}$, the height of the Box increase and the X axis of B shifts upward. Since the position of S_B shifts upward accordingly, the horizontal line passing through S_B or the horizontal axis of B's indifference map will be lifted higher than point S_A. In this situation, there emerges in the Box the α zone in which the indifference maps of A and B intersect. Even in this situation, however, the vertical length of the α zone is quite small unless B's holding of Y exceeds greatly the amount 2 $Y_{min.}$, and therefore it may still happen that B's indifference curve U_B which passes through the initial point i fails to reach S_A, as seen in Figure 3.2.

It is suggested from this that in the case where the goods to be exchanged have certain critical minimum amounts, the market itself would not be established unless the vertical and/or horizontal length of the box reaches a certain size.

If B's holding of Y increases much more to make the vertical length of the Box sufficiently long, then B's indifference curve U_B which passes through the initial position i may be located above S_A as Figure 1.7 and exchanges will begin to take place, although in an imperfect form in which an employment contract is made at the level of minimum subsistence at point S_A. From that

Figure 3.2 THE CASE IN WHICH THE HOLDING OF THE EMPLOYER (MR. B) EXCEEDS THE MINIMUM SUBSISTENCE AMOUNT OF TWO PERSONS



moment the independent workman becomes the employer B and begins to employ worker A. How would the market look if the vertical length of the Box grew still longer?

B will start employing only after B's indifference curve U_B which passes through the initial position i shifts above S_A, however slightly. B's indifference curve U'_B which passes through S_A at the southwest corner of the α zone in this situation will be of higher utility than U_B and will be located below and to the left of U_B. In the case in which indifference curve U_B passes through a point only slightly above S_A, the extrapolation of price line i—S_A, which represents the subsistence wage rate, will soon cut across and get outside above and to the right of the B's indifference curve U'_B which passes through S_A. This implies that the employer B would be willing to employ only one worker A even though he could employ two workers at the subsistence wage level i—S_A.

It is easy to see that the slope of B's indifference curve U'_B which passes through S_A will vary depending on the vertical length of the Box or, in other words, the amount of B's holding, $\overline{Y}_B$. In this example, if good X represents hours, then the holding of hours of A and B are fixed respectively at the level of 24 hours a day or $\overline{X}_A$ and $\overline{X}_B$. If the vertical length of the Box grows large, then the amount of Y that B can consume at point S_A or $(\overline{Y}_B - Y_{\text{min.}})$ will increase. This implies an increase in B's holding of good Y, namely $\overline{Y}_B$, and consequently the longer the vertical length of the box the greater will be Y/X for B at point S_A. Therefore, B's marginal utility of good Y will be small relative to that of good X evaluated at point S_A, that is to say the marginal rate of substitution $\frac{\partial U}{\partial Y}/\frac{\partial U}{\partial X}$ will be small. Accordingly, the slope of B's indifference curve U'_B at point S_A will be steeper the longer is the vertical length of the Box because of the property of the indifference curve such that $-\frac{\partial U}{\partial X}/\frac{\partial U}{\partial Y} = \frac{dY}{dX}$.

When the vertical dimension of the Box is such that B's indifference curve U_B which passes through the initial point i passes barely above S_A, the slope of the indifference curve U'_B which passes through S_A is flat in the neighborhood of S_A. When the vertical length of the Box gets longer and the indifference curve U_B which passes through the initial point i passes well above S_A, then the indifference curve U'_B will have a steeper slope in the neighborhood of S_A. When the vertical length of the Box reaches a certain level due to a further increase in B's holding of fund $\overline{Y}_B$, the slope of indifference curve U'_B at S_A will be steeper than the slope of the price (wage) line i—S_A, and a part of the extrapolation of line i—S_A will pass below and to the left of the indifference curve U'_B. Since B's indifference curve U'_B is concave, the extrapolation of the price line i—S_A will pass first inside the curve U'_B and then get outside the curve beyond a certain point.

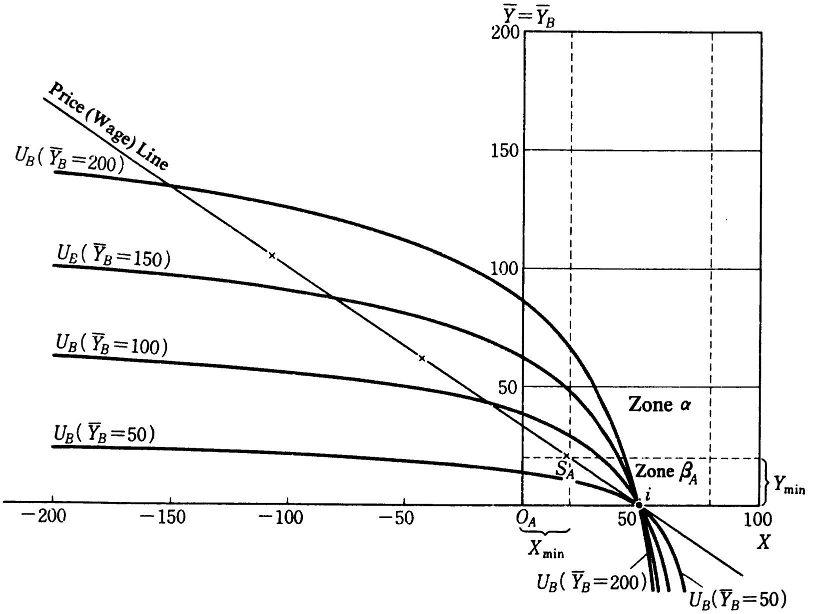
Figures 3.3 and 3.4 illustrate this situation with a concrete numerical example. These diagrams illustrate the relative locations of B's indifference curve U_B which passes through the initial point i, B's indifference curve U'_B which passes through S_A and the price line i—S_A for various alternative cases of different amounts of B's initial holding of Y. The specific form of the utility indicator function used for this example is

$$U = (-20 + X)^{0.4}(-20 + Y)^{0.6}$$

which was used earlier in explaining the example of convergence to the perfectly competitive market equilibrium. The initial conditions are assumed to be such that $\bar{X}_A = \bar{X}_B = 50$, $\bar{Y}_A = 0$. Alternative cases of B's initial holdings are: (1) $\bar{Y}_B = 50$, (2) $\bar{Y}_B = 100$, (3) $\bar{Y}_B = 150$, and (4) $\bar{Y}_B = 200$.

Figure 3.3 illustrates B's indifference curves U_B which pass through the initial point i for alternative vertical lengths of the Box ($\bar{Y} = \bar{Y}_B$); 50, 100,

Figure 3.3 THE VERTICAL LENGTH OF THE EDGEWORTH'S BOX ($\bar{Y} = \bar{Y}_B$) AND THE LOCATIONS OF THE EMPLOYER'S INDIFFERENCE CURVES WHICH PASS THROUGH THE INITIAL POINT i



150, and 200. When Y_B=50, as in case (1), since B's indifference curve U_B passes below S_A, B's utility at S_A is lower than that at the initial point i. In other words, the employer's utility would be lower if he were to employ a worker. B's holding of Y in this situation is 50, which is larger than the critical minimum amount by 30 units. This means that he can still keep a reserve of 10 after paying wages of 20 to an employee. Emplolying a worker is nevertheless disadvantageous in this case.

The amount of $\overline{Y}_B$ is slightly smaller than 75 when B's indifference curve U_B which passes through the initial point i just passes through S_A. That is, it is when the holding of the employer reaches 75 that the employer may profitably employ a worker. A holding of 74 is still short of the critical level above which the employer is willing to employ a worker. A holding of 74 is, however, 3.7 times as much as the minimum amount for subsistence, which is 20. In other words, even though the employer seems to have holdings much more than are necessary to maintain his life, he still would not try to employ a worker. Workman A, on the other hand, would not be able to earn a living unless job opportunities were opened.

This suggests that even in a case in which there existed a sufficient amount of resources to feed more than three persons in a society it may well happen that only one person, the employer, survives eventually due to the failure of market functions because the allocation of the resources was left entirely to the deficient market mechanism. Therefore, a planned allocation without relying on the market mechanism may be more desirable in this case. It is not unreasonable that the naive expectations for allocative functions of resources by the market mechanism based on the experiences of advanced economies are often met with disenchantment in developing economies where available resources are still limited.

However, as the vertical length of the Box increases form 100 to 150 and 200, as seen in Figure 3.3, the employer's indifference curve which passes through the initial point i will begin to pass above point S_A or the point of minimum subsistence of worker A. That is, the employer's incentive for employing a worker will grow stronger as the vertical length of the Box increases. This is because the employer's utility represented by the indifference curve which passes through S_A will be higher than that at the initial point i.

Figure 3.4 illustrates employer B's indifference curves U'_B which pass through the point of minimum subsistence of worker A corresponding to alternative cases of the vertical length of the Box ($\overline{Y} = \overline{Y}_B$).

Each of these curves has a higher level of utility indicator than the corresponding indifference curves U_B (which pass through the initial point i as shown in Figure 3.3) drawn for each of the alternative vertical lengths of the Box. The case in which $\overline{Y}_B$ = 50 is not drawn on the diagram on the ground that a transaction may not be carried out in this case because the utility will decline when a transaction is made.

As discussed earlier, when the initial position is in the β zone, at least the first contract will be made at S_A. This is because it is better for worker A to make a contract than not to make it no matter how poor the terms of the contract may be, and it is most advantageous for employer B to make a contract at S_A among the alternative contracts which are acceptable for A. The price (wage) line in this case will be expressed by line i—S_A, accordingly.

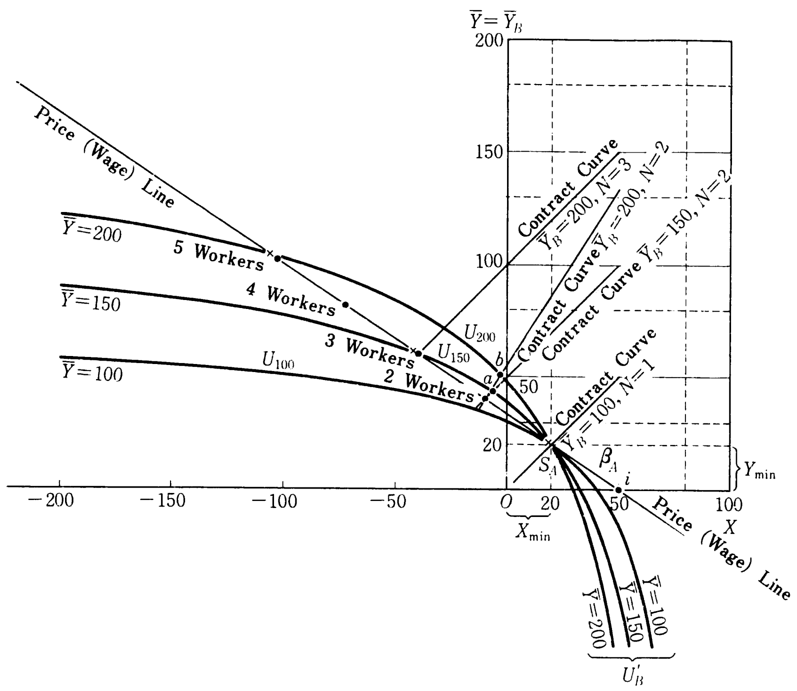
3.4 An Increase in the Fund and the Volume of Employment

In Figure 3.4, the question is whether the extrapolation of price line i—S_A beyond point S_A passes inside or outside of B's indifference curve U'_B. If it passed outside, then it would be disadvantageous for the employer to employ more than one worker. In our example, price line i—S_A happens to be nearly tangential to indifference curve U'_B at point S_A when the vertical length of the Box is $\bar{Y}=\bar{Y}_B=100$. Since an employment contract can be made when the vertical length of the box becomes greater than 75, as seen in the previous section, the extrapolation of the price line i—S_A will pass outside the indifference curve U'_B for the range in which $\bar{Y}_B$ is between 75 and 100. Within this range, the employer B is not willing to employ more than one worker even at the level of the subsistence wage.

As seen in Figure 3.4, when the vertical length of the Box increases to more than 100, say to 150 or 200, then part of the extrapolation of price line i—S_A to the left of S_A will pass inside (below and to the left) of indifference curve U'_B. Since the distance between i and S_A implies employment of one person at the minimum subsistence wage, we may mark on the extrapolation of line i-S_A the points of employment of the second person, third person and so on using this distance as a unit scale. The point of employment of the second person will be outside the indifference curve U'_B when the vertical length of the Box ($\bar{Y}=\bar{Y}_B$) is 100, while it remains inside of the curve when $\bar{Y}=150$ or 200. This means that, for each of the cases $\bar{Y}=150$ and 200, employer B's indifference curve which passes through the contract point of the second person has a higher utility indicator than his indifference curve which passes the contract point S_A of the first worker. In other words, it would be more advantageous for the employer to employ two workers than one at the minimum subsistence wage.

In case (3) where $\bar{Y}_B=150$, both the contract points of the second and the third person will stay inside indifference curve U'_B. But since the point of the second person is further in, it would be better for the employer to employ two workers instead of three. In case (4) where $\bar{Y}_B=200$, the contract points of the second to the fifth person will be inside the curve U'_B, which implies that it would still be more advantageous to employ five workers than to employ

Figure 3.4 THE VERTICAL LENGTH OF EDGEWORTH'S BOX AND THE LOCATIONS OF THE EMPLOYER'S INDIFFERENCE CURVES WHICH PASS THROUGH S_A



only one. But since the point of the third person is on the indifference curve of the highest utility indicator, it would be most advantageous to employ three workers in this case.

The examination of Figure 3.4 above thus suggests the possibility that a contract will be made between one employer B and one worker A in case (2) where $\bar{Y}_B=100$, one employer and two workers in case (3) where $\bar{Y}_B=150$, and one employer and three workers in case (4) where $\bar{Y}_B=200$. In other words, the volume of demand for labor increases as the initial holding $\bar{Y}_B$ of the employer increases.

If there were 10 employers, the demand for workers at the level of the minimum subsistence wage (equivalent to price line i—S_A) would be zero for case (1), 10 workers for case (2), 20 workers for case (3), and 30 workers for case (4). If there existed 15 workers in the labor market in this situation, all workers would be unemployed in case (1), 5 of them would be unemployed in case (2), 5 workers would be demanded in addition to the available workers in case (3), and 15 additional workers would be demanded in case (4). Therefore, even in case (2) where $\overline{Y}_B = 100$, workers would have to compete intensely with each other since 5 of them would have to be unemployed eventually. On the other hand, since there would be no competition among employers, there would be no possibility that wages would be bid up above the level of minimum subsistence.

In contrast, in case (3) where $\overline{Y}_B = 150$, there would be no competition among the workers since they would not lose job opportunities as long as demand is in excess of supply. The employers, on the other hand, would have to compete with each other, which would give rise to higher wage offers. Therefore, there emerges a possibility that the market wage rate will be raised higher than the rate of minimum subsistence wage, which is the slope of line i—S_A and is $\frac{y}{x} = \frac{20}{50-20} = \frac{2}{3}$ in our present example.

In the initial situation of our example, since price line i—S_A cuts across the α zone only at its corner on point S_A, the price line relates to employer B's indifference map to the left of point S_A while it does not relate at all to the indifference map of worker A. This situation will be unchanged so long as the wage rate is the rate of minimum subsitence. However, once the market wage rises above the level of minimum subsitence due to competition among employers, it is obvious that the price line will have to relate to the indifference map of worker A. This is because the price line now cuts across the α zone well above and to the right of S_A since the price line will turn up and to the right, centering at the initial point i.

When the price line passes the intersection of the indifference maps of both bargainers, then the concept of "*Pareto optimum*" in exchange theory becomes relevant, and consequently the commensurate equilibrium conditions need to be examined.

Unlike the case of a one-to-one contract, the equilibrium conditions for the case of a contract between one employer and a plural number N of workers may be expressed generally

$$\frac{U_{AX}(\frac{x}{N}, \frac{y}{N})}{U_{AY}(\frac{x}{N}, \frac{y}{N})} = \frac{U_{BX}(x, y)}{U_{BY}(x, y)},$$

where U_X and U_Y are respectively the marginal utility of good X and good Y. The left-hand side of the equation is the marginal rate of substitution between goods X and Y for each Mr. A, and the right-hand side is the marginal rate of substitution of X and Y for Mr. B.

Assuming the common preference function is of the type specified above for both A and B, we may obtain from the above equilibrium conditions a contract equation between one person of B and N persons of A which is:

$$y = \frac{(\bar{X}_A \bar{X}_B - \bar{X}_B \bar{Y}_A) - a_x(\bar{Y}_A - \bar{Y}_B) + a_y(\bar{X}_A - \bar{X}_B) - (\frac{N+1}{N} a_y + \bar{Y}_A + \frac{\bar{Y}_B}{N})x}{\frac{N+1}{N} a_x + \bar{X}_A + \frac{\bar{X}_B}{N}}.$$

This is a contract equation for B. The contract equation for A may be obtained by dividing both sides of this equation by N. In the case of a one-person-to-N-persons contract, it is required that the tangent to indifference curve U_B at the point where the price line intersects with this indifference curve and the corresponding tangent to indifference curve U_A should be parallel with each other instead of conforming together to make a single line as in the case of a one-to-one contract. Thus the contract equation in the one-to-N-persons case consists of a pair of equations of these parallel lines. Needless to say, this set of contract equations reduce to a single equation for a single contract curve when $N=1$.

Let us consider the case where N=2, or the case in which one employer employs two workers. Assume here that $a_x = a_y = 20$, $\bar{X}_A = \bar{X}_B = 50$, and $\bar{Y}_A = 0$. When the vertical length of the Box ($\bar{Y} = \bar{Y}_B$) is 150, the contract equation for employer B is:

$$y = 100 - x, \quad (\bar{Y}_B = 150, N = 2).$$

When the vertical length of the Box is 200, the contract equation for employer B is

$$y = 133.3 - 1.56x, \quad (\bar{Y}_B = 200, N = 2).$$

On the other hand, the contract equation for each of workers A will be

$$\frac{y}{2} = 50 - \frac{x}{2}, \quad (\bar{Y}_B = 150,\ N = 2),$$

$$\frac{y}{2} = 66.7 - 1.56\,\frac{x}{2}, \quad (\bar{Y}_B = 200,\ N = 2).$$

The points of intersection of these contract lines and the price line indicate the equilibrium amounts of transactions for A and B, respectively, corresponding to alternative vertical lengths of the Box. When the wage level equals the minimum subsistence, the intersection of the contract curve for B and the price line i-S_A passes the point of the 2 persons contract in Figure 3.4 both in the case of $\bar{Y}_B = 150$ and of $\bar{Y}_B = 200$.

If each of 10 employers tries to employ 2 workers, while there exist altogether only 15 workers, wages would be bid up in the process of competition and consequently the slope of the price line would grow steeper. Since the contract curve of employer B in this situation is sloping down and to the left, its intersection with the price line would shift up and to the right with an increase in wage rate. This implies that the working hours per worker would be shortened since the horizontal distance between the intersection point and the initial point i would be shorter. Therefore, wage rate increases due to a shift of the contract point together with a decrease in working hours per worker, and thus the working conditions are improved on both sides.

In the case where the vertical length of the Box ($\overline{Y} = \overline{Y}_B$) is 150, the contract point between an employer and two workers which has been pushed up and to the right due to an increase in wage rate will have to cut across employer B's indifference curve U'_B eventually at some point like a in Figure 3.4. Once the contract point shifts to the right or outside of indifference curve U'_B, then each of the employers will no longer compete with others in trying to employ two workers since the employer's utility associated with employing two workers at that wage rate will be lower than that of employing one worker. If, on the other hand, each employer employed only one worker, workers would then compete with each other by bidding down wages since five of them would have to be unemployed eventually. Therefore, the price line would not turn further up and to the right than the position of line i—a. If the price line happened to turn around further than line i—a, then it would be pushed back by the competitive pressure among the workers.

Thus, in the case where $\overline{Y}_B = 150$, the price line shifts from the position of i—S_A to i—a. Then, some employers employ one worker each and others employ two workers each, and when all 15 workers are employed a kind of market equilibrium may be reached. To summarize the process, when wages are at the level of minimum subsistence, each employer wants to employ two workers and they compete with each other. Consequently, wage rates are bid up to reach the position of line i—a where a market equilibrium is attained where the numbers of workers demanded and supplied are balanced. The equilibrium attained at point a may be regarded as a "full employment equilibrium" in a contemporary sense. This is because in contrast to the contract at point S_A, which reflects only the preference of employers, the contract attained at market equilibrium point a reflects not only the preference of employers but also of workers as well.

Similarly, when the vertical length of the Box reaches 200, the point of contract will shift along the employer B's contract curve further up and to the right to reach point b, where a full employment equilibrium is attained. The employer's indifference curve which passes through S_A in the case where $\overline{Y}_B = 200$ is located in a position above and to the right of its counterpart in the case where $\overline{Y}_B = 150$. Therefore, the equilibrium wage rate when the price line reaches the position i-b is higher and the equilibrium working hours are shorter than in the case where $\overline{Y}_B = 150$.

As can be seen from Figure 3.4, generally speaking, the greater the initial holding of the employer or in other words, the longer the vertical length of the Box, the total number of workers employable at the subsistence wage level will be greater, and consequently wages will be more likely to rise due to excess demand for the same number of available workers in the market. For instance, in our numerical example, when a full employment equilibrium is attained at point b, daily wage rate is 24 units and working hours are 26 units. These are much improved working conditions compared to the subsistence wage 20 and the longest possible working hours 30 which were obtained in the contract made at point S_A.

3.5 The Irreversibility of the Effect of Wage Changes on Employment

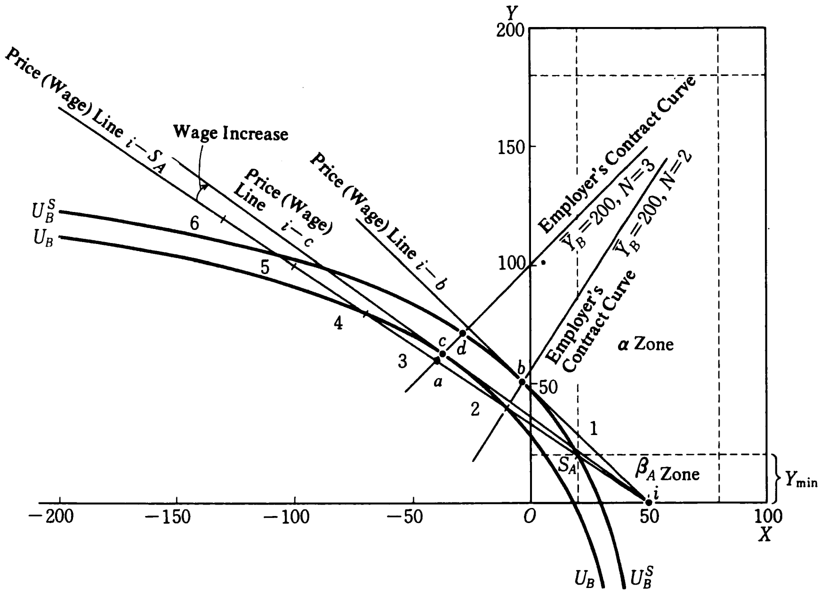
In order to ascertain the meaning of unemployment let us examine once again the situation in which the accumulated fund $\overline{Y}_B$ of each of the employers is 200. In the case where $\overline{Y}_B = 200$, the indifference curve U_B^s which passes through S_A may be illustrated as in Figure 3.5.

The points of employment for the second to fifth persons marked on the extrapolation of the price line i—S_A are located inside the indifference curve U_B^s. This means that it would be more advantageous for the employer to employ 2 to 5 workers than to employ only one when he can employ them at the level of minimum subsistence wage. To find out the most advantageous number of employees, let us draw an indifference curve of the employer B which passes through the point of the second person's employment. We will find then that the employment point of the third person will be inside the curve U_B, the point of the fourth person on the curve, and the point of the fifth person outside the curve. Therefore, we can see that employment of three workers would be the most advantageous for each employer.

Suppose there exist more than 30 workers, say 40 workers, for 10 employers. Each employer can in this case employ three workers without any difficulty or without competition among employers. The level of wages is at the minimum subsistence level and 10 workers remain unemployed in this case.

However, if there are 28 workers, then the demand of 30 persons would be in excess of the supply by 2 persons. Employers would then have to compete with each other and bid up wages if they still want to employ 3 workers each. Therefore the price line would turn up and to the right along the employer's

Figure 3.5 THE INDIFFERENCE CURVE WHICH PASSES THROUGH S_A AND THE VOLUME OF EMPLOYMENT WHEN THE ACCUMULATED WEALTH IS 200



contract curve as shown in Figure 3.5,

$$y = 150 - x, \quad (\bar{Y}_B = 200, \ N = 3).$$

The price line would reach point c where this contract curve and the employer's indifference curve U_B which passes through the employment point of the second person intersect. In other words, the price line would shift from the position i—S_A to i—c. If the contract points were to shift further up and to the right, it would no longer be advantageous for each employer to employ 3 workers. This is because an employer can enjoy a higher level of utility by employing 2 workers at lower wage rate than by employing 3 persons at this high wage rate.

However, if each employer decided to employ only 2 workers, 8 of the 28 workers would be unemployed since the aggregate demand would be only 20 persons. The unemployed workers then would have to compete fiercely for jobs since they would otherwise have to return to point i at which they would not be able to survive. We have already explained that this kind of competition which takes place when the initial position was in the β zone tends to become very severe or "cutthroat" competition, quite unlike the kind of competition which occurs when the initial position was in the α zone. At any rate, this kind of competition among workers can reduce wages down to the level of the subsistence minimum or even lower.

Even though wages may have increased because of competition among the employers who altogether demanded 30 workers while there existed only 28, the point of contract would not shift to the right of point c since workers would now compete with each other fiercely once the contract point shifts to the right of c. Since point c is on the same indifference curve on which the employment point of the second person at subsistence wage rate is located, it would be almost indifferent for each employer whether to employ 2 workers or 3 workers at a point between a c at the wage rate represented by a price line which locates between i—S_A and i—c. Some of the employers would employ 2 workers and others employ 3 workers each. All of 28 workers will be employed in this situation at a wage rate higher than the level of minimum subsistence. In fact, each employer could optimally employ even 4 workers if he wished to, as can be seen in Figure 3.5. This means that there remains still a potential pressure of excess demand, and accordingly the wage rate should rise nearly to the level represented by the price line i—c.

If on the other hand there were 18 workers altogether and each employer held 200 units of funds, wages would be bid up further since each employer may not be able to secure even 2 workers. If an employer wishes to employ 3 workers, the contract point would reach point d at which the contract curve of $N=3$ and the indifference curve $U_B{}^s$ which passes through S_A intersect and the wage rate would then be represented by the slope of i—d. However, he would probably cease to try to employ 3 workers and instead try to secure 2 workers. If he changes his mind this way, then he would proceed along the contract curve of $N=2$, which may be expressed as $y = 133.3 - 1.56x$, ($\bar{Y}_B = 200$, $N=2$). The competition among employers would stop when the contract reached point b at which this contract curve and the indifference curve $U_B{}^s$ intersect. As seen in Figure 3.5, the wage rate i—b is even higher than i—d. Since each employer would be indifferent at this point between employing 2 workers at wage rate i—b and 1 worker at wage rate i—S_A, each employer would employ either 1 or 2 at wage rate little less than i—b without any particular preference and eventually all 18 workers would be employed.

If each of the 10 employers has a greater amount of funds for example 300, 500, 1000 or even more, then the wage level for the 18 workers would accordingly be much higher and the working hours much shorter.

As we can see from this example, even for the same level of funds, say 200, the labor market situation would vary depending upon the total number of workers in the labor market: (1) if there exist more than 30 workers, only 30 workers would be employed at the subsistence wage level i—S_A, and the rest of the workers would be unemployed, (2) if there are 20 or more but less than 30 workers, all of them would be employed at the market wage rate which would eventually rise up nearly to the level *i—c,* and (3) if there are less than 20 workers, then the market wage rate would increase up to the slope little less than *i—b* and full employment would be attained. Case (1) would seem to correspond to the situation of the labor market depicted by Ricardo and Marx, and (2) and (3) seem to correspond to the situation of the American labor market as described by Adam Smith.

Now let us reconsider in terms of our example shown by Figure 3.5 the meaning of the problematical proposition of the school of thought referred to by Keynes as the "Classical" school concerning the emergence of unemployment. According to this school of thought the reason why unemployment exists is that workers are reluctant to accept lower real wages.

In case (1) where there existed, for example, 40 workers, we have ascertained that 10 workers would have been unemployed even at the level of minimum subsistence wage. To digress for a moment, when we compare point *c* and the employment point of the fourth person in Figure 3.5, we will see that these points are on the same indifference curve. Then, each employer would be indifferent between employment of 3 workers at the wage rate *i—c* on point *c* and employment of 4 workers at the wage rate $i—S_A$ on the point of the fourth person. Therefore, in the latter case there would seem to be no unemployment since the total employment would be 40. However, we have seen earlier that there existed unemployment of 10 workers at the level of minimum subsistence wage $i—S_A$. The reason for this unemployment was that when the price line was $i—S_A$ the indifference curve passing the employment point of the third person has higher utility than the curve passing the employment point of the fourth person on the same price line.

In view of this fact it seems more likely that even though workers accepted a reduction in wage rate from *i—c* down to $i—S_A$ the volume of employment would not increase and unemployment would not disappear.

Why do we have such an apparent contradiction? It was the case in which wages were bid up by competition among employers for which we compared price lines $i—S_A$ and *i—c*. In contrast, we have just now attempted to use this diagram in explaining the case in which workers are bidding down wages from the price line *i—c* to line $i—S_A$. Therefore, the fact that the conclusion of the former case is valid implies that the comparison between price lines in Figure 3.5 may be applied to explain the process of bidding up wages due to competition among employers but not to the process of bidding down of wages due to competition among workers. The effect of bidding down of wages due to competition among workers has been incorporated in our diagramatical explanation only in a limited and passive sense. That is, the competition among workers has an effect of stopping the bidding up of wages by employers at point *c* when there are less than 30 but no less than 20 workers, and at point *b* when there are less than 20 workers. The competition among workers, however, was not considered to have the effect of bidding down wages in the form of turning the price line down and to the left.

The same thing may be said to the case of comparison of points *b* and *d*. If the market wage rate declined from *i—b* to *i—d*, it appears that each employer would increase employment from 2 to 3 workers, and hence the employers would increase their total employment from 20 to 30 workers since both *b* and *d* are on the same indifference curve for each employer. But in reality this would not be the case. This is because the point of intersection between the contract curve of the employer who employs 2 workers and the price line *i—d* will be located inside the indifference curve which passes through point *b* and *d*. In other words, it would be more advantageous for each employer to employ 2 workers than 3 workers at the wage rate *i—d*.

Thus, Figure 3.5 indicates that a reduction in market wage rates would not increase the number of employed workers even if workers accepted the reduction.

In the case where the optimal number of employees for an employer is 3, as illustrated in Figure 3.5, whether or not unemployment emerges depends on whether or not the total number of workers exceeds 30 and not on the level of the wage rate. Of course, it is possible that the optimal level of employment would increase to 4 workers if the wage rate was reduced lower than the level of minimum subsistence. But even if all of the 40 workers were employed in this way, it would not mean full employment for reasons we have discussed earlier.

We have seen that when the total number of workers is less than 30, wages are bid up by competition among employers. We have also seen that if the price line were to shift higher than the position of *i—c* as a consequence of employers' competition, then each employer would reduce his demand to 2 workers. Thus, when wages are bid up above a certain level the demand for labor will be reduced. However, it should be noted that the demand for labor would not necessarily increase when wages are reduced. In other words, the effect of changes in wage rate on demand for labor in this case is irreversible, and the consequences are asymmetrical.

Because of this irreversibility, whether or not unemployment emerges in the labor market would depend upon the relationship between the total number of optimal employment $n \cdot N_s^*$, which is the optimal number of employment N_s^* for an employer for his holding of fund $\overline{Y}_B$ at the level of minimum subsistence wage multiplied by the number of employers n and the total number of workers $n \cdot N$, or

$$n \cdot N_s^* \lesseqgtr n \cdot \bar{N} \text{ or } N_s^* \lesseqgtr \bar{N},$$

and not upon the subsequent changes in the level of wages. If the total number of workers $n \cdot \bar{N}$ exceeds the total optimal number $n \cdot N_s^*$, then unemployment would be created and there would be no way of absorbing it. If on the other hand the total number of workers $n \cdot \bar{N}$ is less than the total optimal number of employment $n \cdot N_s^*$, wages would be bid up according to the degree of labor shortage and eventually full employment would be attained. The volume of employment in this case would be equal to the total number of workers, which is smaller than the optimal employment at the minimum subsistence wage i—S_A. This being the case, unemployed workers would not be absorbed merely by reducing wage rates.

There would be three cases in which unemployment may be eliminated. The first is a classical case along the lines of Malthus and Ricardo in which unemployed workers die out. However, the realistic policy suggested by this case would be to take precautionary measures to prevent unemployment occurring by controlling population increase on the basis of the predicted capacity of the economy to feed the population, as Neo-Malthusianism has asserted.

The second is the case in which employers are forced extraneously by the governmental policy interventions to employ more workers than the optimal numbers they wish to employ. As seen in Figure 3.5, each employer is not willing to employ 4 workers simply because employment of 3 workers is more advantageous.

Nevertheless, the policy intervention of forcing each employer to employ 4 workers should not cause a disastrous loss to the employer since employment of 4 workers is more advantageous than employing one worker, and much more so than employing none. It is also possible in this situation, as seen in Figure 3.5, to enforce a wage rate such as represented by the price line i—c which is higher than the level of minimum subsistence.

In the case where 10 employers and 40 workers exist in the labor market, if the optimal volume of employment N_s^* was realized, then 10 workers would have to be unemployed. In this situation, each employer would keep in hand a stock of wealth of 140 units after paying subsistence wages for 3 workers, or $3\ Y_{\text{min.}} = 60$, out of his initial holding of 200 units. While 10 workers are starving to death, each of the 10 employers on the other hand enjoys holdings of 7 times the minimum subsistence wage of a worker or $7\ Y_{\text{min.}} = 140$. In view of this inequitable distribution of resources, few would think it objectionable to make policy interventions as mentioned above in this kind of situation. If it is technically difficult to force employers to employ more workers than they are willing to, it may be possible for the government to siphon off part of employers' holdings in the form of taxes and use this fund directly to employ unemployed workers.

The third case for elimination of unemployment is the case in which the vertical length $\overline{Y}=\overline{Y}_B$ of the box becomes greater than 200. If the vertical length of the box extends to 250 or 300, the slope of the indifference curve which passes through S_A would become steeper than it would in the case illustrated by Figure 3.5. Thus, the distance between the indifference curve and the price line i—S_A would grow larger, and accordingly the optimal level of employment for each employer at the level of minimum subsistence wage would increase from 3 to 4 or 5. In this situation all of the 40 workers will be employed easily and probably at a wage rate higher than the level of minimum subsistence.

Once it is recognized that the effect of wage changes on labor demand is "irreversible," it is easy to see that the proposition of the Neo-Classical school that unemployment persistently exists because workers do not accept wage reductions is erroneous. It seems that Keynes advocated the achievement of full employment by means of demand control policies probably because he was subconsciously aware of this problem.

Notes to Chapter 3

1. Keynes (1936).
2. Galbraith (1975).
3. Smith (1776).
4. Ricardo (1821) and Marx (1867).
5. Lewis (1954) and Fei and Ranis (1964).
6. Smith (1776), pp. 68-69.
7. Smith *op.cit.* p. 64.
8. Smith, *op.cit.* p. 69.

Chapter 4
Effective Demand and Market Functions

4.1 Expansion of Frontiers and Money Supply

The "New World" as viewed by Adam Smith may be characterized as an economy in which the wealth of the nation is increasing and wages also are increasing above the level of minimum subsistence. All this was possible because the rate of increase of funds was greater than the rate of increase of population. In the case of North America, for example, it was possible to maintain wages at a level higher than minimum subsistence since the increase in available funds due to continuous expansion of frontiers was faster than the increase of population due to births and immigration. However, in connection with Keynesian theory, we need to pay attention to a condition which helped the situation of North America as observed by Adam Smith to be realized. This was the fact that those owners of farm lands, merchants and master craftsmen who successfully developed their business in America possessed a vigorous "frontier spirit". Remarks of Benjamin Franklin as quoted by Max Weber eloquently suggest the vigor of Americans in those days.[1]

Although not all immigrants to America in those days were like Franklin, it would not be an exaggeration to say that most employers probably shared a spirit similar to that of Franklin. If the fund increased from 200 to 250 or to 300, such people would have attempted to expand their business taking full advantage of it. In other words, the vertical length of Edgeworth's Box (*GEBD*) as discussed in the previous chapter would have been expanded as much as possible.

What would have happened if, in contrast to the way things actually happened, the pioneers had hoarded the profits of their activities for the purpose of spending comfortable retirements back in the countries of their origins, in Wales or Scotland or wherever, instead of investing for further expansion of their businesses in America? In this case, the development of frontiers would have slowed down and the increase in the funds would have been caught up by the increase in population, and consequently the labor market would have remained stagnant as in the "Old Continent" dominated by minimum subsistence wages in a stationary equilibrium in the Classical school sense. Spanish colonies in Latin America appear to have been this latter type.

It is not the lack of available funds for wages, but rather the lack of a vigorous motivation to use the available funds that gave rise to this kind of stagnation. The expansion of frontiers would stop unless the profits obtained from cultivated frontiers were invested back to expand the frontiers further. The North American colonies expanded flourishingly, as Smith described. This expansion was made possible by the active people motivated by "frontier spirit" which may be regarded as a concrete expression of "Protestant work ethic" described by Weber. However, was the frontier spirit a sufficient condition for this vigorous development? Galbraith suggests that it was not, quoting an example of bank notes issued in Pensylvania in 1723.[2] Galbraith points to the fact that Benjamin Franklin himself was fully aware of the fact that not only the "frontier spirit" of capitalism but also the systematization of rules and the framework of the market were necessary for the economy to prosper.

Although the pure theory of exchange using the concept of Edgeworth's Box-Diagram takes a form in which goods and services are exchanged directly, actual exchanges in civilized societies take place with money as an intermediary. The intermediation of money itself serves as a necessary condition for the formation of exchange markets for multiple goods and services. A simple quantity theory of money suggests that if the quantity of money did not increase when output increased due to expansion in the frontiers, then the exchange ratio between money and goods or equivalently the price of the goods would fall. If the quantity of money as purchasing power (nominal income) is reduced to a half while the quantities of goods and services remaining unchanged, then prices of goods and services would be cut down to a half. From the viewpoint of the homogeneity of degree zero of the demand function in the Neo-Classical general equilibrium theory, no substantial change should occur in this case except for the figures on the price tag, since proportionate changes in nominal income and prices would not change the level or structure of demand. However, in the real world, such changes as described above are called "deflation" and are usually regarded as implying a deterioration in market conditions and a reduction in the quantity of transactions. This fact is irreconcilable with the viewpoint of the homogeneity of degree zero of the demand function. To reconcile this contradiction, the concept of "money illusion" is often introduced. This concept gives the interpretation that, since economic decision making units are under the illusion that things have changed while in fact they have not, the illusion becomes the reality.

However, it would damage the methodological consistency seriously if one dares to introduce for the purpose of explaining the reality such a shaky concept as "illusion" into the analytical system of the Neo-Classical school, which is built on the basis of the premise of rational behavior of economic decision making units. It is with regard to this discrepancy that the theory of liquidity preference developed in Keynes' *General Theory* plays an important role.

4.2 Exchange Between One and N Persons in the α Zone

When we examined the possibility of underemployment equilibrium using the concept of the Generalized Edgeworth's Box-Diagram (*GEBD*), we set the initial point at the edge of the β zone. Some readers may have felt this treatment unrealistic, since there are only few in our contemporary society who could not survive more than a week once unemployed. For example, as far as the unemployed worker can receive unemployment benefit greater than the level of minimum subsistence he should be within the α zone at least during the period for which he is a beneficiary. The unemployed should also be in the α zone during the period in which he can live on his own saving. Also, those dependents in a family who can maintain their living without working but are willing to work if there is a good job certainly live in the α zone. In other workds, when the worker's initial holding $\overline{Y}_A$ exceeds the minimum subsistence $Y_{\text{min.}}$ the employer and the worker will have the same kind of bargaining as that between Robinson Crusoe and Friday as illustrated by Edgeworth.

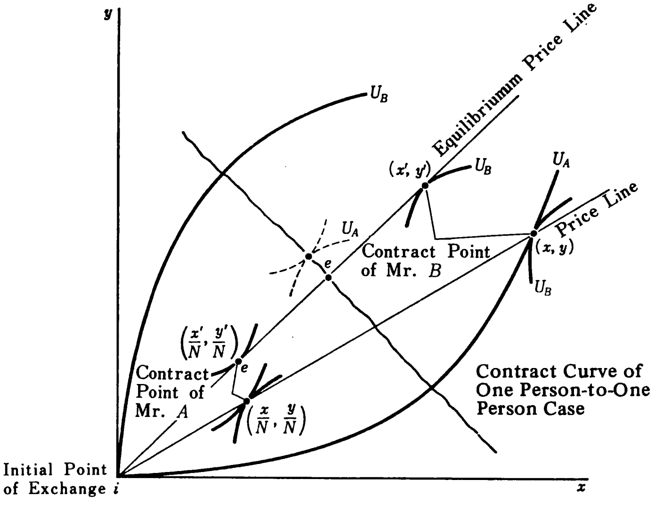
All this has been the case of one-to-one bargaining. It was not rare, however, even in the times of Adam Smith for an employer to employ 20 workers. In modern industrial societies, the number of workers is much greater than the number of firms which employ workers. Thus, the labor market does not usually consist of n pairs of a firm and a worker, or equivalently n firms and n workers altogether, but rather of n sets of one firm and N workers combinations. That is, there exist in total n firms and $n \times N$ workers in the labor market. This is also true for the relationship between the firm as a producer and consumers. In many cases, there exist N consumers who purchase products of a single firm.

In the case of exchange between one person Mr. B and N persons Mr. A, the equilibrium condition is given by

$$\frac{U_{BX}(x, y)}{U_{BY}(x, y)} = \frac{U_{AX}(\frac{x}{N}, \frac{y}{N})}{U_{AY}(\frac{x}{N}, \frac{y}{N})} ,$$

where the left-hand side of the equation is the marginal rate of substitution between good X and good Y on the part of Mr. B and the right-hand side is the marginal rate of substitution between good X and good Y on the part of Messers A. We have seen that in the case of one-to-one bargaining, an equilibrium is reached on a single contract curve. In the case of $1(B)$ to $N(A)$, where $N>1$, an equilibrium will hold when the tangent to B's indifference curve U_B at point (x, y) (where the price line intersects with the curve) is parallel with the tangent to A's indifference curve U_A at point $(x/N, y/N)$. When a perfectly competitive equilibrium holds, however, the respective tangents to U_A, and U_B will coincide with the price line. Given the vertical length of the Box as being constant, the equilibrium price line would incline further to the southeast direction as the number N of Messrs A increases. In terms of an employment contract, this would mean a lower equilibrium wage rate. Accordingly, the equilibrium quantity or working hours x will be shorter and the absolute amount of wage earnings will be smaller. In terms of a transaction between producer B and consumer A, it would mean a higher equilibrium commodity price. Figure 4.1 illustrates this kind of shift in the price line.

Figure 4.1 EXCHANGE BETWEEN $n \cdot N$ PERSONS OF A and n PERSONS OF B



In the case in which the initial position is in the α zone, an equilibrium holds smoothly through the contract-recontract process as described by Edgeworth.

The equilibrating process in this sense implies that bargainers agree with terms of contract considering relative advantages associated with the contract. The market would be cleared in this situation and no unemployment would remain since the *B* side would presumably be satisfied with the amount of working hours the *A* side agreed to supply. It appears therefore that full employment is always attained regardless of the size of *N*.

It should be added, however, that there are cases in which unemployment emerges inevitably. One such example is the case in which *N* workers are organized and decide not to work for less than a certain wage rate. As far as this wage rate is above the competitive equilibrium wage rate, the firm would not employ all of the *N* workers since it would make losses if it did so. Therefore some of the *N* workers would remain unemployed. This was the view of the Neo-Classical school, as represented typically by Pigou and Hicks.[3] And it is a view which was certainly logically consistent.

When the number *N* of *A* for each of *B* falls within an adequate range, earnings y, which is the product of wage rate and hours worked, would exceed the level of minimum subsistence $Y_{\text{min.}}$. This situation may properly be called the situation of full employment. However, if the number *N* is too large relative to the vertical length of the Box. then the slope y/x of the price line would be flatter and the equilibrium hours of work would be shorter, and consequently earnings y would be smaller. If these earnings y were lower than the normal level of consumption, then the worker would run a deficit. The deficit would eventually suppress his normal standard of living. If the wage earnings continued to be less than the level of minimum standard, then the saving would be used up sooner or later. These situations cannot be regarded as situations of full employment. In underdeveloped economies, this kind of situation is known by the name of "disguised unemployment," a situation which is not unusual. Unemployment in advanced economies, on the other hand, has been regarded conventionally as being distinct from this type of situation, although the reasons for making the distinction are obscure.

Unemployment in advanced countries and underemployment in underdeveloped countries are in fact not essentially different. In underdeveloped countries, even after the middle of the 20th century, the labor market is such that the initial position of workers is often in the β zone and the vertical length of the Edgeworth's Box (*GEBD*) is short while the size of labor force per firm *N* is large. Under these circumstances, minimum subsistence wages would inevitably be dominant. On the other hand in advanced countries, the initial position is usually in the α zone, the minimum standard level $Y_{\min.}$ itself tends to be high because of the effect of habit formation, and the number N of A is small relative to the vertical length of the Edgeworth's Box (*GEBD*). Under these conditions, the normal level of wages tends to be high. Differences in these conditions help to make the difference in the nature of the labor market seemingly large.

A mass of explicit unemployment can exist in advanced countries simply because unemployed workers can survive for a considerable period of time on unemployment benefits or savings even though they have no earnings. Unemployment in developing countries cannot take an explicit form for the simple reason that unemployed workers cannot survive unless they work even in very poor working conditions, in a situation similar to that in Europe until the first half of 19th century. As such, the reasons for the seeming difference in the labor market between advanced and developing economies are evident, and no mystical reasons are involved.

Given the vertical length of the *GEBD* as being constant, if the size of labor force per firm, N, approaches infinity $N \rightarrow \infty$, the quantity of transaction per employee (x/N, y/N) would be infinitely small so long as the equilibrium quantities (x, y) for firm B can not be infinite. The price line in this case would be tangential, in the neighborhood of the initial point, to A's indifference curve U_A which passes through the initial point. No exchanges would be made at a price lower than this. This is because an equilibrium should be attained between A and B, no matter how large N may be before the transaction price (or wage rate) gets down to this level.

An equilibrium of this kind is not incompatible with voluntary decision-making of bargainers on both sides. This is, both A and B trade the quantities they wish to and at the price they like. In this sense, this equilibrium may be said to be a full employment equilibrium since neither side should be dissatisfied with respect to the exchanges they have made. This was the typical Neo-Classical interpretation of a full employment equilibrium, as represented for example by Pigou's *The Theory of Unemployment*.

However, even in equilibrium, there is no guarantee that earnings per worker y/N will always be above the level of minimum standard of living $Y_{\min.}$, since y/N can be smaller as the number N grows greater. What would happen if $y/N < Y_{\min.}$? If the initial position of a job seeker in zone α was supported by an unemployment benefit greater than the minimum subsistence level, he would naturally be unwilling to take up employment at a wage level lower than the unemployment benefit since the benefit would be terminated once he accepts the job offer. This is the case in which the initial position may vary depending upon whether or not the contract is agreed to, unlike the case of Friday in Edgeworth's example, who could always choose to live on his own, in which case the initial position is unaffected by whether or not the contract is settled.

The situation would be similar also in a case where the initial position of a job seeker is maintained by drawing from savings. If he were employed at a wage level (y/N) lower than the normal level of expenditures, his savings would be used up eventually if he had to supplement his earnings from them. Once the saving is used up, then it would be anticipated that the initial position returns to the β zone and that the wage rate would be pulled down to the level of minimum subsistence.

In summary, when the initial position of the job-seeker is maintained by possibilities of self-employment or family support, low wages and short working hours can constitute the terms of an "equilibrium" employment contract, and hence the labor market in this case may be interpreted as realizing full-employment. Small farmers who also engage in by-employment, say in local public offices, or housewives who work as part-time workers are such examples. However, this interpretation does not apply to the majority of breadwinners in urban households in highly industrialized modern societies. If the initial point is only maintained in the α zone by means of drawing from savings, which means that net income is zero or through unemployment benefits, which will be terminated once the worker is employed, the competitive equilibrium attained in such a situation would only be apparent since it would not differ in substance from a situation under which the initial position is in the β zone. The basic condition upon which the competitive equilibrium in Edgeworth's sense may be attained is that a worker can still secure a certain amount of net income even when he fails to find employment.

The equilibrium of low wages and short working hours which is insufficient to maintain the accustomed standard of living, and which may be attained in cases where the number N is too large relative to the vertical length of the Box $\overline{Y}$, is also only in its outlook and does not deserve the name of full employment in its true sense of the word. The assertion of the Neo-Classical school that a full employment equilibrium should always be attained is erroneous even from the viewpoint of the logic of Edgeworth's exchange theory. It is erroneous because the initial point in the α zone is interpreted as being fixed in theory while in fact it is not.

In a real world, not only workers but also employers are keenly aware of this fact. It is for this reason that employers try to reduce employment while maintaining the accustomed level of wages in times of recession instead of offering lower wages to maintain employment. The explicit occurrence of unemployment is merely the consequence of such actions.

4.3 Absolute Unbalances in the Quantities of Goods for Exchange and Paralysis of Market Functions

While Edgeworth's Box is concerned with and is expressed in terms of physical factors, it has been known from the days of Smith that the determination of the vertical length of the Box involves monetary factors in addition to physical factors. Even when the vertical length of the Box can potentially grow large, thanks to increased physical and natural resources and an increased stock of knowledge, it would on the other hand shrink if money supply lags behind the increase in the physical quantity. This would happen because, due to the deficient increase in money supply, the marginal efficiency of investment (interest rate as a demand price of money) would decline on the one hand while the marginal utility of liquidity of money (interest rate as a supply price of money) would not decline on the other. As a consequence of these unconcerted movements, the equality between savings and investments would be lost and part of income would be hoarded without being expended. Therefore, part of the productive capacity of the economy would remain idle under deficient effective demand. This is the process by which in our terminology the vertical length of the Box would shrink.

If unemployment occurs as a result of the shrinkage of the vertical length of the Box to less than the physically possible level, the basic remedy to eliminate unemployment in this situation should be to expand the vertical length of the Box up to the physically possible maximum by increasing the effective demand by means of increasing money supply. One way to increase effective demand is to reduce the rate of interest as a supply price of money. However this method would not be effective if the firms' propensity to invest is depressed, a situation experienced for example in the United States during the 1930s. The Keynesian remedy for this situation was to trigger the multiplier-accelerator processes by increasing demand directly by means of government fiscal expenditures.

Needless to say, Keynes fully recognized that the vertical length of Edgeworth's Box in physical terms is not limitless. He pointed to the fact that if money supply increases beyond the level compatible with full employment, then "true inflation" would take place according to the extent of excess money supply.

Although it is suspected that Keynes might have developed further insights into the relationships between excess and deficient effective demand, inflation, and deflation, he did not explain clearly the relationship between these concepts and the market mechanism. After effective demand is controlled by fiscal policies, he simply says that appropriate adjustments will be made by the market mechanism as expected by the Neo-Classical school.[4]

Galbraith criticizes Keynes on the ground that he has underestimated the power of large firms and trade unions in controlling the market which was already quite strong in advanced countries even prior to the time when Keynes' *General Theory* was written.[5] Theories of monopoly, especially the modern theories of imperfect competition developed initially by Joan Robinson and Edward Chamberlin offer neat explanations for the market phenomena with which we are concerned. Those theories suggest that when a large firm excercises its monopolistic power, prices will be raised in the commodity market by the restriction of supplies, and factor prices will be depressed by restricted demand in factor markets. Likewise, when a trade union excercises its power to control the market, wages will be pushed up by restricted labor supply. Since the quantity of transactions is suppressed in either case, it is reasonable to expect that reductions in quantities and increases in prices and wages will occur simultaneously in the markets. This analysis appears quite useful and obvious in the sense that it offers a comprehensive explanation to the annoying phenomenon of stagflation in the 1970's. At least, it is undeniable that a considerable part of the actual market outcomes is being generated by such a mechanism as described above.

However, when we try to relate once again the Keynesian theory and the real world in the 1930s with which Keynes was confronted, there seems to be something more to be said. The subject of economic theory which Keynes was trying to resolve does not seem to be resolved so easily by the Galbraithian explanation of industrial organization. This is in part because Keynes himself was not unconscious of the problem of industrial organization. Indeed, it is well known that he was rather keenly concerned with the harmful effects of monopoly. The other reason is that the conditions of economies during the 1930's for which Keynes tried to find a remedy were not "stagflation" but "stagnation."

The modern theories of imperfect competition, which serve as a basis for Galbraith's contention, primarily focus on the power of a small number of buyers or sellers to control the market which affects demand-supply balances in the market. These theories, however, pay little attention to the adverse cause-effect relationship that the demand-supply balance in turn affects the extent of market controlling power. We need to remind ourselves of the fact here that Adam Smith evidently knew that competitive conditions among sellers or buyers themselves depend upon demand-supply balances in the market, and that Edgeworth implied this relationship implicitly in explaining the process of attainment of a competitive equilibrium. Generally speaking, contemporary price theory seems to have made it a rule, perhaps in the tradition of Walras, to describe only the competitive equilibrium situation in the market, skipping the important process in which the competitive equilibrium is attained. Such concepts as "price given" or a "price-taker" symbolically express this tradition. Since the theory of imperfect competition has been developed compatibly with this basic methodology, it also ignores the equilibrating process of competition itself. The theoretical scheme of Edgeworth seems to serve merely as a preface to contemporary price theory.

This preface remains valid so long as the Edgeworth's Box consists only of the α zone as he implicitly assumed. This is because, insofar as the number n is plural, whether the bargaining is N-to-N or $n \times N$-to-N, the convergence toward an approximately competitive equilibrium is theoretically guaranteed, unless collusion is involved.

However, what does the fact mean in terms of economic theory that Adam Smith was conscious of the concept of buyer's market which implies competition among sellers in an excess-supply situation and the concept of seller's market which implies competition among buyers in an excess-demand situation? Also what is the significance in terms of economic theory of the fact that buyers and sellers participating in actual market transactions do share Smith's feeling in their daily life?

Needless to say, Edgeworth's theory allows for the fact that if the price of Y relative to X is lower than the competitive equilibrium price then competition within the group of Mr. A, which offers good X, would take place and would bid up the relative price of Y, and also, conversely, the fact that if the price of Y is higher than the competitive equilibrium price, then competition within the group of Mr. B which offers good Y would take place and would bid up the relative price of X. The former is the case in which good X is in excess-supply relative to Y, while the latter is the case in which good Y is in excess-supply relative to X. These cases represent the market conditions behind Smith's concepts as mentioned above. However, does this formulation fully encompass the breadth of the problem suggested by Smith's conceptualization?

It has been mentioned earlier that the problem of unemployment is in fact not merely the problem of relative unbalance between demand and supply as taken up in the Edgeworth's theoretical scheme. We have emphasized that the situation under which unemployment can take place is the situation in which the absolute amount of worker's earnings y is reduced below the minimum subsistence level $Y_{\text{min.}}$ by a reduction in the Edgeworth-type competitive equilibrium price (wage rate) which is arrived at depending on the size of N, the number of existing workers A for each single firm B. In this situation, Mr. A's earnings y is absolutely deficient compared to the critical minimum amount, and not merely deficient relative to the amount which he wished to receive through the transaction at that wage rate. Although the quantities of x and y agreed by A and B to be exchanged are balanced in Edgeworth's sense, it is important to note that the equilibrium set of quantities (x^*, y^*), especially y^*, are short of the absolute minimum amount $Y_{\text{min.}}$ for Mr. A.

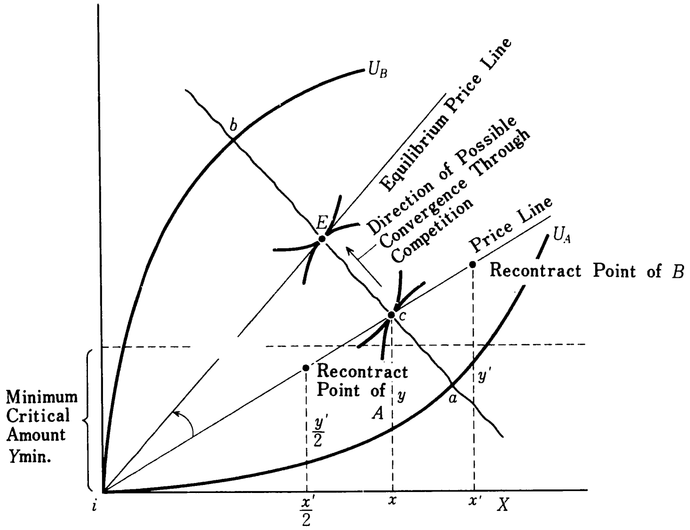
This kind of situation is not peculiar to the labor market. Let us consider the case in which consumer A offers money x and commodity supplier B offers commodity y. If the vertical length of the Box $\overline{Y}$ was insufficient relative to N, the number of consumers A per seller B, then the slope of the equilibrium price line y^*/x^* in the case of $n{\times}N$-to-N contract could be quite small. In other words, only a small quantity of commodities y^* can be obtained for a large amount of money x^*. If there existed the minimum critical necessity $Y_{\text{min.}}$ for the commodity, then it might happen that the equilibrium quantity y^* is short of $Y_{\text{min.}}$. It is interesting to recall the situation toward the end of 1973 and the beginning of 1974 which followed the shocking announcement of an oil embargo by Arab oil producing countries. The shortage of oil, for example, aggravated the danger of death from cold in cold regions and in urban areas caused hardship of life for many self employed taxidrivers.

We have so far discussed the case of $N>1$ on the ground that this is more realistic in both the labor and commodity markets. However, let us now return to the case of $N = 1$ for the sake of simplicity without altering the essentials.

Figure 4.2 illustrates the case of $N = 1$, or the case of n-to-n contract. The initial point is within the α zone, which means that both A and B hold more than the necessary minimum of goods X and Y. Suppose here that a contract happened to be reached temporarily at point c. According to the logic of the Edgeworth's diagram, since point c deviates to the southeast direction from the perfectly competitive equilibrium point E, the contract point would shift toward E through the process in which $(n-1)$ persons of Mr. B recontract with n persons of Messrs. A. This process proceeds for example in such a way that one of Messrs. B, say Mr. B_1 surpasses Mr. B_2 and makes a contract with two of Messrs. A. In this supplementary contract, B_1 offers y amount of Y and receives x amount of X, while each of the two Messrs. A receives $y'/2$ amount of Y and offers $x'/2$ amount of X to Mr. B_1. Needless to say, the relationship $U_A(x'/2, y'/2) > U_A(x, y)$ holds in this case.

However, what would happen if Mr. A's receipt $y'/2$ of good Y was below the critical minimum amount $Y_{\text{min.}}$? Note that $y > y'/2$ even though Mr. A's utility indicator at $(x'/2, y'/2)$ is higher than at (x, y). Even if Mr. A's holding of good Y at the beginning of the current period, $Y_{A,t}$ was greater

Figure 4.2 THE EXISTENCE OF THE MINIMUM CRITICAL AMOUNT WITHIN THE α ZONE AND THE POSSIBILITY OF RECONTRACT



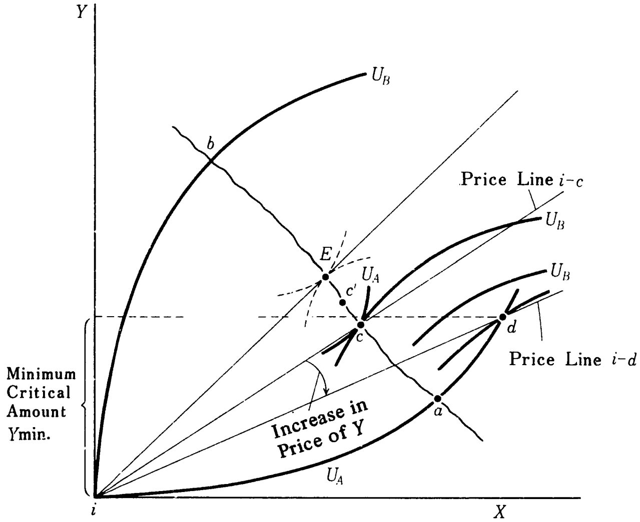
than the critical minimum amount, his holding at the beginning of the subsequent period $\overline{Y}_{A,t+1}$ would be smaller compared to the present period if the amount of Y he receives during the current period is less than the critical minimum amount to be consumed. It might well be anticipated that, after having experienced this kind of situation repeatedly, his initial holding at some point in future could fall below the critical minimum amount $Y_{min.}$, which means that the initial position in that period would fall within the β zone of *GEBD*. Therefore, each of Mr. A would hesitate to make a recontract with Mr. B_1 in the current period, or the t-th period, even if Mr. A's utility associated with that supplementary contract at $(x'/2, y'/2)$ happend to be temporarily higher than his utility at the normal contract point c. He would rather choose to maintain the contract at c at which he can secure amount y which is obviously much large than $y'/2$. He would surely do so when amount y is greater than the critical minimum amount $Y_{min.}$.

When the amount which each Mr. A receives at point c is greater than $Y_{min.}$ and then $n-1$ persons of Messrs. B try to make a recontract with n persons of Mr. A, the contract point may still shift from c to E as long as the number n, which determines the amount of supplementary contract $\frac{n-1}{n}y'$ for each Mr. A, is large enough. This is because although $n = 2$ may be too small to let sides A and B engage in a supplementary contract, $n = 10$ would be sufficiently large to make the recontract possible since the relationship $\frac{n-1}{n}y' > Y_{min.}$ might hold in that case. However, generally speaking, the speed of convergence toward equilibrium would be slower the greater the number n in Edgeworth's scheme. Moreover, in the case where the difference between the quantity y contracted at c and $Y_{min.}$ is very small, the relationship $\frac{n-1}{n}y' > Y_{min.}$ would not hold unless the number of participants n in the market is very large. For these reasons, the contract point is likely to stay at c. This implies, for the side of Mr. B, that no matter how many persons exist on the side of B, competition within the group of B would be meaningless and unnecessary. In short, each B will enjoy in this situation a similar position to that of a monopolist.

In the case where even the quantity y contracted at c falls below the critical minimum amount $Y_{min.}$ for Mr. A, there would remain no room to make a recontract which provides even smaller amount $\frac{n-1}{n}y'$. Rather, a competition which reverses the direction would occur. That is, each A would offer to exchange with any one of B greater amount of X and Y at the same price, or on the extrapolation of price line i—c, for fear of carrying forward the negative balance to the next period even though his utility would decline temporarily. Part of the extended price line i—c will get outside Mr. A's indifference curve U_A which passes through point c. But Mr. B should have no reason to reject the proposal of Mr. A for increasing the quantities of transactions since it is advantageous for Mr. B because the extended price line would pass inside Mr. B's indifference curve.
However, the market for each Mr. B is now in an almost non-competitive situation in which he does not have to worry about competitive pressure of his fellow B's. Each seller B would not, therefore, necessarily accept the proposal of buyer A since B enjoys a position similar to a monopolist against A no matter how many sellers exist on the B side.

If each Mr. B is bullish and tries to take full advantage of this temporary quasi-monopolistic position, then he would hesitate to increase the quantity of transaction on the price line i—c. In such a situation, each buyer A would desperately try to obtain an agreement with the seller B's side even at a higher price since it is an emergency for the buyer A to secure the critical minimum amount $Y_{min.}$. It would then be possible for seller B to raise his supply price at least up to the neighborhood of price line i—d in Figure 4.3. Point d is the intersection of the critical minimum line which is the horizontal

Figure 4.3 A CASE IN WHICH THE CONTRACTED QUANTITY OF COMMODITY Y FALLS SHORT OF ITS MINIMUM CRITICAL AMOUNT WITHIN ZONE α



line at the height of the minimum necessity $Y_{min.}$, with buyer A's indifference curve U_A which passes through the initial point of exchange i. Buyer A's utility would therefore be reduced by this exchange since the shift of the contract point to the right of point d means a shift to the outside of the indifference curve U_A. For this reason, buyers A would probably resist a further increase in the price of Y beyond the price line i—d. Nevertheless, buyers in this situation are obliged to take into account not only their utilities during the present period but also in future periods. In other words, the buyers are now forced to make "over time choices." It is for this reason that they might accept even higher prices of Y (a smaller exchange ratio y/x) while knowing that their utility during the current period may decline below that prior to the exchange.

The extent to which the price of Y increases will depend in part upon how long each seller B anticipates this temporary non-competitive situation, due to shortage of supplies of commodity Y, will last, and in part upon how impatient buyer A is. However, if the bargainers on both sides, both buyers and sellers, are aware of the fact that this situation is only temporary, then the increase in the price of Y would be likely to be satiated in the neighborhood of point d.

Thus, in the case in which buyer A cannot obtain the critical minimum amount $Y_{min.}$ on point c which is on the contract curve of n-to-n person exchange, each of the sellers B occupies a *de facto* quasi-monopolist position and obliges buyer A to accept conditions which are advantageous only to B. Let us call this situation "acute polypoly" which differs from the familiar concept of monopoly which is chronic in the sense of industrial organization.[6]

We need to note here that the contract point has deviated from the contract curve, in our example, simply because the quantity of exchange y_c at point c on the contract curve at which the bargaining happened to start was smaller than the critical minimum amount $Y_{min.}$. This result does not contradict with the fact that the quantity of exchange y_E at the point of perfectly competitive equilibrium E can be greater than the critical minimum amount $Y_{min.}$. Since y_E is generally greater than y_c, even though $y_c < Y_{min.}$ it can be $y_E > Y_{min.}$ or $y_E > Y_{min.} > y_c$ holds. Therefore, if the bargaining happened to start at c' which locates closer to E than c on the contract curve and if $y_c > Y_{min.}$, it would be possible that the point of contract converges to equilibrium point E through the normal competition which takes the form of repetition of Edgeworth type contract and recontract.

In this way, unlike the case of "chronic monopoly", the emergence of "acute polypoly" depends often on coincidental factors. That is to say, while the competitive mechanism of the market would operate normally if the bargaining started at point c', the normal operation would be disturbed if the bargaining happened to start at point c. The same logic applies basically to the case in which each of n sellers is confronted with N buyers, or equivalently the case in which there exist n persons of B and $N \times n$ persons of A. It should be added, however, that in the case of bargaining of one-to-N persons the forces which bid up the price of Y would be stronger since the forces would be fortified by the competitive pressure among N buyers of the A group.

In either case, generally speaking, if the quantity of Y which is exchanged at the normal contract point on the contract curve happened to be less than the critical minimum amount, buyer A would be obliged to act taking into account his utility not only in the present period but also in future periods, and hence the contract point would deviate from the contract curve and the normal competitive market would no longer operate. As a result, the price of Y would be determined by polypoly, which is the market situation analogous to monopoly. This type of situation arises in cases where all or part of the contract curve sinks below the horizontal critical minimum line because the vertical length of the Box shrinks for some reason. In the case in which only the southeast part of the contract curve falls below the critical minimum line, whether or not "polypoly" emerges would depend upon whether or not the starting point of the bargaining happens to be above or below the critical minimum line.

When the supply of kerosene became short in the winter following the world oil crisis of 1973, consumers in cold regions were obliged to take the position of *A* and sellers of kerosene the position of *B*. The fact that the consumers were caught up in the fear of hardship as a result of the kerosene shortage eloquently indicates that there does exist the critical minimum amount $Y_{min.}$ in the case of kerosene. In view also of the fact that prices of such items as toilet paper and cleansers rose rapidly with the abnormal phenomena of hoarding of goods for future sales or cornering, one may reasonably infer that the concept of the critical minimum amount $Y_{min.}$ applies not only to a limited number of special commodities but to almost any goods. The concept will apply not only to consumers' goods but also to productive goods ranging from gasoline for taxis, construction materials for builders to crude oil for petroleum refineries and iron ore and coal for iron and steel corporations, and even to a national economy as a whole.

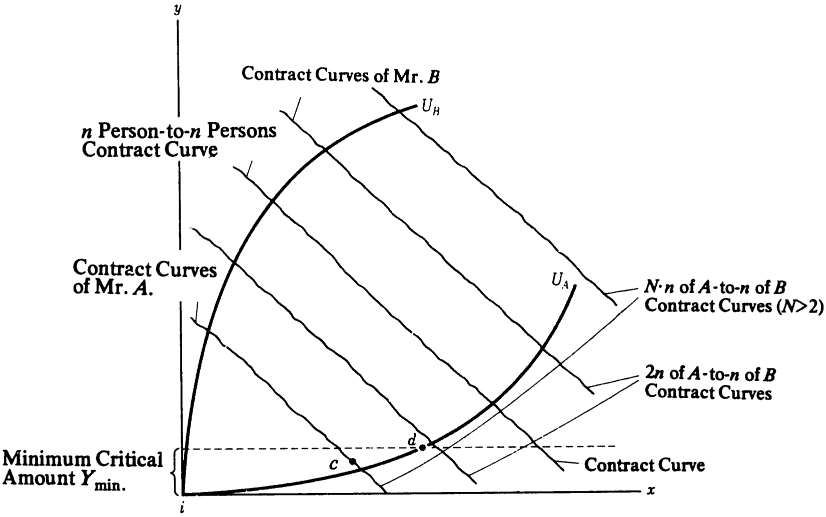
The disturbance in the competitive market mechanism discussed in the previous section is therefore not the market imperfection due to the familiar problems of monopoly or oligopolistic collusions in industrial organization but rather is due to "acute polypoly" which arises from temporary distortions of market balances due to failures in securing the critical minimum amounts of relevant commodities.

4.4 "Polyopsony" in the Labor Market and Unemployment

The situation described above can be applied intact to the labor market by simply replacing Mr. *B* with the employer, Mr. *A* with the worker, good *X* with working hours, and good *Y* with wages. In the previous section, discussed the problem of shortage of good *Y*, but we now replace this with the problem of reduction in employment opportunities.

Suppose now that the vertical length of the Edgeworth's Box shrinks for some reason. This will naturally have the effect of pressing the contract curve between *A* and *B* downward in the direction of the *x* axis. Let us once again generalize the situation in such a way that each employer *B* is confronted with *N* workers called *A*. As explained earlier, a pair of contract curves at each side of *A* and *B* in the case of 1-to-*N* bargaining is located above and below the single common contract curve of *A* and *B* which is drawn for the case of one-to-one bargaining. The vertical distance between the pair of

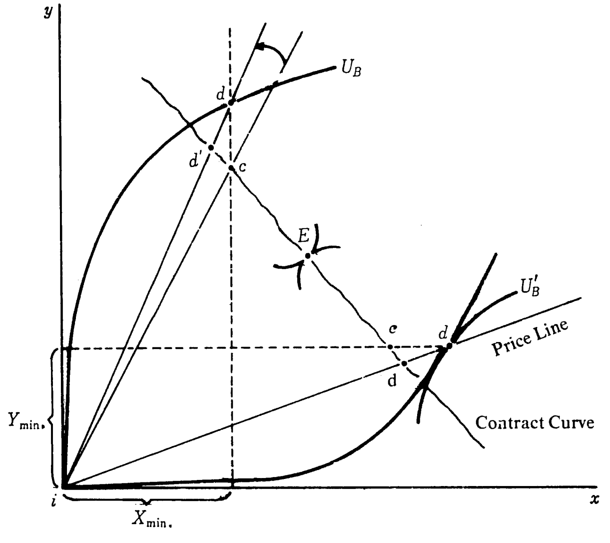
Figure 4.4 THE RELATIONSHIP BETWEEN ALTERNATIVE SIZES OF N AND THE LOCATIONS OF CONTRACT CURVES IN $n \cdot N$ PERSONS OF A-TO-n PERSONS OF B CONTRACTS WITHIN THE α ZONE.



curves widens as the number N grows larger. Generally speaking, the greater the number N of workers A, the more will the contract curve for A deviate downward from the contract curve in the case of one-to-one bargaining, as seen in Figure 4.4. Moreover, the price line in the case of perfectly competitive equilibrium will be flatter, and accordingly the equilibrium price y^*/x^* will also be low.

Therefore, given the vertical length of the Edgeworth's Box $\bar{Y}$, if the number N of workers A exceeds a certain level, then the southeast part (or the whole in the extreme case) of the contract curve for each A would fall below the critical minimum line. And if point c at which the bargaining starts happened to be on the part below the critical minimum line, then employers would temporarily occupy the position of "polyopsony." In this situation, the Edgeworth type competitive mechanism would no longer operate and the equilibrium of polyopsony would be attained at point d where the critical minimum line $Y_{\text{min.}}$ and worker A's indifference curve $U_A{}'$, which passes through the initial point i, intersect. This means that the bargain is made independently from the contract curve for worker A and, consequently, from that for employer B. This situation is illustrated by Figure 4.5.

Figure 4.5 A CASE IN WHICH THE CONTRACT CURVE WITHIN THE ZONE α IS LOCATED BELOW THE LEVEL OF MINIMUM CRITICAL AMOUNT



As seen in Figure 4.5, since the initial point i is within the α zone, however temporarily, the wage rate i—d generally would not fall as low as the minimum subsistence level even though the labor market is in a situation of a buyer's market. This means that although earnings y equals the critical minimum amount $Y_{min.}$ working hours would not extend to the longest possible hours.

In this situation, the extended price line i—d will surely pass inside employer B's indifference curve U_B which passes through the initial point i and also generally passes inside the indifference curve U'_B which passes through point d. In many cases, the point which marks N times the interval i—d, the length corresponding to employment of one person, on the extended price line, will also remain inside employer B's indifference curve U'_B. However, employers can now decide the number of their employees arbitrarily since they enjoy non-competitive positions. Therefore, each of the employers would probably choose his optimal number N^* of employees in the neighborhood of the point at which the price line i—d touches his highest possible indifference curve U^*_B.

If the number of employees N^* which gives the highest utility for each employer at the wage rate i—d exceeds the number N, then the wage rate would be bid up by competition among employers. However, since each employer has no reason to secure the number N^*, it is unlikely that the optimal number of employees N^* will exceed the number of existing workers N. However, it is possible, on the other hand, that the optimal number N^* is smaller than N. Let us consider the latter situation in some detail.

Suppose the case in which the contract curve for each of workers A in a 1 employer-to-N workers bargaining is located above the critical minimum line $Y_{min.}$ and that the labor market may thus be regarded as competitive. When the price line happens to fall on the position $i—d$ temporarily in the process of convergence toward competitive equilibrium point E, the equilibrium quantities of demand and supply for one worker will be indicated not at point d but at point d' at which the price line and the contract curve for each worker A intersect. Generally, the length of $i—d$ is greater than the length of $i—d$. In this situation, the distance from point i to the intersection of the price line and the contract curve for each employer B will be N times the length of the segment of line $i—d'$. The distance from i to this point of employment of the N-th person is generally shorter than the distance to the point at which this price line touches employer B's highest possible indifference curve U^*_B. However, the length of $i—d$ which indicates the employment of one worker in the case mentioned above where the price line falls on the position $i—d$ because employers' "polyopsony" jeopardizes the market competition, is longer than the length of line $i—d'$ in a competitive situation. Consequently, the distance from i to the employment point of the N-th person, which is marked by the length of the segment of line $i—d$ multiplied by N, will necessarily be longer than the length of $i—d'$ multiplied by N. Therefore, although the distance from i to the N-th person point in the latter competitive case is shorter than the distance to the tangent point of the price line with the indifference curve U_B, the distance to the N-th person point in the former "polyopsony" case may or may not be shorter than the distance to the tangent point. There may be polyopsonical cases in which the distance from i to the N-th person point exceeds the distance to the tangent point. It is in this situation that the optimal number of employees N^* for each employer falls short of the number of existing workers N. Thus, if $N^* < N$ there would emerge unemployment of $N-N^*$ workers for each employer and $n(N-N^*)$workers in total.

This is the case of "underemployment equilibrium" due to "polyopsony" in the employment market. The situation of "polyopsony" takes place, as explained above, when a part of the contract curve for each A falls below the critical minimum line because the vertical length of the Edgeworth's Box is too short relative to the number N of workers A who are to be employed by each employer B. Therefore, this situation should be clearly distinguished from the underemployment equilibrium which occurs as a result of monopsonization of the market, in the sense of industrial organization as pointed out by Galbraith etc.

Since the formation of underemployment equilibrium due to chronic monopsonization of the market means the formation of monopsonistic equilibrium at either end of the effective region of the contract curve, it is unrelated to expansion or contraction of the vertical length of the Edgeworth's Box. In contrast, the underemployment equilibrium under "acute polyopsony" many or may not be attained depending on the relationship between the vertical length of the Box Y and the number of workers N.

If the vertical length of the Box is sufficiently long for a given number N of workers A, and thus the contract curve for each A is above the critical minimum line, then "polyopsony" would not emerge and the convergence toward the Edgeworth type competitive equilibrium would proceed. In this situation, "full employment equilibrium" in its true sense will be established.

If, on the other hand, a deficient effective demand prevented the potentially realizeable vertical length of the Edgeworth's Box from being realized and the temporarily shrunk Box in this way were too small relative to the given size of the labor force N, then unemployment would be created by the emergence of "polyopsony" in the market. This unempolyment, thus created, may be eliminated by restoring an adequate vertical length of the Box by means of effective demand control policies. It must be this problem which Keynes wanted to point out 40 years ago.

It has been inevitable for the Neo-Classical economics to maintain that the full employment equilibrium is guaranteed since the concept of the "critical minimum" has been assumed out of its perspective. However, even in the case where the initial position of exchange is within the α zone, it is possible that this position may not be able to be maintained for future periods if the quantity acquired during the current period is less than the critical minimum amount. In view of this possibility, the danger of "polypoly" or "polyopsony" which jeopardize the competitive functions of the market may not be ignored, because this makes unemployment due to deficient effective demand quite probable.

In the previous section, we considered an example in which consumer A pays amount of money X to buy amount of good Y from seller B. For simplicity, we assumed a market of n sellers and n buyers. This example can be handled easily since there exists only one contract curve insofar as both A and B are of similar numbers there are the same number n of A and B. Even in this case, however, the southeast part of the contract curve can be below the critical minimum line of good Y for Mr. A, if the vertical length of the Box is insufficient. For the given level of the vertical length of the Box, the greater the number N of consumers A who are confronted with each seller B, the further the contract curve for each A will be located below the contract curve in the case of one-to-one bargaining. Thus, the contract curve in the case of one-to-N bargaining is more likely to be below the critical minimum $Y_{\text{min.}}$. Even if the contract curve in the case of one-to-one bargaining is located above the critical minimum line, it is possible that the contract curve for the N side in the case of one-to-N bargaining will be located below the critical minimum line, when N increases beyond a certain number. In such a situation, in the aforementioned case of kerosene for example, there may be cases in which it would be more advantageous for seller B to limit the number of buyers N^* below the number N. Viewed from the side of consumers A, this means that some of them might have to loose the chance to make a purchase. Needless to say, this would aggravate the psychological panic on the part of consumers.

In the case of the kerosene market, seller B was in the position of "polypoly", and in the case of the labor market, employer B was in the position of "polyopsony". Both cases are similar, however, in that the competitive market mechanism is paralyzed and the formation of prices and the determination of transaction quantities are made in a quasi-monopolistic situation. These cases are also similar in that some of the buyers are excluded from the contract in the temporary seller's market situation (polypoly) and some of the sellers are excluded from the contract in the temporary buyer's market situation (polyopsony). Within the realm of Neo-Classical economic theory, this phenomenon is usually handled as a problem of divisibility of the commodity. But the issue discussed above is an entirely different problem from the question of "divisibility."

Needless to say, our heuristic example of unemployment in the labor market relates to "stagnation" accompanying serious deflation which provoked Keynes, and our example of shortage of commodities relates to the abnormal market phenomena which we experienced in the explosive inflation during 1973 and 1974. The former is the case in which unemployment was created while the wage rate y/x (or line i—d) was pressed below the competitive equilibrium wage level by "polyopsony" and the latter is the case in which some buyers fail to purchase the commodity while the price x/y (or line i—d) is maintained higher than the competitive price level by "polypoly." The former is the case of quantity and price restrictions similar to the case of monopsony and the latter is the case of a quantity restriction and price hike similar to the case of monopoly. Both are cases of "acute paralysis of market functions" which takes place only temporarily due to changes in the relationship between the real possibility of supply and the effective nominal demand; the deficient effective demand for the former case and the excessive effective demand for the latter case. Therefore both of these cases have to be distinguished from the "chronic paralysis of market functions" due to structural problems in industrial organization. In view of the fact, however, that "polypoly" also imposes quantity restrictions, just like monopoly, it would not be surprising if inflation and reduction in capacity utilization occur simultaneously when "polypoly" occurs in the market due to excessive inflation generated by an excess effective demand. In other words, "stagflation" may well take place when the market is caught up by "polypoly".

If we generalize the Keynesian explanation of the emergence of unemployment, the possibility of stagflation under an excessively large excess demand would reasonably be explained with the theory of "acute polypoly."

4.5 Aggregate Demand Control to Make the Market Function Effective

Excessive effective demand will give rise to polypoly not only in the commodity market but also in the labor market (i.e. shortage of manpower) while deficient demand will give rise to polyopsony not only in the labor market but also in the commodity market (i.e. market stagnation) when the degree of demand-supply imbalance grows excessively large. Polypoly itself emerges as a result of the existence of large excess demand in the market, but once it emerges it necessarily aggravates the magnitude of excess demand acceleratedly. It is with this respect of self-generated expansion of market imbalances that the symptom of "polypoly" deserves special attention. By the same token, "polyopsony" itself is a result of large excess supply, but once it emerges in the market it in turn increases the excess supply acceleratedly. In fact, in times of deflation, as in the Great Depression during the 1930s, in which unemployment increases in the labor market, the commodity market also usually becomes stagnant, and conversely, in times of staggering inflation, as for example in the period 1972 to 1974, in which commodities are in short supplies, labor also usually tends to be in short supply in the labor market because of polypoly.

We have explained earlier that when the starting point of the bargaining c on the contract curve went below the critical minimum line $\overline{Y}_{\text{min.}}$, the point of transaction would shift to point d deviating from the contract curve, because over-competition takes place among suppliers of labor A while under-competition takes place among purchasers of services B.

Also on the side of the firm which demands labor services there exists a critical minimum level. No employer would deny the fact that a firm always needs labor services at least to the extent of a certain minimum amount $X_{min.}$, in order to maintain its operations. If this is the case, when the vertical length of the Edgeworth's Box $\overline{Y}$ is excessively large, the northwest part or the whole of the contract curve for B would fall to the left of the critical minimum vertical line $X_{min.}$ in Figure 4.5. Also, if the starting point of the bargaining happened to be on that part of the contract curve, firms B would bid up the wage rate y/x from i—c to i—d because of their desperate need to secure the critical minimum employment $X_{min.}$. In this situation, each supplier of labor A now enjoys the position of "polypoly" and each purchaser of labor B is obliged to take the position of "negative-polyopsony" or in short "negopsony", in the sense that there exists over-competition among purchasers and under-competition among suppliers in the labor market deviating far from the normal situation of competition on the contract curve.

If the market is in a normal competitive situation, firm B's equilibrium employment point $(N{\cdot}x,\ N{\cdot}y)$ corresponding to wage rate i—d will be at d', and the quantities for each of suppliers $A(x, y)$ will be determined on the contract curve for A. Thus, a temporary equilibrium of one-tò-N contract between A and B will be attained. But if the contract point of B was located at d deviating from the contract curve, and worker A could decide his optimal amount of labor supply x^* at wage rate i—d without competition, then it would not be guaranteed that $N \cdot x^* = N \cdot x$ or it may even happen that $N \cdot x^* < N \cdot x$. In such a situation, there might be competitive pressure among purchasers which would push up the wage rate even higher than the rate i—d, since firms would suffer from absolute shortage of manpower. It is through this mechanism that labor supply falls short of demand and wages jump in times of "stagflation".

Suppose a commodity market where seller B sells good Y to buyer A in return for money X and also the seller has to secure the critical minimum amount $X_{min.}$ in order to maintain his business. If the effective demand was inadequate in this case, as indicated by the northwest corner of Figure 4.5, each seller would sell at a loss deviating from the normal competition in order to secure the critical minimum amount $X_{min.}$ and consequently bring down the sales price x/y to the position i—d. In this situation, over-competition would take place among sellers B while under competition would prevail among buyers A, and therefore each seller B would fall in the position of "negative-polypoly," or in short "negapoly," and each buyer A would enjoy the position of polyopsony. Since, as we have noted earlier, there would be no guarantee that each seller B can secure the revenue he wishes to get $x = X_{min.}$ at the price $i—d$, it may happen that some of the commodities, $N{\cdot}y - N{\cdot}y^*$, remain unsold in spite of the abnormal price reduction.

Thus, there do exist situations under which "over-competition" among sellers and "under-competition" among buyers occur simultaneously due to deficient effective demand. For example, in the commodity market a large reduction in price would not help clear the market and similarly in the labor market a large reduction in wage rate would not help clear the market.

Conversely, an excessively large effective demand may give rise to simultaneous occurrence of "polypoly" and "negopsony" in the commodity market as well as in the labor market. Under such circumstances, buyers in the commodity market may not be able to obtain the desired amount of commodities even if they accepted price hikes, and employers in the labor market may find it difficult to employ a necessary number of workers even if they increased wages.

In the Neo-Classical theory of market competition, it is postulated that excess demand will be cleared by an increase in the price and that the excess supply will to be cleared by a decrease in the price. It is assured in the theory that this process operates so long as there is a plural number of sellers and buyers and no collusions among them. Note, however, that it is true only in those cases in which the contract curve for Mr. A is above the critical minimum line $Y_{min.}$ and the contract curve for Mr. B is located to the right of the critical minimum line $X_{min.}$, even when the initial position of exchange falls within the α zone. There are cases in which the nominal effective demand is either deficient or excessive relative to the physically realizable size of the Edgeworth's Box and, as a consequence, part of the contract curve is located below the critical minimum line. In these cases, the balance between demand and supply would not be restored by relevant changes in prices and the contract may well diverge from instead of converging to an equilibrium point. Needless to say, one extreme case of this situation would be "stagnation" and the other would be "stagflation".

It is difficult to assess how far Keynes was conscious of the relationship between the size of effective demand and the extent to which the market mechanism is effective, when he advocated demand control. Nevertheless, one may reasonably suspect that he might have an intuitive insight into the importance of the concept of critical minimum in view of the fact that he was particularly concerned with the rigidity of money wages or liquidity preference (the downward rigidity of interest rates as the supply price of money). At any rate, although seemingly unrelated to the market mechanism, the Keynesian policy implications of the aggregate demand control should be interpreted, in substance, as emphasizing the necessity to prepare conditions on which market competition can operate normally as expected by the Neo-Classical school.

Notes to Chapter 4

1. Weber (1904).
2. Galbraith (1976)
3. Pigou (1933) and Hicks (1932).
4. Keynes (1936).
5. Galbraith (1967).
6. The word "polypoly" was first used by Ragnar Frisch (1933b). Also see his "Monopoly-polypoly—The Concept of Force in the Economy," *International Economic Papers*, No. 1, London. pp. 23-36. We use the concept, however, in a slightly different way in this book. The theory of "polypoly" was first developed in a formulation analogous to Lerner's (1934) type. Usually, the condition of producer's profit maximization is expressed in a equality between the marginal revenue and the marginal cost: $P(1 - \frac{1}{\eta}) = M.C.$ where η donotes the price elasticity of demand and $|\eta| \to \infty$ is assumed in a competitive market. We generalized this equation as $M.R. = p(1 - \frac{\lambda}{|\eta|})$ where λ stands for the "seller's elasticity of conjuctural market response" which express the seller's anticipation about the response of the total supply X in a particular commodity market to a change of quantity supplied x by himself, or $\lambda = \frac{\partial X}{\partial x} \frac{x}{X}$. The value of λ will vary depending not only on his market share but also on the total demand-supply balance in that market in Smith's sense. In a phase of large excess demand in the market each seller will be free from the competitive pressure of his fellow sellers no matter how many competitors exist, and he can temporarily enjoy quasi-monopolistic position, then λ will take a value near to unity. This is the situation called " aute polypoly" here. On the other hand, for a firm as a buyer of factors of production and raw materials, the "buyer's elasticity of conjuctural market response" can be defined analogously.

   In the short run we have

   $\frac{\partial C}{\partial x} = w\frac{\partial l}{\partial x}(1 + \frac{\lambda_l}{\nu_w}) + \sum_i p_i a_{ij}(1 + \frac{\lambda_{mi}}{\nu_{pi}})$, where w is the wage rate and p_i are prices of raw materials and V_s are price elasticities of supply of cost elements. Also for a household both "buyer's " and "seller's" elasticities are defined as a consumer and as an unit of labor supply respectively. For a particular commodity j, the "buyer's elasticity of conjectural market response" of a household is defined in a utility maximization equation: $\frac{\partial U}{\partial q_k} / p_k = \frac{\partial U}{\partial q_j} / p_j(1 + \frac{\lambda_{qj}}{\nu_j}) = \mu$,

where ν_j denotes the price elasticity of supply of j. In usual cases $\lambda = 0$ will hold. When a large excess supply takes place, however, λ can take a value larger than zero reflecting the under-competition among consumers. When a severe short supply takes place, on the other hand, λ may take negative finite value. The former case corresponds to "polyopsony" and the latter situation corresponds to "negopsony".

Empirical validity of the theory developed above was tested by Sakiko Tsuzuki making use of monthly time-series data related to the toilet-paper market in Japan. Estimating first the consumers' demand function and the marginal cost curves of suppliers for normal periods she observed significant upward shifts of both demand and supply curves caused by "acute negopsony" of consumers and "acute polypoly" of supliers respectively for months immediate before and after Arab's oil embargo in the fall of 1973. Negative values of consumers' λ and positive values of suppliers' λ were measured for more than ten moths, and so the outbreaks and terminations of "negopsony" and "polypoly" were indentified in the toilet-paper market. Negative λ of consumers have also been meausred in other commodity markets.

Theory of "acute polypoly" developed in terms of the Edgeworth Box in this chapter is another type of expression of the same content and wholly based on the research mentioned above. Details are given in K. Tsujimura and S. Tsuzuki, "Theory and Measurement of Acute Polypoly and Polyopsony; A Reconsideration of the Theory of Price", *Keio Economic Observatory Review* No. 1, July 1975. (Originally printed in Japan)

The "anticipated demand function" used in the model building in Chapters 7 and 11 of this volume is a first approximation to the concept of "polypoly".

Chapter 5
General Equilibrium Theory and Empirical Analysis

5.1 "Quantitative Theory of Price" and the Construction of Our Empirical Model

What we have developed so far may be symbolically summarized, as a "quantitative theory of price." We emphasize the term "quantitative" because we mean to distinguish our price theory from the abstract Neo-Classical price theory.

Our theory differs from the conventional theory in which it takes into account explicitly as a corner stone of the theoretical set up the quantitative concept of "minimum necessity" which played an important role in Adam Smith's theorizing of competitive market. Although the existence of necessities for human life is self-evident and does not require any empirical verification, we do have empirically measured evidence of their existence which is presented through the use of a specific form of preference function which we call a "Bernoulli-Laplace" type preference function. A part of the evidence will be presented in Chapter 10 of this book. It is on the basis of the empirical validity of this specific type of preference function, as a first approximation, that we have generated the crucial building blocks of our price theory, e.g. the concept of minimum critical amount and the numerical relation between changes in the indeterminateness of final setting of a contract and changes in the number of competitors. Since in this sense our price theory has been constructed strictly on the basis of quantitative concepts with measured evidence, we stress its quantitative nature and distinguish it from the abstract treatment of conventional price theory.

The single most important implication derived from the "quantitative theory of price" is the point, clearly demonstrated in the previous chapter, that appropriate types and intensity of policy interventions are indispensable to assure proper market operation. Once it is admitted that adequate policy interventions are imperative to make the market work appropriately, the central role of economics should be to measure the cause-effect structure of the actual economy for the purpose of presenting a menu of economic consequences of alternative policy choices in precise quantitative terms so that people can choose from the list of optional policies the appropriate one to attain their goal. This kind of policy menu can be obtained only from an empirical quantitative model which is a correct and close description of a real economy. We will construct such a model in Part II of this book using the Japanese economy as an example.

An outstanding feature of the multi-sectoral model of the Japanese economy described in the following eight chapters (Chapters 6 to 13) is the fact that the model is formulated on the basis of theoretical specifications which are supported strictly by empirical evidence. In specifying theoretical components of the model, no assumptions are made *a priori* such as mutual independence of individuals' preferences in consumption and linear homogeneity of production functions, which are usually assumed for the purpose of assuring the existence of unique and stable solutions for a purely theoretical general equilibrium system.

Since the theoretical perspective of the market encompassed by our "quantitative theory of price" is not limited within the confines of the narrowly defined Neo-Classical concept of the market, the specification of our model is free from such conventional theoretical constraints. Instead, the model is built incorporating such elements as shifts in consumer preference which allow for interdependence of preferences, and non-homogeneous production functions which are convenient to take into account explicity economies of scale in production. Recognition of economies of scale in production, however, will not always be compatible with the presumption of perfectly competitive equilibrium. In order to circumvent this difficulty we have devised an instrumental concept of "anticipated demand." With the help of this concept, the model can now properly deal with the possibility of imperfect market competition in a broader sense which has been discussed in the previous chapter.

All these theoretical components are specified on the basis of empirical findings obtained from many research studies conducted by the authors and their colleagues. The unique approach for specification of the model such as this has been adopted primarily for the purpose of making the model as close a description of the real economic system as possible. This is an essential requirement for a model to be capable of providing realistic policy implications in precise quantitative terms.

Evaluated in the light of the development of empirical economic analysis, our multi-sectoral model is built by critically incorporating methodological achievements of various approaches of quantitative analysis. Before presenting our critical review and evaluations of the merits of previous development upon which our model building is based, let us introduce briefly the major streams of such development. In retrospect, the development may be summarized in the following three streams.

The first stream of empirical analysis is the analysis of the behavior of economic decision making units which has been developed on the basis of Neo-Classical micro theory, such as the analysis of consumption, production, investment etc. Although this type of analysis is compatible, in principle, with "general equilibrium" type analysis, it has nevertheless been carried out mostly in the context of "partial equilibrium" analysis largely due to inadequate development of appropriate methodology by which to handle empirical analysis of general equilibrium.

The second stream of empirical economics may be found in Input-Output analysis initiated by Leontief. Although he intended to develop a scheme of analysis to encompass all economic activities endogenously within its model, or in short, a "closed model," the subsequent development of input-output analysis has not sufficiently attained this goal. Most input-output models currently being put to use are of the "open model" type.

The third stream of development is found in the so-called "macro econometric models" which have been formulated primarily on the basis of Keynesian theory. While macro models are used these days not only as a major tool of empirical analysis of economies but also as the most popular tool for short-term economic predictions and policy planning, one of their shortcomings from our point of veiw is that they are not amenable to the specification of multi-commodity markets. This drawback makes macro models incapable of dealing with the determination of relative prices of commodities.

The multi-sectoral model developed in this book takes full advantage of merits of all three streams while minimizing the shortcomings associated with them. The model contains as its building blocks micro behavioral components of consumption, production, and investment etc. all of which are specified on the basis of our intensive micro empirical analyses. The model, which is subdivided into four sectors, can enjoy the advantage of Input-Output analysis in describing the interdependent structure of the economy. Not only this, the model is also capable of analyzing the determination of relative prices of commodities thanks to its inclusion of demand and supply schedules specified for composite commodities classified by industry sectors. Moreover, the model enjoys practically all the virtues of conventional macro models in conducting simulation analysis of various macro economic policies. With this preview in mind, let us now make a detailed critical review of the theoretical and methodological development of quantitative economic analysis as a preparatory step for introducing our quantitative model.

5.2 Partial Equilibrium Analysis and General Equilibrium Analysis

When we interpret general equilibrium theory as a theory of empirical science rather than a gospel of *laissez faire,* we necessarily have to qualify its relationship with partial equilibrium analysis. As Schumpeter pointed out repeatedly, the difference between Walras and Marshall was not the difference in the way they recognize economic phenomena but the difference in their approaches to the interpretation of complex phenomena in the theoretical framework.

Cournot, a predecessor of Marshall in partial equilibrium analysis, had already exhibited a clear-cut recognition of general interdependence of economic phenomena as early as 1838.[1]

> "So far we have studied how, for each commodity by itself, the law of demand, in connection with the conditions of production of that commodity, determines the price of it and regulates the incomes of its producers. We considered as given and invariable the prices of other commodities and the incomes of other producers; but in reality the economic system is a whole of which all the parts are connected and react on each other. An increase in the income of the producers of commodities *B, C,* etc., and the incomes of their producers, and , by its reactions, will involve a change in the demand for commodity *A.* It seems therefore, as if, for a complete and rigorous solution of the problems relative to some parts of the economic system, it were indispensable to take the entire system into consideration. But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically."

As can be seen clearly from this passage, Cournot, while recognizing fully that an economy is a system of general interdependence, chose the method of partial equilibrium analysis because of its feasibility. It is evident from the above quotation that Cournot was interested in specific form of economic equations and numerical values in parameters, and this indicates eloquently that Cournot's understanding of economic phenomena was highly emprical in nature.

In 1874, the year when he published the first part of his general equilibrium theory, Leon Walras, being aware of this inclination of Cournot, wrote to him telling that he was sympathetic to Cournot type approach though he himself took another road.

All this seems to suggest that early Neo-Classical economists clearly recognized that partial and general equilibrium analyses are the outgrowth from the same root and that they knew fully that it was meaningless to suggest that one was superior than the other although it was not meaningless to discuss differences between the two. However, once these theories started to develop along their own paths, it was probably inevitable that, while partial equilibrium analysis kept a strong interest in empiricism, general equilibrium analysis reinforced its abstractness.

It is well known that Marshall, who emerged as the grand master of partial equilibrium analysis when he published the first edition of *Principles of Economics* in 1890, which systematized the theory comprehensively utilizing the famous *ceteris paribus* clauses, had a strong inclination toward empiricism. This can be seen, for example, from the fact that when he presented a speech entitled "*New and Old Generations of Economists*", he preached that economics should proceed from traditional qualitative analysis to quantitative analysis. Marshall knew well the general interdependence of economic relations, but since he also understood the difficulty of analyzing the whole system as it is, he invented his own method of analysis of economic relations without jeopardizing the empirical applicability of the analysis. Thus, Marshall advocated the postulate of "the negligibility of indirect effects", linkening it to the oft-used method in celestial mechanics, introduced by Leipnitz and Newton, of approximating the interaction effect of the gravitation between two celestial bodies through direct and indirect effects. And he asserted the effectiveness of partial analysis as the first approximation.

The effectiveness of partial equilibrium analysis is limited when evaluated from the view-point of general equilibrium analysis since the postulate of "the negligibility of indirect effects" is not as powerful as Marshall has expected. Nevertheless, the method of drawing either a demand or a supply curve on the assumption of "other things being constant" has undoubtedly proved to be useful in analyzing real economic conditions. On the other hand the general equilibrium analysis, while having contributed to the clarification of the limit in the degree of approximation of partial equilibrium analysis, was not effective by itself in empirical analysis.

As Henry Moore, the outstanding pioneer who attempted quantification of general equilibrium analysis, quotes in his book *Synthetic Economics*, the general equilibrium analysis has been viewed as "castello incantata" (an imaginary magic palace) or "a theory which has hardly anything to do with the reality," even by Umberto Ricci, a successor of Pareto, as late as 1924.[2]

Henry Moore points to several factors as reasons why general equilibrium theory can not be realistic, such as: the assumption of perfect competition, the assumption of statics, the assumption of instantaneous adjustment, unspecified forms of mathematical equations, and the difficulty associated with obtaining actual solutions of a system of simultaneous equations.

Despite his ambitious attempts, Moore was not so successful to realize his goal. Many works by Moore and his successor Henry Schults and others, which tried to preserve the standpoint of general equilibrium analysis by introducing the concept of *cross-elasticity*, can be regarded as having contributed in the improvement of the methodology of partial equilibrium analysis in the course of development of econometrics.

From the viewpoint of general equilibrium analysis, the introduction of the concepts of cross-elasticity and income elasticity, which enables us to analyze shifts in the Marshallian demand curve in response to changes in prices of other commodities and income levels of purchasers, is certainly an improvement. Nevertheless, the method of analysis which concentrated on markets of individual commodities has traditionally been that of partial equilibrium analysis.

5.3 The Development of Quantitative General Equilibrium Analysis

It was Wassily Leontief's *The Structure of American Economy* which, some ten years after the publication of Moore's *Synthetic Economics*, brought about substantial progress in the quantification of general equilibrium analysis.[3] Leontief has first successfully formulated a Walrasian framework of empirical analysis by specifying input-output linkages of intermediate goods among production sectors. Leontief certainly had good reasons to criticize conventional partial equilibrium analysis, describing it as "shop-worn partial-equilibrium concepts."[4] At the same time, it should not be forgotten that improvements in the collection of basic statistical data and the development of electronic data-processing systems have made considerable contribution to this type of analysis.

Ten years after Leontief's book, a macro econometric model was formulated by Lawrence Klein which was another epochal achievement in the history of econometrics. His book, *Economic Fluctuations in the United States, 1921-41*, is based theoretically on general equilibrium theory in the tradition of the Lausanne School.[5]

However, the actual form of his analytical model, made specific through aggregation procedure, may appropriately be classified in the category of macro analysis which was originated by Cournot. The development of econometric method through a series of works in statistics promoted under the auspices of the Cowles Commission should also be remembered side by side with the Keynesian theory as a contributory factor to Klein's achievement.

In the 1950s and early in the 1960s, as depicted above, three streams of quantitative economic analyses were developed simultaneously: partial analysis in the tradition of Moore and Schultz, input-output analysis originated by Leontief, and macro models constructed earlier by Tinbergen and Klein. The enrichment of basic statistical data systematized notably in the form of national economic accounts by Simon Kuznets and others, refinement of economic theories including mathematical method, and development of electronic computers were indispensable for these quantitative economic analyses to be developed to such an extent that they could be put to daily use in the public interest. Conversely, it should also be born in mind that the development of econometric analyses itself have promoted innovation in these technological areas.

Shumpeter has aptly pointed to the fact that the concept of Keynesian macro income analysis had already been contained in the perspective of Walrasian general equilibrium analysis. More specifically, this point is well exhibited in the process of theoretical specification of Klein's model. Chenery and Clark, in the first chapter of their book *Inter-industry Economics*, classify three specific types of analysis which may be used to quantify general equilibrium analysis: partial analysis, input-output analysis and macro income analysis.[6]

Partial analysis deals with market determination of price and output by explaining actions and interactions of producers and consumers in a commodity market. In other words, partial analysis focuses on one of many sectors composing the Walrasian system and analyzes the interactive relationship of actors in singled out market, by means of specifying supply and demand functions. Each of the functions is specified on the assumption that variables in other markets are constant, and because of this treatment, income levels of purchasers are left without being explained explicitly in the demand function. By the same taken, factor prices, such as wages, remain as an unexplained variable which shifts the supply function.

In contrast, input-output analysis focuses on changes in factors themselves which are assumed to be constant in partial analysis. The input-output analysis of intermediate goods, in particular, presumes that their quantities change in response to changes in the levels of production of various industrial sectors through a set of fixed input coefficients and not through the pricing mechanism of the market.

Macro income analysis, on the other hand, analyzes how the aggregate volumes of consumption demand, investment, production and income are determined corresponding to alternative values of exogenous and policy variables without getting involved too much in analysis of the determination of prices and quantities in commodity markets.

Although the three types of analyses reviewed above utilize in common analytical concepts contained in the perspective of general equilibrium analysis, they employ distinctive approaches most useful for their own objectives. They employ different approaches because it has been difficult to translate the general equilibrium theory into the scheme of empirical analysis in its full-fledged form. Each of the above listed approaches naturally has its own merits and demerits depending upon the way it is specified. Therefore, these approaches have been used alternatively in the light of specific objectives of analysis.

However, going back to the original thoughts of Leontief when he first formulated input-output analysis the open-model analysis might not have been the eventual goal. It seems that his original aim was to construct a closed-model in which the final demand is determined endogenously. But for instrumental reasons, he developed an open-model analysis which utilizes fixed input coefficients as the first approximation. It was found in the subsequent empirical analyses that input coefficients of intermediate goods are highly stable at a point in time and therefore the concept of the fixed coefficient approximates reality quite well. However, in terms of usefulness in analyzing factor inputs, the assumption of fixed capital or labor input coefficients may not be necessarily regarded as the best specification. In view of the progress in the methodology of empirical analysis, enrichment of basic data and development of electronic computers in the last two decades, we have now probably reached a stage where we could and should reformulate the theoretical framework of input-output analysis on the basis of Leontief's original thought and use it as a quantified scheme of general equilibrium analysis. Similar progress is needed in quantification of macro economic analysis, which was initiated by Tinbergen and developed significantly by Klein.[7]

5.4 Keynesian Macro Model and General Equilibrium Theory

In constructing his quantitative model, Klein first explained theoretically the character of the equations contained in the model in terms of general equilibrium concepts previously developed by such forerunners as Walras and Pareto up to recent theoreticians as Hicks and Samuelson, and then proceeded to quantitative analysis. His book, *Economic Fluctuations in the United States*, presents an example of highly sophisticated and well thought-out design of experiment of economic science, particularly in its careful match between abstract theoretical constructs and statistical data.

Klein explicitly took up the difficult problem of aggregation; how the structural equations of a macro model may be derived from the two basic behavioral theories of the consumer (household) and the producer (firm) which together constitute the basic content of Neo-Classical general equilibrium theory.

The procedure of aggregation may be described as follows. First, a consumption function for a household is derived assuming that each household will maximize its utility according to its preference function and budgetary constraints. The consumption demand function is derived for each of various present and future commodities. Although the form of the preference function is not specified, the consumption demand function for each good is approximated by a linear equation at least in the neighborhood of the equilibrium point.

The left-hand side of each demand function is expressed in value terms (price × quantity) and the values for all the present commodites are summed through equations. The left-hand side of the aggregated equation thus will become the total consumption value for households. The right-hand side of the linearly approximated equation of a general equilibrium-type demand function inevitably contains a linear combination of prices of all goods as an independent variable together with the variable of household income. Klein replaces this by a general price index in the second step of approximation.

As a result, the Keynesian consumption function is obtained in which the real consumption value is expressed as a linear function of real income. By eliminating the demand for future commodities from his aggregation procedure, Klein in effect equates the demand for future goods as saving.

By aggregating respective terms of household consumption functions thus derived for all households in a country, Klein relates the real consumption in national income as a linear function of real personal income, and makes this as the macro consumption function. The constant term (intercept) of this consumption function is the sum of constant terms of individual household consumption functions and the coefficient associated with the real income is the weighted average of coefficients of individual household consumption functions by a factor of real household incomes. Thus, Klein in effect gave a general equilibrium-type interpretation to the macro Keynesian consumption function.

In short, the macro consumption function is derived by first aggregating commodities and then households, on the assumption that consumption demand functions of individual households for each commodity are of a linear form. As mentioned earlier, it is a feature of this method that a linear combination of prices is replaced by a general (consumers') price index. Klein's aggregation is made bearing national income statistics and various composite indices in mind.

Let us now see how the investment function, the other side of the coin of the Keynesian macro model, is treated in Klein's model. The investment function is viewed generally as a factor demand function within the framework of marginal productivity theory. The production function under the given technological constraint is assumed to be approximately by a log-linear (Cobb-Douglas form) function of machine-hour capital input and man-hour labor input. If an individual firm is assumed to maximize profits under such technological constraints, the theoretical proposition will be derived that, as a condition of profit maximization with respect to fixed capital input the marginal value productivity of capital service is equal to the price of capital service. If we assume that the capital input during the present period is a linear function of the capital stock existing at the end of the previous period and investment during the current period, then we can distinguish a portion of the demand for fixed capital for this period which is satisfied by investment.

If we assume that the prospective price of a product during the time horizon of investment is approximated by a linear function of prices of the current and previous periods, the amount of investment during the current period will be determined as a linear function of values of production in the current and previous periods and the capital stock at the end of the previous period.

Actual individual firms may well be producing more than one kind of products and employing more than one type of capital and labor. Where this is the case, there inevitably remains the problem of intra-firm aggregation; i.e. each firm is treated just as though it is producing a single kind of product using only one type of capital and labor, assuming that the composition of different types of products and factors remains intact.

At any rate, insofar as the investment function of each firm is approximated linearly, the macro investment function can be obtained by merely aggregating respective terms of individual firm's investment functions. The coefficient associated with each of the independent variables of the aggregate investment function will be, likewise the case of the aforementioned consumption function, the weighted average of the corresponding coefficients in individual firms' investment functions. According to this macro investment function, which is measured in terms of national income and wealth, the real private investment during the present period is obtained as a linear function of real national product of the current and the previous periods and the real capital stock at the end of the previous period. The real national product in this case is of the form that the nominal value inflated by the product price index and deflated by capital price index.

The fact that either the macro consumption function or macro investment function is expressed in terms of real values is the consequence of the treatment that the linear polynomials of prices which always accompany a linear approxition of a general equilibrium system are replaced by a general price index. The kinds of variables contained in structural equations of macro models obtained as aggregated theoretical equations may vary depending upon the underlying micro theoretical formulation. It is nevertheless true that the consumption and investment functions as described above constitute the prototypes of the basic structural equations of macro economic models which have been measured and developed later in many countries.

When Klein derived behavioral equations in accordance with theories of utility maximization in the household and profit maximization in the firm, he assumed competitive market, i.e. market prices are taken as given to individual behavioral units. However, Klein considered also the possibility of imperfect markets. The possibility of imperfect competition can be dealt with, in his framework of analysis, without altering functional forms since he assumed that the price elasticity of demand was constant and thus that the constant elasticity was merely reflected in the value of structural parameters.

The supply function of commodities can also be derived as an application of the marginal productivity theory of profit maximization behavior. Klein did not attempt this but simply derived factor demand functions. This is probably a reflection of demand oriented viewpoint underlying the Keynesian macro models.

After having derived structual equations in the demand side of a macro economy, such as consumption demand based on the theory of household behavior, demand for labor, fixed capital equipment and inventories based on the theory of behavior in the firm, Klein also touches upon the market determination of prices.

In dealing with the determination of prices, he shifted his focus away from the micro empirical viewpoint of demand and supply behavior and their interactions to the abstract problem of market stability conditions in pure theory of general equilibrium. In other words, Klein derived market adjustment functions making use of the pure theoretical market stability conditions which had been developed by Hicks, Samuelson, Lange and Metzler and on this basis he suggested relationships between price and inventory fluctuations. This aspect of his model has also influenced the method of constructing macro economic models. However, this aspect seems to have remained one of secondary importance in the general stream of demand oriented Keynesian models.

According to Schumpeter, the Keynesian macro economic analysis belongs to the general type of aggregate concept developed by Cournot and Marshall. It is interesting that Klein consistently tried to deal with the problem of theoretical aggregation from the viepoint of Walrasian general equilibrium theory.

There remains one problem in this regard; i.e.: whether it is valid to use the linear approximation as a rationale for linear approximation of structural equations on the ground that it is acceptable in the neighborhood of the equilibrium point. It is usually acceptable when purely theoretical analysis is made focusing strictly on one point of equilibrium and its neighborhood. It is the method conventionally used also in physics. However, in the case of an econometric model corresponding to actually observed statistical time-series data, it is the locus which combines more than several points of equilibria rather than an isolated equilibrium point and its neighborhood that is really relevant, since our concern is with the movement from one equilibrium point to another. Take the case of consumption function for example, the problem is to check empirically whether a linear approximation may be accepted throughout the actual domain of variation in incomes and prices. This is not the kind of question which may be proved making use of mathematical *a priori* notions.

Notes to Chapter 5

1. Cournot (1838) in English edition translated by N.T. Bacon, p.127.
2. Moore (1929).
3. Leontief (1941).
4. Leontief *op.cit.* p. 34.
5. Klein (1950).
6. Chenery and Clark (1959).
7. Tinbergen (1939).

Part II Empirical Analysis of a General Equilibrium System
— A Four Sector Model —

Chapter 6
The Theoretical Setting and Model Building

6.1 Theoretical and Methodological Backgrounds

1. The Principle of Maxima

We have said earlier that the pursuit of norms in the pure theory of general equilibrium does not necessarily imply the advocacy of an unlimited degree of *laissez faire*. In other words, we ascertained that *laissez faire* and perfectly competitive market are not synonymous. We therefore need to analyze empirically the working of an actual economy and to examine the effects of institutions and policies regulating the market. This task could not be accomplished by merely accumulating statistical data. What is needed is to analyze the observed data theoretically and to clarify the network of cause-effect relationships. It is here that general equilibrium theory as an empirical science is called for.

Cautious economists have often used an expression "general interdependence" instead of "general equilibrium" for the purpose of avoiding the normative connotation associated with the latter and in order to emphasize their intention of empirical analysis. The basic analytical tools for analyzing general interdependence in this situation are provided by the analytical framework of Neo-Classical economics, including not only the general equilibrium analysis of Walras and Parato but also the partial equilibrium analysis of Cournot and Marshall. In terms of empirical analysis, the latter group has played a rather important role.

Among theoretical variations of Neo-Classical economics, the theories developed by the Lausanne School, which elaborated on the concepts of the existence and stability of market equilibrium, may not be dependable for empirical analysis since they contain too many assumptions without the support of empirical evidence. We need to have a theoretical framework which can describe theoretically the process in which incomes, prices, employment, output etc. in an actual economy are determined for each year.

The basic standpoint of our methodology is to make full use of Neo-Classical micro theory and Marshallian market concepts as tools to analyze general interdependence.

In criticizing the Neo-Classical economic theory, Galbraith expresses skepticism not only about market theories but also about behavioral theories of actors.[1] He maintains that both the utility maximization principle of consumer's behavior and profit maximization principle of firm's behavior do not reflect the reality. Galbraith's criticism is valid insofar as the Neo-Classical theory asserts the normative principle of *laissez faire*. His accusation is understandable particularly when it is directed against the utilitarian interpretation of the concept of utility. However, his criticism is pointless when the concepts such as utility maximization or profit maximization are interpreted as purely analytical concepts of empirical science.

There is a criticism, for instance, that the principle of utility maximization does not apply because the preference of the consumer in contemporary society is distorted by advertising. While this criticism contains an element of truth in that the sovereignty of consumers is being eroded by the pressure imposed by sellers, whether sellers take such an action in order to influence consumers' preference and whether the consumer acts according to his own preference are separate questions which must be clearly distinguished.

The assertion that modern large firms do not aim to maximize profits but rather to stabilize and increase the volume of sales is often made drawing upon the results of opinion or attitude surveys of managers of large firms. This is however only a new expression of the old concept of the full cost principle. That the cost accounting and the determination of sales prices are perceived by many of managers as being governed by the full-cost principle, and that the principle of profit maximization is used as an objective analytical principle of the observed behavior are clearly two separate questions. It has often been pointed out that the full-cost principle itself is incapable of explaining the gap between the cost and the selling price. Unless the problem of arbitrariness involved in mark-up ratios is resolved, the full-cost principle can not serve as an objective analytical principle.

As pointed out by Irving Fisher and stressed by Ragner Frisch, while the sense of muscular movement gives one the tangible notion of power intuitively, it is different from the concept of force in dynamics.[2] The latter as an analytical concept may be defined only by a certain relationship between abstract mathematical notions of space, time and mass.

Specification of analytical concepts and theories in economics as an empirical science may not be carried out by merely conceptualizing day-to-day casual experiences. Maximization of utility or profits and minimization of costs as analytical principles are not the kind of questions whose validity can be examined by introspections of a consumer or a firm manager.

It is common to all empirical science that the principle of maxima is convenient in analyzing theoretically the observed facts. The merit associated with the presumption of maximization of utility and profit is no more than this. Therefore, it is not the analytical principle that empirical validity is questioned, but the degree of approximation of the entire theoretical system which is constructed on the basis of the analytical principles.

Needless to say, the utility maximization principle is used only as an analytical principle and not with any normative implications. The utility maximization principle as an analytical concept has nothing to do with the argument, for example, that a consumer should enjoy freedom to maximize his utility within the constraints of a given income and relative prices. Likewise, the labor demand function derived from the relationship between wages and value marginal productivity on the basis of the profit maximization principle has nothing to do with verifying or nullifying the incidence of exploitation.

Taking advantage of principles of maxima and minima, we may thus make use of many of the concepts contained in theories of consumer's preference and marginal productivity for the purpose of empirical analysis. From the theory of consumer's preference we may derive a system of consumer demand functions, and from the theory of firm's behavior based on the marginal productivity thesis we may derive labor demand functions and investment functions as we have seen in the case of Klein's model.[3]

2. Demand and Supply Curves

Klein has suggested a method of introducing market adjustment functions which is distinct from the approach as described above. Unlike Klein, we have chosen an approach by which to derive short-run commodity supply functions together with demand functions for production factors and raw materials from the theory of behavior of a firm. The supply function for each commodity thus derived will then be confronted with the corresponding demand function and the determination of price and quantity will be explained by the interaction of supply and demand. We use this approach because it is more convenient to follow the analytical framework of partial equilibrium analysis as developed by Cournot and Marshall for our immediate purpose of describing in concrete terms the determination of price and quantity which are expressed in the form of statistical data rather than of describing the existence of stable equilibrium in the market in abstract theoretical terms.

In our model, demand and supply functions for various commodities contain common variables. Therefore, the market equilibrium for each commodity will not be determined independently from other commodities, but rather prices and quantities of all commodities will be determined simultaneously through mutual interactions. This feature shows up clearly in the system of consumer demand functions. Since the demand for each commodity is a function not only of its price but also of prices of other commodities, the demand-supply equilibrium for the commodity will not remain independent from other commodities even though the supply curve for each commodity is independent of the others.

Moreover, since in the system of general equilibrium, sectors are mutually related by the input-output relationship of intermediate goods as described by Leontief, changes in the amount of supply (production) of a commodity will lead to changes in demand for other commodities through concomitant changes in inputs of intermediate goods. The supply curve of a commodity, therefore, can not remain independent from the prices of other commodities and, consequently, from the supply curves of other commodities.[4]

The supply price of each commodity depends not only on prices of raw materials used in its production but also on labor costs. Since labor markets for various industries are inter-related, as are wage levels in the various markets, commodity supply curves for different industries are not independent of each other. The system of general interdependence is such that a set of commodity demand schedules and a set of commodity supply schedules are composed together and the prices and quantities of all commodities are determined simultaneously. Our intention is to describe such a complex system quantitatively and to facilitate understanding of the movements of the actual economy in relation to basic theories.

3. The framework of Input-Output Table and Theoretical Components of the Model

It is convenient to make use of an Input-Output Table for the purpose of over-viewing the general interdependence of the entire economy. Reading the Input-Output Table vertically, one can see how much of intermediate goods and other factors are put into each industrial sector, and by examining the Table horizontally one can see how much of the produced output from each industrial sector is distributed as intermediate goods for use in other industries or as final commodities for consumers. The final demand is usually classified into household consumption, firm investment, government spending and exports.

The household is a consumer as well as a supplier of labor services while the firm is a purchaser of investment goods, short-run productive factors such as labor and capital services and also is a producer and supplier of commodities. Government demand is viewed as exogenous since it is determined outside the market system. The external demand for export is interpreted as being quasi-exogenous since it depends largely on levels of income and economic conditions of foreign countries, although it is influenced to some extent by an endogenous factor, i.e. export prices.

The so-called "open model" is a frequently used method of input-output analysis by which output levels of industrial sectors are determined under the given level of aggregate final demand. While this method is relatively easy to handle, it gives only a limited amount of information, as we have mentioned earlier. Instead, we try to envisage a closed model in the perspective of the input-output table. We try to describe, under the given exogenous and quasi-exogenous constraints and initial conditions such as capital stock and labor force at the beginning of each period, how prices and quantities of commodities and incomes of economic actors are determined by the market equilibrating processes of interactions between households and firms.

The endogenous demand for each commodity will be determined in this framework as a composite of consumer demand, investment demand and intermediate good demand. The demand schedule is given consequently by the system of consumer demand functions, sectoral investment demand functions and input functions of intermediate goods. In contrast, the supply schedule is given by the short-run supply function of each industrial sector.

Our model resembles models developed, for example, by R. Stone in which the system of consumer demand functions is derived from the preference functions of households.[5] However, we are fully aware, in view of the findings of our previous research, that it is impossible to measure theoretically consistent consumer demand functions without explicitly allowing for shifts of preference functions due to habit formation. Our consumer demand functions, therefore, do not presume the constancy of preference which has been assumed implicitly by the conventional approach of Neo-Classical school. As will be shown later, the structure of consumption will change if consumers' preferences change due to self-generated shifts in habit formation even though income levels and prices remain unchanged. This change in consumption structure will feed back through interactions with supply conditions and consequently affect relative prices and income levels. In other words, a change in consumption structure generated by shifts in consumers' habits will have a disequilibrating effect in the actual passage of time and will jeopardize the maintenance of the Walrasian stationary state.

In this sense, our consumer demand functions will contain non-Neo-Classical theoretical elements.

The firm's short-run supply function and investment function are derived from a production function which expresses technological constraints on production. In our model, we use the SFS (Semi-Factor-Substitution) production function instead of the well-known CES and CSE production function.[6] While the latters usually utilize as a homothetic function, which include Cobb-Douglas type function as its special case, our SFS function is principally of a heterogeneous form. The Neo-Classical marginal productivity theory usually handles the problem of capital-labor substitution counting labor input in terms of man-hour units. We, however, distinguish between man (the number of workers allocated to machines) and hour (hours of operating the machine). We presume that the capital equipment and workers attached to it are combined perfectly complementarily by a certain technologically determined ratio for a given level of production capacity per time unit. The relationship between capital equipment and labor is already technologically determined when the equipment is designed. In other words, there is no substitutability between capital equipment and the number of workers for equipment of a given productive capacity, even at what L. Johansen refers to as the *ex ante* stage.[7] Johansen gave an interpretation to the Neo-Classical theory of production by viewing that while labor and capital are substitutable at the stage where the equipment is being designed the substitutability will be lost once the investment takes a concrete form of equipment. The presumption behind our SFS production function is clearly different from such an interpretation.

In the SFS production function, when the level of production capacity per time unit is determined by the equipment and workers attached to it, the level of output will be changeable by hours of operation (either per week or month). Therefore, there remains some room for substitution between the capacity and hours of operation and between capital input and man-hour labor input for a given level of output. It is for this reason that we call our production function the "Semi-Factor-Substitution" type. The technological economies of scale with respect to labor input are actually observed in the relationship between the output capacity of equipment and allocated labor force. Increases in outputs in the process of economic growth shows the effect of increasing labor productivity through expansion of productive capacity. Our SFS production function has been developed in an attempt to introduce the actually observed economies of scale into the theoretical specification of the production function without inviting contradictions with the Euler's theorem on the distribution of products among factors of production.

4. The Treatment of Elements of Imperfect Competition

It is also in the treatment of elements of imperfect competition that our model differs from the conventional specification of Neo-Classical general equilibrium theory.

In textbooks, the equilibrium condition for a producer facing a competitive market is given by price = marginal cost, and for a producer facing an imperfect market is given by marginal revenue = marginal cost. While the existence of general equilibrium and its stability conditions have been studied in mathematical economics assuming competitive markets, it has been considered difficult to do the equivalent analysis for the case of imperfect competition. This is because in the case of imperfect competition the suppliers react not to given market prices but to the demand schedules of purchasers, and the thus determined supply has to interact with the actual demand schedules to reach a market equilibrium. In this case, the demand curve which is reflected in the supply action of the producer and the demand curve which interacts with the supply curve in the market are not usually conceptually distinguished.

The difference between theories of perfect and imperfect competition depends on whether the supplier considers the market price as being given or not. To use Marshall's expression, the difference depends on whether an increase in supply would make the supplier worry about "softening" of the market. If the demand curve in the actual market is parallel with the horizontal (quantity) axis, then the price would not fluctuate. The theory of competitive market, however, does not exclude the possibility of price fluctuations. That the supplier considers the price as given and that the actual demand curve is horizontal are two separate matters which should be clearly distinguished theoretically. In other words, the former is the demand curve conceptualized in the mind of a producer when he evaluates possibilities of softening of the market which accompanies increases in his supply to the market, or perhaps what might be called the "anticipated demand curve."

Once we distinguish the anticipated demand curve from the actual demand curve in the market, it will not be as difficult as it used to be to obtain a general equilibrium solution since it is the former which is involved in the determination of the supply curve and it is the latter which interacts with the supply curve to determine jointly equilibrium price and quantity in the market.

Our model of market equilibrium thus allows for possibilities of imperfect competition. Therefore, we measured the anticipated demand function which is contained in the supply function for each commodity separately in addition to the set of market demand functions composed of the three groups of demand functions: consumer demand function, investment good demand functions, and intermediate input functions. Note that the parameters of the anticipated demand function are obtained through the structural estimation method applied to the action of the supplier instead of that of the purchaser. We did so because otherwise we would in the process of actual measurement nullify the concept which we have carefully distinguished in theoretical specification.

5. The Design of Experiment

As has been stressed, we have tried to make use of micro economic theories not to support the doctrine of *laissez faire* but as building blocks of a model which can be used to evaluate the effects of institutions and policies in the actual economy. To achieve this goal, we need to show explicitly the relationship between theoretical constructs and observable data. We call this process "design of experiment" in economic science.[8]

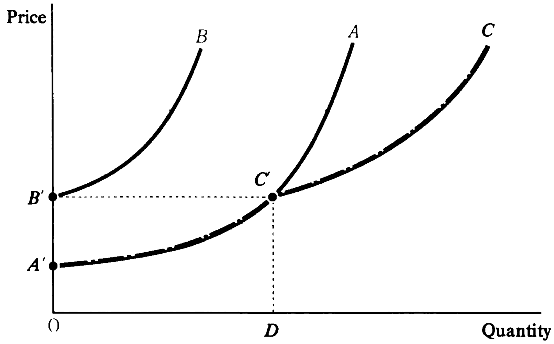
When we attempt to analyze general inter-dependence within the entire economy, the basic information we have to rely on will be provided by such tables of national accounting as *National Income Statistics*, *Input-Output Table*, and *the Table of Money Flow*.

As for the consumer demand function, aggregate amounts of personal consumption classified by expenditure categories are listed in National Income Statistics. Dividing them by the number of household we estimate the values of consumption items and use them as measurable counterparts of household consumption in the theory. The preference function in this case is therefore the preference function which corresponds to the itemized consumption of "an average household", as suggested by Jevons when he put forward his marginal utility theory for the first time. Thus, the problem of comparison of preference patterns between different households is excluded from the beginning.

A similar treatment is used in dealing with the firm's behavior. Each industry sector contains many firms. The distribution of firms by size would present an important problem in this situation particularly when the assumption of linear homogeneity of the production function is relaxed. However, it is quite difficult to analyze the activities of each industry sector by means of decomposing it down to the level of individual firms.

Suppose that the short-run supply functions of firms A and B are as those described in Figure 6.1. Since Firm B will not be in a position to compete with Firm A until the market demand reaches the level D, the supply curve of Firm A serves as the market supply curve. Once the market demand exceeds the level of D, Firm B will be able to compete with Firm A at each

Figure 6.1 AN ILLUSTRATION OF AGGREGATION PROBLEM—SUPPLY SCHEDULES OF INDIVIDUAL FIRMS AND THE MARKET SUPPLY SCHEDULE



Note: Lines AA' and BB' represent the supply schedules of Firm A and Firm B, respectively, and line $CC'A'$ represents the aggregate supply schedule in the market.

price level and thus the market supply at each price level will be the sum of supplies of Firm A and B. Consequently, the market supply curve will have the wavelike shape shown by the thick curve. As the number of firms increases, the market supply curve will take a wavelike shape with many more troughs and will eventually be approximated quite well by a continuous upward sloping curve. It is this kind of continuous curve of approximation which corresponds to our aggregate data of commodities for each sector.

Therefore, in theory we treat an industry as equivalent to a single firm and regard the continuous curve of approximation described above as the supply curve of the firm. We then obtain the corresponding production function and profit maximization behavior in a similar manner. This method may appear strange to eyes accustomed to abstract theories for economic doctrines unaccompanied by an appropriate design of experiment. However, it is not an unusual method in empirical science. For example, to make an approximation of the density of the earth it is often presumed that its internal structure is homogeneous while knowing that it is in fact not homogeneous. At any rate, we think of the behavior of a single representative firm in our model as a theoretical counterpart to the aggregate data of each industry sector. In this sense, our analysis of production corresponds quite well with our theoretical specification of the behavior of an average household in consumption.

The micro theory has attracted attention in macro model building ever since the pioneering work of Klein. Unfortunately, however, the tendency has grown increasingly dominant recently by which the goodness of fit is so emphasized in specifying quantitative equations that their link with basic theories are often ignored. Because of this tendency it has often been the case that the precision of the model is judged only by statistical interpolation or extrapolation tests without due regard to its theoretical basis.

Since time-series data observed in the process of economic growth contain elements of secular trend and, moreover, the data series are mutually related by threads of general inter-dependence, they often exhibit much stronger correlations than would be expected from their mutual causal relationships. To illustrate the situation by an example, when there is a causal sequence among variables such as $A \rightarrow B \rightarrow C \rightarrow D\ldots$ it may happen that a sequence such as $A \rightarrow D \rightarrow B \rightarrow C\ldots$ or any other arbitrarily chosen sequence will show correlations which are just as high and are indistinguishable in terms of statistical significance. Unfortunately, statistical data can not be perfect even though they are sufficiently refined to satisfy the logical and quantitative consistencies suggested by the System of National Accounting of the United Nations. They are bound to contain not only probabilistic sampling errors but also conceptual and other errors. As Ragnar Frisch has pointed out, when measurement errors are involved in a situation where serial correlations are already high because of interdependence among the data series, it may even happen that the apparent goodness of fit of structural equations of the model becomes higher in the case where the equations do not represent the true causal relationships than it does in the cases where the equations do represent the true relationships.

The goodness of fit of a model is evaluated not only by interpolation tests but also by final tests which take into account the lag structure of structural equations and by extrapolation tests. It is true therefore that the validity of a model is not judged naively by mere statistical significance. However, such judgements may nevertheless be unduly influenced by the apparent goodness of fit especially when the theoretical reasoning behind each structural equation is weak.

For models developed with undue emphasis on a mere statistical fit, it is often difficult to find causes of the failure when they fail to predict, even in cases of unconditional predictions. They are more prone to failure in the case of conditional predictions such as policy simulations. Even though inadequate variables may be chosen for structural equations or the signs associated with structural parameters may be incorrect theoretically, it may happen that the model still shows an agreeable degree of goodness of fit in interpolation tests or simple extrapolation tests for some coincidental reasons. For instance, it may happen that the effect of a theoretically contradictory sign of one variable is offset by the effect of another such variable. However, in cases where the structural parameter which represents the effect of changes in an exogenous variable has an incorrect sign, it can not be assumed that the effects of incorrect signs of parameters offset with each other when conditional predictions are made in the area apart from the past experience. It is, therefore, quite risky either to make policy simulations using a model degenerated from theoretical grounds or to make simple predictions in cases where environmental conditions have changed significantly.

6. Problems in Estimation

It has been widely recognized, ever since the warning made early by Ragnar Frisch, that the most difficult problem in estimation of economic relationship is multi-collinearity.[9] In cases where the data of variables are highly correlated, estimated regression coefficients may well be biased because of errors in measurement and the biases may sometimes be as large as to alter even signs associated with the estimated coefficients. A probabilistic approach to this problem was developed by T. C. Koopmans and T. Haavelmo, and towards the end of the 1940s it was completed in the form of a structural estimation method through the endeavor of a large scale joint research project under the auspices of the Cowles Commission.[10]

As Frisch pointed out, the biases involved in estimated parameters due to collinearity are basically due to the fact that economic analysis can not resort to controlled experiments as in natural sciences.[11] In the structural estimation method which was devised as an alternative to a controlled experiment, the entire system of general interdependence is expressed by a set of structural equations. In the system, exogenous variables which are theoretically regarded as being outside the network of interdependence are distinguished from endogenous variables which are deemed as being determined interdependently under certain constraints imposed by the aforementioned exogenous variables. To facilitate statistical estimation of economic relationships, the system is designed in such a way that each of the endogenous variables may be expressed as a function solely of a set of exogenous variables rather than of other endogenous variables.

It was in Klein's macro model that the structural estimation method was fully applied for the first time. Since then the estimation of economic models has basically relied on this method although a number of convenient devices developed later have been incorporatd in the subsequent process of enriching the method.[12]

However, the structural estimation method developed in the tradition of the Cowles Commission formulates the stochastic character of economic relationships in terms of a shock model and the aspect which may be appropriately understood in terms of an error model has not been given due considerations. Moreover, when models are actually being estimated, it is not unusual for some of the variables which are not necessarily regarded as exogenous variables from the theoretical standpoint have to be treated formally as exogenous variables due to some constraint, for example, a limit in computational capacity. Consequently, it may often happen that biases in parameter estimates due to collinearity can not be eliminated even though the structural estimation method is being applied formally.

Furthermore, since the structural estimation method is designed relying basically on linear models, the application of it becomes difficult when non-linear models are required from the underlying economic theory. We have pointed out earlier that the rationales for linear approximation, once put forth by Klein could not be supported when the values of variables observed from actual time-series data are interpreted theoretically not as points in the neighborhood of equilibrium but rather as the locus of shifts from one equilibrium point to another. Indeed, our model includes many equations which are theoretically required to be formulated in non-linear forms, We of course try as far as possible to make use of the principles of the structural estimation method. Nevertheless, it is difficult to eliminate biases in estimates due to collinearity completely.

We have tried to show explicitly the range and sign conditions of structural parameters not only in terms of the first order conditions (necessary conditions) but also of the second order conditions (sufficient conditions) which are required from the theoretical specification of optimization whether the estimation is linear or non-linear. We have tried to discern the correct quantitative system of structural equations, avoiding meaningless parameter estimates by incorporating explicitly the theoretically designated conditions of parameters as mentioned above in the statistical estimation procedure, wherever it is necessary to do so.

This kind of method is impossible unless the theoretical rigor of the structural equations is strictly maintained. There do exist cases in which the seeming goodness of fit is improved by so choosing variables at the expense of theoretical rigor. We, however, have taken the alternative course to preserve theoretical rigor even at the expense of apparent goodness of fit. By doing so, we intended to examine the correspondence between the data and the theory rigorously, hoping to get insights into improvements in theoretical specification and in the precision of statistical data.

We also intend to make it possible to understand the impact of institutions and policies upon a number of endogenous variables, not merely in terms of statistical relationships but rather in terms of theoretically clarified causal relationships.

6.2 General Interdependence and the Framework of Social Accounting

The basic aim of our research is to analyze the general interdependence of the economic system in Japan using the data of 1955 to 1967.

From 1964 till 1967, a large scale project was carried out in the United Nations under the chairmanship of Richard Stone for the purpose of revising *the System of National Accounts and Supporting Tables*.[13] This revised system of National Accounting provides a framework of systematizing economic data which encompasses interdependent economic relationships more broadly than in the past that it includes in its production account commodity transactions as depicted by Leontief-type input-output table. In addition to expanding the current system this revision aims at combining systematically the three kinds of flow-concept accounts, namely input-output table, national income statistics and flow-of-fund table, and also incorporating balance sheet account though it is still insufficient.

In Japan, too, prior to the introduction of this comprehensive new SNA, the system of national income accounting was revised considerably in May 1969 for the purpose of facilitating integration with other accounting systems. For three years since 1974 efforts have been payed in Japan along the guidelines recommended by the United Nations in 1968 for systematization of national accounting systems towards the system of full integration of (1) national income statistics, (2) input-output table, (3) flow-of-fund table, (4) balance of payments table, (5) balance sheet account. The systematic data under this new system are made available in December 1977 retrospectively up to 1970. This new set of data were, however, not available for our use in 1972 when the earlier Japanese version of this book was published, and so we had to systematize in our own way the then existed partially integrated data.

An example of rigorous study of a multi-sectoral model using explicitly the system of national accounts may be found in a series of articles produced in the process of constructing Stone's Cambridge model.[14] Stone's study not only utilizes the present national accounting system as a useful device in systematically combining economic data with theoretical constructs but also suggests the directions of future improvements in the system itself to be useful for more elaborate description of economic phenomena.[15]

Table 6.1 is the basic table of comprehensive accounts constructed for the purpose of classifying the variables of our model. As noted earlier, the scheme of each accounting had not yet systematized in Japan along the lines of a complete system as intended by the new SNA when we started our project of data editing and estimation. Therefore, our attempt in constructing this

TABLE 6.1 THE BASIC TABLE OF COMPREHENSIVE ACCOUNTS

INCOMINGS \ OUTGOINGS			NO.	PRODUCTION: INDUSTRIES	CONSUMPTION: EXP.	CONSUMPTION: INCOME & OUTLAY		
NUMBERS				1 ~ 9	10 ~ 18	19~36	37~41	42~50
PRODUCTION		INDUSTRIES (COMMODITIES)	1~9	$T_{1\cdot1}$	$T_{1\cdot2}$	$T_{1\cdot3}$		
CONSUMPTION		EXPENDITURE: HOUSEHOLDS AND GOVERNMENT PURPOSES	10~18					$T_{2\cdot5}$
	INCOME AND OUTLAY	VALUE ADDED	19~36	$T_{3\cdot1}$				
		TRANSFER INCOME	37~41					$T_{4\cdot5}$
		INSTITUTIONAL SECTORS	42~50			$T_{5\cdot3}$	$T_{5\cdot4}$	
ACCUMULATION	CAPITAL FORMATION	INVENTORY INCREASES by PRIVATE ENTERPRISES	51~58					
		INVENTORY INCREASES by GENERAL GOVERNMENT	59					
		FIXED CAPITAL FORMATION by PRIVATE ENTERPRISES	60~67					
		FIXED CAPITAL FORMATION by GENERAL GOVERNMENT	68					
		HOUSING INVESTMENT	69					
		AMOUNTS OF CAPITAL FOR.	70					
	CAPITAL FINANCE	(Dummy) CAPITAL FORMATION	71					
		FINANCIAL CLAIMS	72					
		CAPITAL TRANSFER	73					
		INSTITUTIONAL SECTORS	74~82					$T_{15\cdot5}$
REST OF THE WORLD		CURRENT TRANSACTION	83~86					
		TRANSFER INCOME	87~88			$T_{17\cdot3}$	$T_{17\cdot4}$	$T_{17\cdot5}$
		CAPITAL TRANSACTION	89~90					
TOTAL			91	T_1	T_2	T_3	T_4	T_5

ACCUMULATION										REST OF WORLD			TOTAL
CAPITAL FORMATION						CAPITAL FINANCE							
51~58	59	60~67	68	69	70	71	72	73	74~82	83~86	87~88	89~90	91
$T_{1\cdot 6}$	$T_{1\cdot 7}$	$T_{1\cdot 8}$	$T_{1\cdot 9}$	$T_{1\cdot 10}$						$T_{1\cdot 16}$			$T_{1\cdot}$
													$T_{2\cdot}$
											$T_{3\cdot 17}$		$T_{3\cdot}$
											$T_{4\cdot 17}$		$T_{4\cdot}$
											$T_{5\cdot 17}$		$T_{5\cdot}$
						$T_{6\cdot 12}$							$T_{6\cdot}$
						$T_{7\cdot 12}$							$T_{7\cdot}$
						$T_{8\cdot 12}$							$T_{8\cdot}$
						$T_{9\cdot 12}$							$T_{9\cdot}$
						$T_{10\cdot 12}$							$T_{10\cdot}$
													$T_{11\cdot}$
									$T_{12\cdot 15}$				$T_{12\cdot}$
									$T_{13\cdot 15}$				$T_{13\cdot}$
												$T_{14\cdot 18}$	$T_{14\cdot}$
							$T_{15\cdot 13}$					$T_{15\cdot 18}$	$T_{15\cdot}$
													$T_{16\cdot}$
													$T_{17\cdot}$
								$T_{18\cdot 14}$	$T_{18\cdot 15}$				$T_{18\cdot}$
$T_{\cdot 6}$	$T_{\cdot 7}$	$T_{\cdot 8}$	$T_{\cdot 9}$	$T_{\cdot 10}$	$T_{\cdot 11}$	$T_{\cdot 12}$	$T_{\cdot 13}$	$T_{\cdot 14}$	$T_{\cdot 15}$	$T_{\cdot 16}$	$T_{\cdot 17}$	$T_{\cdot 18}$	$T_{\cdot\cdot}$

TABLE 6.1 THE BASIC TABLE OF COMPREHENSIVE ACCOUNTS

Notes: Number attached at column and row represents respectively,

Production Accounts

Industries (Commodities)
(1) Agriculture, forestry and fisheries
(2) Light Manufacturing Industries
(3) Heavy Manufacturing Industries
(4) Constructions
(5) Whole sale and retail trade
(6) Finance and insurance
(7) Service industries
(8) Whole Industries
(9) Commodity Taxes

Consumption Accounts

Expenditure: Household Goods and Services, and Government Purposes
(10) Food
(11) Clothing
(12) Fuel and Light
(13) Housing
(14) Miscellaneous
(15) Sub-total of Consumption Expenditure
(16) Statistical Discrepancy
(17) Government Consumption Expenditure
(18) Sub-total of Expenditures

Income and Outlay

Value Added
(19) Business Consumption Expenditure
(20) Compemsation of Employees
(21) Income from Unincorporated Enterprises
(22) Income from Property
(23) Personal Income taxes
(24) Income from Corporations
(25) Corporation Income Taxes and Charges
(26) General Government Income from Entrepreneurship
(27) General Government Income from Property
(28) (less) Interest on Public Debt
(29) (less) Interest on Consumer's Debt
(30) (less) Stock Valuation Adjustment
(31) (less) Imputed Service Charges
(32) Operating Surplus
(33) Provisions for the Consumption of Fixed Capital
(34) Indirect Taxes
(35) (less) Current subsidies
(36) Sub-total of Value Added

Transfer Income
(37) Transfer from General Government to Households
(38) Social Insurance Contribution
(39) Transfers from Households to General Government
(40) Transfers from Corporation to households
(41) Sub-total of Transfer Income

Institutional Sectors
(42) Non-financial Corporations
(43) Government Enterprises
(44) The Bank of Japan
(45) Financial sector
(46) Households
(47) Non-Profit Institutions
(48) General Government
(49) Sub-total of Institutional Sectors
(50) Total of Income-Outlay Accounts

Accumulation Accounts

Capital Formation Accounts
Inventory Increases
To (51) through (57) corresponds to the j-th sector where j=1. . . 7
(58) Sub-total of Inventory Increases
(59) Inventory Increases by General Government

Fixed Capaital Formation
To (60) through (66) corresponds to the j-th sector where j=1. . . 7
(67) Sub-total of Fixed capital Formation
(68) Fixed Capital Formation by General Government
(69) Private Housing Investment
(70) Total of Fixed Capital Formation

Capital Finance Accounts

(71) (Dummy) Capital Formation
(72) Financial Claims
(73) Capital Transfer

Institutional Sectors
To (74) through (80) corresponds to each institutional sector
(81) Sub-total of Institutional Sectors
(82) Total of Capital Financial Accounts

Rest of The Worlds Accounts

(83) Export
(84) (less) Import
(85) (less) Custom Duties
(86) Sub-total of Current Accounts
(87) Factor Income from Abroad
(88) Transfer from the Rest of the World
(89) Capital Transfer
(90) Total of the Rest of the World Accounts
(91) Total of Whole Accounts

table may be interpreted as an attempt to integrate National Income Statistics and Input-Output Table following the general scheme of the comprehensive basic table of the new SNA.

The comprehensive Table 6.1 sub-divides the system of social accounting basically into five accounts: production account, consumption account, capital formation account, capital finance account, and account of the rest of the world. In contrast, the new SNA recommended by the United Nations in 1968 divides the entire system largely into four accounts such as production account, consumption account, accumulation account and account of the rest of the world, and then sub-divides each of the first three accounts further into such items as commodity-activity, consumption expenditure-income and outlay, capital formation-capital finance. Thus, its basic structure of accounting system does not differ much from ours.

The Structure of the System

In the case of the new SNA, the production account is sub-divided into commodity and activity accounts. The commodity account consists of commodities defined in terms of principal products of industries. The activity account consists of three categories: (1) industries, (2) producers of government services and private domestic services, and (3) producers of private non-profit services to households. The Japanese Input-Output Tables are based either on the commodity concept in terms of the concept defined in the new SNA. Therefore, we do not distinguish commodity from activity in our system of accounting unlike new SNA. This principle will be maintained even if commodity classifications are consolidated.

While for the sake of expository convenience each of the seven consolidated categories of commodities is attached an industry name in Table 6.1, we must note that each of these categories is nevertheless the consolidation of commodities. In the ensuing explanations we will call each of such sectors of integrated commodities "industry." But it must be emphasized that the industry in our sense is different from the usual concept of industry which is a composite of industrial establishments. Therefore, $T_{1\cdot 1}$ in Table 6.1 is interpreted to imply the intermediate input transactions between a commodity to another commodity in the new SNA.

Private consumption expenditure is divided into five consumption items the same as in National Income Statistics (food, clothing, fuel and light, housing and miscellaneous items). Notation $T_{1\cdot 2}$ shows correspondence between commodity and usage. The private consumption expenditure in $T_{1\cdot 2}$ corresponds to elements of institutional sectors in $T_{2\cdot 5}$: one relating to household sector having column number 46 and the other relating to private non-profit institutions having number 47. Of elements of $T_{1\cdot2}$, general government consumption expenditure with column number 17 corresponds to column number 48, general government sector of $T_{2\cdot5}$.

$T_{1\cdot3}$, $T_{1\cdot6}$, $T_{1\cdot7}$, $T_{1\cdot8}$, $T_{1\cdot9}$, and $T_{1\cdot10}$ correspond to items of domestic final demand in the input-output table except for expenditure of households, expenditure of private non-profit institution expenditure and general government consumption. $T_{1\cdot3}$ indicates that the column sum equals the row sum of business consumption expenditure having number 19. Other elements of $T_{1\cdot3}$ are zero.

Each element of $T_{1\cdot6}$ to $T_{1\cdot10}$ indicates inventory increases and fixed capital formation of both the private sector and the government. In particular, elements of private inventory increases and private fixed capital formation of $T_{1\cdot6}$, and $T_{1\cdot8}$ respectively represent transaction matrix of industry (commodity) vs. industry (commodity). This is to indicate the correspondence between the production sector of commodities used for inventory or equipment investment and the sector in which the commodities are used. It is desirable to estimate this element as is done in the case of consumption converter $T_{1\cdot2}$. However, we estimated in our model only the vectors representing the sub-totals of columns 58 and 67 only because of the lack of the necessary data. We have presented them here in the form of a matrix simply for the purpose of displaying the basic form.

$T_{1\cdot16}$ indicates export, import and customs. Following the scheme of competitive import-type input-output table, imports and customs are eliminated.

The row item, $T_{3\cdot1}$ in the production account indicates the value added generated by industrial activity. Elements of value added are classified into various forms of factor incomes ranging from item 19, business consumption expenditure to item 35, subsidies; item 32, operating surplus, is a sub-total of items 21 to 31 which is devised to correspond to a classification of value added in input-output tables. Of components of operating surplus, tax items such as personal income tax, corporation income tax and charges are specially listed items. Double accounting is avoided by means of including dividends in corporate incomes and also interest and rents in income from property. Item 31, imputed service charges, is treated in the input-output table as an intermediate transaction between financial sectors and industries except for transaction within the financial sector itself. Consequently, imputed service charges is not included in operating surplus as an item of value added.

The expenditure and income-outlay accounts in our basic table correspond to one of the sub-items of the consumption account of the new

SNA, namely, income expenditure. Similarly as in the case of value added generated by industrial activity $T_{3\cdot1}$, let us express income categories in terms of income sources classified by institutional sectors $T_{5\cdot3}$. The table of transaction of transfers between different institutional sectors may be set up in such a form as to correspond to $T_{4\cdot5}$ and $T_{5\cdot4}$.

Ignoring the transactions with the rest of the world, each item of the expenditure and income-outlay accounts will attain a balance in relation to domestic accounts as follows:

Value added $T_{3\cdot1} = T_{1\cdot3} + T_{5\cdot3}$,

where $T_{3\cdot1}$ is value added classified by form of income in each industry,

$T_{1,3}$ is business consumption expenditure,

$T_{5\cdot3}$ is incomes classified by institutional sectors.

Transfer income $T_{4\cdot5} = T_{5\cdot4}$.

Institutional sectors $T_{5\cdot3} + T_{5\cdot4} = T_{2\cdot5} + T_{4\cdot5} + T_{15\cdot5}$,

where $T_{5\cdot3}$ is incomes classified by institutional sectors,

$T_{5\cdot4}$ is receipts of transfer incomes by institutional sectors,

$T_{2\cdot5}$ is consumption expenditures classified by institutional sectors.

$T_{4\cdot5}$ is payment of transfer incomes classified by institutional sectors,

$T_{15\cdot5}$ is saving classified by institutional sectors.

In addition to domestic balances as described above, each of the factors $T_{3\cdot17}$, $T_{4\cdot17}$, $T_{5\cdot17}$ and $T_{17\cdot3}$, $T_{17\cdot4}$, $T_{17\cdot5}$ indicates receipt, payment, and transfer of factor incomes from the rest of the world and their relationships with domestic institutional sectors.

Transactions between industrial sectors of the production account and each sector of accumulation accounts are represented by $T_{1\cdot6}$, $T_{1\cdot7}$, $T_{1\cdot8}$, $T_{1\cdot9}$, $T_{1\cdot10}$. Balances with each of those factors are treated by means of setting up a dummy account called "capital formation" within the capital-finance account. Thus each of the row sums of $T_{1\cdot6}$, $T_{1\cdot7}$, $T_{1\cdot8}$, $T_{1\cdot9}$, $T_{1\cdot10}$ corresponds to each of the dummies $T_{6\cdot12}$, $T_{7\cdot12}$, $T_{8\cdot12}$, $T_{9\cdot12}$, $T_{10\cdot12}$, respectively.

This treatment is comparable with the use of the dummy accounts known as "capital formation of industry" in the new SNA.

The sum of the dummy accounts of capital finance $i=6 \ldots 12$ corresponds to elements of $T_{12\cdot15}$ which reveal capital appropriation by institutional sectors.

For institutional sectors 74 to 82, the capital-finance on balance will hold for each actor such as follows:

$$T_{15\cdot 5} + T_{15\cdot 13} + T_{15\cdot 18} \gtreqless T_{12\cdot 15} + T_{13\cdot 15} + T_{18\cdot 15},$$

where $T_{15\cdot 5}$ is saving classified by institutional sectors,

$T_{15\cdot 3}$ is increases in financial liabilities classified by institutional sectors,

$T_{15\cdot 18}$ is capital borrowing from overseas the rest of the world classified by institutional sectors,

$T_{12\cdot 15}$ is capital raising classified by institutional sectors,

$T_{13\cdot 15}$ is increases in financial assets classified by institutional sectors,

$T_{18\cdot 15}$ is capital loan to the rest of the world by institutional sectors.

These balances will not necessarily be expressed by equalities for all the sectors. In case of inequality, the difference which emerged during the present period will be added either in the form of an increase in liability or of an asset to the balance of liability or of an asset remaining at the beginning of this period. Such differences are not dealt with explicitly in our accounting system although they are explicitly listed in the new SNA.

In our accounting system, the overseas account is subdivided into trade balance such as export, import, and customs, non-trade balance, income transfer, and capital account of lending and borrowing. Note that the row sum and column sum of income transfer and capital lending and borrowing in items 87, 88, 89 and 90 formally suffer from double accounting because institutional sectors in the rest of the world are not listed explicitly.

Thus far, we have occasionally explained the accounting system in our model using the comprehensive basic table presented in Table 6.1, in comparison with the new SNA. The principal objective of this basic accounting table is to integrate the three accounting systems : Input-Output Table, National Income Statistics and Flow-of-Fund Table. However, as we have pointed out earlier there exist considerable conceptual differences between these three accounting systems in Japan since they are not based on a common framework for consolidation. One of the important differences is the fact that the National Income Statistics and based on a "national" concept while the Input-Output Table depends on "domestic" concept. Since our model is basically dependent on the scheme of input-output table because of its theoretical character, our data have to be edited on the basis of the "domestic" concept.

The editing of time-series data will be carried out by means of relating fragments of partially integrated data according to the framework of our social accounting system which we have developed thus far. For this purpose, it is necessary that the data sets are systematized in the form of mutually consistent time-series. In Japan, the first input-output table was constructed in 1951. Since then input-output tables have been made for 1955, 1960, 1965 and 1970. In other words, comparable input-output tables are available every five years. In chapter 9 of this volume, we will report on our estimation of input-output tables for intermediate periods between the years when the official tables are available. Two kinds of time-series of input-output tables will be estimated; one based on 1965 constant prices evaluated in terms of producer's prices and the other based on nominal prices. The national income statistics and flow-of-fund tables, on the other hand, are available every year. We can therefore check the consistencies of them together with the estimated time-series of input-output tables according to the framework of our comprehensive system of social accounting, which has been explained above.

In compiling the comprehensive accounting table by reconstructing sets of partially integrated data, we cannot completely avoid discrepancies in the data even after having made necessary conceptual adjustments. This type of error can be expected to be reduced if the system of organizing statistical information is improved on the basis of comprehensive consolidation. At this stage, however, we did not make further adjustments even though there remain errors unless they exceed 10 percent and instead left them as they are, as statistical discrepancies.

6.3 The Composition of the Model

In our comprehensive social accounting table, each row and column are supposed, in principle, to balance with each other, although, as we have said earlier, some degree of statistical discrepancy is unavoidable. Figures of the basic table at a time, therefore, may be interpreted as describing in the form of a one-shot picture of the on-going dynamics of interdependent economic relationships. The time-series combination of such one-shot pictures will provide a description of the dynamic operation of the interdependent behaviors of economic actors. Our theoretical model should be capable of explaining such behaviors of economic actors on the basis of the observed data.

In our model, the endogenous variables are confined within the category of variables which are described mainly by national income statistics and input-output tables. Because of this limitation, the treatment of the relationships described by the flow-of-fund table is necessarily simplified and some of the relevant variables are incorporated in the model as exogenous variables when necessary.

Before presenting the results of empirical verification of components of our model, it will be useful and perhaps necessary to explain briefly the following three aspects of the model construction: (1) the sub-division of industry (commodity) sectors and institutional sectors, (2) classification of endogenous and exogenous variables, and (3) the skeleton of the model.

1. Sectoral Classsification

Table 6.2 compares our four sector classification with the two-digit industries of the Japan Standard Industrial Classification. Our four sector classification may be summarized as: Sector 1: agriculture, forestry and fisheries; Sector 2: light manufacturing industries; Sector 3: heavy manufacturing industries; Sector 4: commercial and service industries.

In our comprehensive basic table explained in the previous section, we classified all industries into seven categoris instead of four, by distinguishing specially some of the components of these four broadly defined sectors, namely, the construction industry from the second sector, the whole-sale and retail industries and the financial and insurance industries from the fourth sector. Institutional sectors in the same basic table were classified into six:

Table 6.2 Sectoral Classification

Sector No.	Name	Industries
Sector 1	Primary	Agriculture, forestry and fishery.
Sector 2	Light Manufacturing Industry	Food and processing food, textile products, pulp, paper products, publishing printing and allied, rubber products, stone, clay and glass products, precision instruments and other light manufacturing industries.
Sector 3	Heavy Manufacturing Industry	Chemical and related products, petroleum and coal products, iron and steel, nonferous metals, fabricated metal products, machinery, electrical machinery and transportation equipment.
Sector 4	Commercial and Service Industry	Electricity, gas and water supply, wholesales and retail trade, transportation and communication, banking and insurance, real estate, service and public administration.

private corporations, the central bank of Japan, private financial intermediaries, households, private non-profit institutions, general government. To simplify the treatment of the model with regard to money flow, we specified in our model only the four types of the institutional sectors as endogenous actors: firms (including both juridical persons and others), households, private non-profit organizations, and the government. Other institutional sectors such as the Bank of Japan and private financial intermediaries are incorporated into the model as exogenous variables when it is necessary to deal with them explicitly for theoretical reasons.

2. Endogenous vs. Exogenous Variables

Let us specify the distinction between endogenous and exogenous variables.

Exogenous variables consist of two types: one type consists of those which are determined exogenously from the viewpoint of domestic economic actors such as demographic factors, technological factors, variables of economic changes outside of Japan, and policy variables, and the other type consists of those variables which we are obliged to treat as exogenous variables because there is inadequate information based on empirical analyses to treat them as endogenous variables. The latter type includes, for example, business consumption expenditure, transfer incomes, private housing investments, etc. This latter type of exogenous variables may be divided further into two categories: one is those which are given nominal values exogenously and the other the values of which are regarded as being determined proportionately to those of certain endogenous variables on the ground that the exogenous variables change passively with changes in the endogenous variables. The exogenous variables with asterisk * in Table 6.4 are those which are given certain ratios vis-a-vis the values of certain endogenous variables. Details of what are the ratios and how they are determined will be exlained in the following chapters, wherever such questions are relevant. Including all this, there are 97 exogenous variables considered explicitly in our model. On the other hand, the endogenous variables are listed in Table 6.3.

Our model employs altogether 263 variables, of which 166 are endogenous variables and 97 are exogenous variables. To denote these variables in the following chapters we will use the notations listed in Tables 6.3 and 6.4.

Table 6 – 3 List of Endogenous Variables

Variable Number	Symbol	Name of Variables	Sector
1–3	h_j	Hours of Operation	2,3,4
4–6	p_j	Output Deflator	2,3,4
7	$w_1 h_1$	Value Marginal Productivity	1
8–9	w_j	Hourly Wage Rate	2.3,4
10–13	V_j	Value Added	1,2,3,4
14	η_b	Deflator of Business Consumption Expenditure	
15	η_{C_1}	Deflator of Personal Consumption; Food	
16	η_{C_2}	Deflator of Personal Consumption; Clothing	
17	η_{C_3}	Deflator of Personal Consumption; Fuel and Light	
18	η_{C_4}	Deflator of Personal Consumption; Housing	
19	η_{C_5}	Deflator of Personal Consumption; Miscellaneous	
20	η_{CP}	Deflator of Expenditure of Private Non-profit Institutions	
21	η_G	Deflator of General Government Expenditure	
22	η_{KP}	Deflator of Fixed Capital Formation (Private)	
23	η_H	Deflator of Housing Investment	
24	η_{KG}	Deflator of Fixed Capital Formation (Government)	
25	η_{INV}	Deflator of Inventory Increase	
26	η_{EX}	Deflator of Exported Goods	
27–30	E_{yj}	The Number of Employees	1,2,3,4
31–34	E_{Ij}	Compensation of Employees	1,2,3,4
35–37	U_{cIj}	Income from Unincorporated Enterprises	1,2.3,4
38–40	P_{Ij}	Income from Property	1,2.3,4
41–43	C_{Ij}	Income from Private Corporations	1,2.3,4
44–46	B_{Sj}	Operating Surplus	1,2.3,4
47–49	D_{ej}	Provisions for the Consumption of Fixed Capital	1,2.3,4
50–53	T_{Ij}	Indirect Taxes	1,2,3,4
54	E	Total of Private Consumption Expenditure	
55	T_P	Personal Direct Taxes and Charges	
56	S_P	Personal Saving	
57	Y_P	Personal Income	
58	Y_D	Disposable Income	
59	E_I	Total of Compensation of Employees	
60	U_{cI}	Total of Income from Unincorporated Enterprises	
61	P_I	Total of Income from Property	
62	C_I	Total of Income from Private Corporations	
63–64	C_{Ij}	Income from Private Corporations	2,3,
65	S_T	Total Saving	
66–69	T_{Cj}	Corporation Income Taxes and Charges	1,2,3,4
70–73	M_j	Retained Earnings	1,2,3,4
74	T_C	Total of Corporation Income Taxes and Charges	
75	T_I	Total of Indirect Taxes	
76	q_1	Private Consumption Expenditure; Food	
77	q_2	Private Consumption Expenditure; Clothing	
78	q_3	Private Consumption Expenditure; Fuel and Light	

Table 6 – 3 List of Endogenous Variables (Continued)

Variable Number	Symbol	Name of Variables	Sector
79	q_4	Private Consumption Expenditure; Housing	
80	q_5	Private Consumption Expenditure; Miscellaneous	
81–83	$SINV_j$	Normal Inventory Stock	2,3,4
84–86	$USINV_j$	Other Inventory Stock	2,3,4
87	M_P	Money Demand for Persons	
88	M_C	Money Demand for Private Corporations	
89	M_D	Total Money Demand	
90–93	K_j	Capital Stock	1,2,3,4
94–97	I_j	Net Investment	1,2,3,4
98	D_T	Total Depreciation Cost	
99	I_{GT}	Total Gross Investment	
100	I_{NT}	Total Net Investment	
101	E_{X_1}	Real Export; Food and Processed Food	
102	E_{X_2}	Real Export; Textiles	
103	E_{X_3}	Real Export; Chemical Products	
104	E_{X_4}	Real Export; Metal Products	
105	E_{X_5}	Real Export; Machinery	
106	E_{X_6}	Real Export; Miscellaneous	
107–112	η_{EXj}	Deflator of Exported Goods, Food, Textiles, Chemical Products, Metal Products, Machinery and Miscellaneous.	
113–118	EX_{Nj}	Nominal Export; Food, Textiles, Chemical Products, Metal Products, Machinery and Miscellaneous.	
119–122	X_j	Output	1,2,3,4
123–125	GW_j	Growth Rate for Output	2,3,4
126–128	Q_j	Production Capacity	2,3,4
129–132	L_j	Number of Workers	1,2,3,4
133	GR	Total Revenue of General Government	
134	GRE	Saving of General Government	
135–138	INV_j	Inventory Increases	1,2,3,4
139–142	IM_j	Real Imports	1,2,3,4
143	IM_T	Total of Nominal Import	
144	EX_T	Total of Nominal Export	
145	$RGDP$	Real Gross Domestic Product	
146	$NGDP$	Nominal Gross Domestic Product	
147–150	F_j	Final Demand	1,2,3,4
151–155	H_j	Habit Potential for Private Consumption Expenditure	
156	PP	General Price Index	
157–159	D_j	Debt	2,3,4
160	D_e	Total of Provisions for the Consumption of Fixed Capital	
161–163	YY_j	Anticipated Demand for Investment Behavior	2,3,4
164	P_{A_1}	Anticipated Price for Investment Behavior	
165	I_T	Total Investment	
166	λ	Marginal Utility of Money	

Table 6 – 4 List of Exogenous Variables

Variable Number	Symbol	Name of Variables	Sector
1	P_1	Output Deflator of Sector 1	1
2	A_1	Cultivated Acreage	1
3	K_{g1}	General Government Capital Stock of Sector 1	1
4*–7*	B_{cj}	Business Consumption / Value Added	1,2,3,4
8–11	S_{cj}	Current Subsides	1,2,3,4
12*–14*	A_{Pj}	Stock Variation Adjustment / Value Added	1,2,3,4
15*–17*	Y_{Gj}	General Government Income from Property and Entrepreneurship / Value Added	1,2,3,4
18*–20*	D_{cGj}	Interest on Public Debt and Consumers' Debt / Value Added	1,2,3,4
21	TR_{PG}	Transfers from Households and Private Non-Profit Institutions to Government	
22	TR_{GP}	Transfers from Government to Households	
23	TR_{SI}	Social Insurance Contributions	
24	TR_{PR}	Transfers from Households and Private Non-Profit Institutions to the Rest of the World	
25	TR_{rp}	Transfers from the Rest of the World to Households	
26*–29*	I_{ccj}	Imputed Service Charges by Private Corporations / Value Added	1,2,3,4
30*	I_{cP}	Imputed Service charges by Persons / Value Added	
31*–34*	D_{Vj}	Dividend Payments / Value Added	1,2,3,4
35*–38*	TR_{CPj}	Transfers from Private Corporations to Households / Value Added	1,2,3,4
39	TR_{GR}	Transfers from Government to the Rest of the World	
40	TR_{RG}	Transfers from the Rest of the World to Government	
41–44	TR_{INj}	Net Factor Income from Abroad	1,2,3,4
45	t_c	Rate of Corporation Income Taxes	
46–49	t_{Ij}	Rate of Indirect Taxes	1,2,3,4
50	M_G	Money Demand for Government	
51	a_{ij}	Input coefficients	1,2,3,4
52	m_j	Import Coefficients	1,2,3,4
53–55	I_{NVGj}	Inventory Increase in General Government	2,3,4
56–59	B_{FCj}	Business Consumption Expenditure (Final Demand)	1,2,3,4
60–63	IH_j	Private Housing Investment	1,2,3,4
64–67	G_j	General Government Consumption Expenditure	1,2,3,4
68–71	C_{Nj}	Consumption Expenditure of Non-Profit Institutions	1,2,3,4
72–75	I_{Gj}	Gross Fixed Capital Formation by Government	1,2,3,4
76	M	The Number of Families	
77	m	Average Family Size	
78–83	W_j	Quality Index of World Trade; Food and processed Food, Textiles, Chemical Products, Metal Products, Machinery and Miscellaneous.	
84–89	P_j	Price Index of World Trade	
90	i_O	Prime Rate	
91	i	Interest Rate of Loan Discounts of All Banks	
92	h_1	Labor Hours in Sector 1	1
93	t_P	Rate of Income Taxes for Persons	
94	L	Total Labor Force	
95–97	h^*_j	Normal Operation Hours	2,3,4

3. The Skeleton of the Model

Although a detailed presentation of the structure of our model may be found in the flow-chart appended at the end of this volume, it is useful at this point to give a brief picture of how the model would look.

For the sake of expository convenience, the model may be segmented into four blocks. These blocks and the major subjects which they deal with by them are:

the first block:	the structure of short-run supply and the determination of employment and wages,
the second block:	the distribution of factor incomes,
the third block:	the determination of final demand,
the fourth block:	demand-supply balances of commodity and money market.

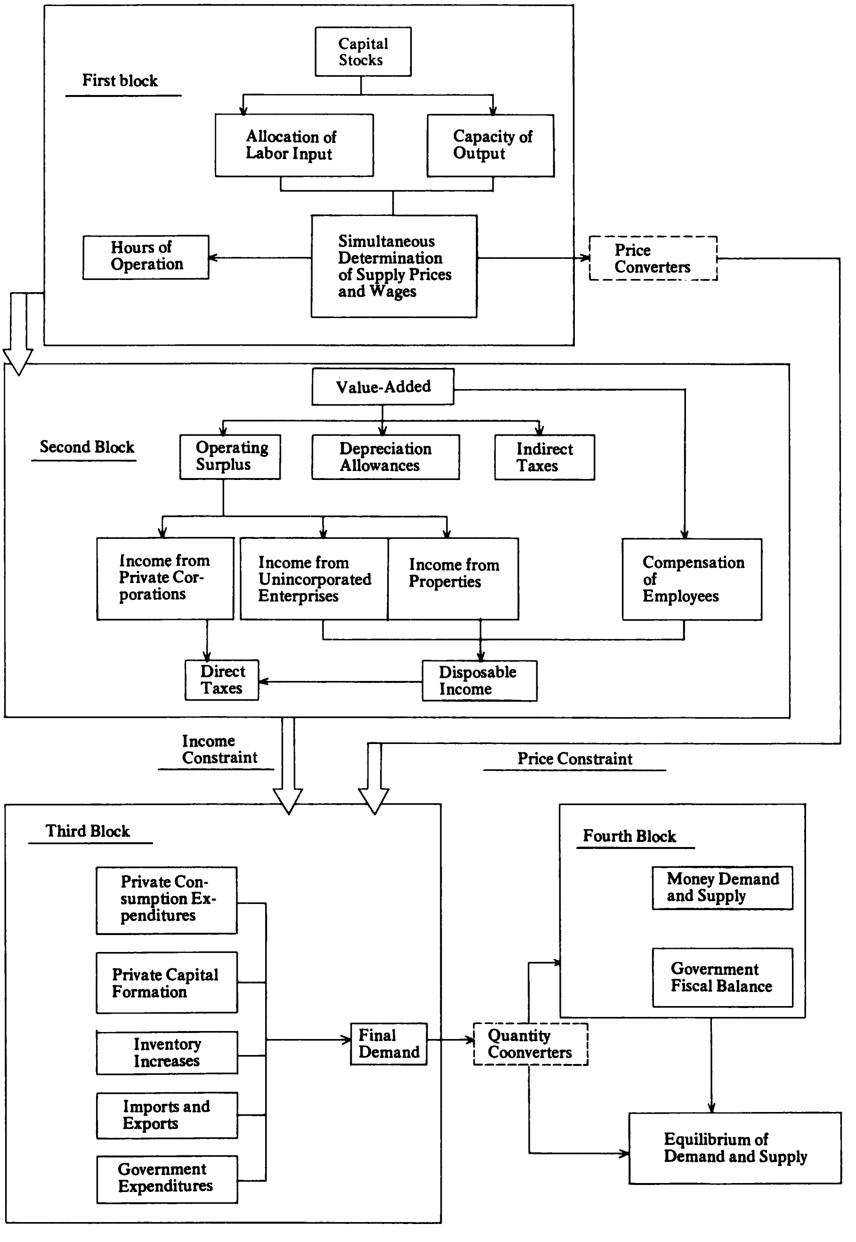
Figure 6.2 is made for the purpose of visualizing the skeleton of our model in a simple way. To facilitate quick understanding, blocks are combined together by arrows. But these arrows do not imply the relationship of recursive determination. All the endogenous variables, 166 of them, are determined in our model simultaneously when the markets of goods and services of the four sectors are cleared.

The First Block

In block 1, on the one hand, the patterns of short-run supply behavior of firms in three of the four sectors are described. These firms are supposed to act in order to maximize profits under the given amount of capital stock at the beginning of each period and having anticipated reactions in the market which the firms calculate in terms of their own anticipated demand functions of their respective markets. Estimation of supply curves based on what we call "Semi-Factor-Substitution" production functions will be explained in detail in chapter 7. On the other hand, the structure of production in the first sector, namely agriculture, is approximated by the conventional Cobb-Douglas type production function. The price of agricultural product is given exogenously as a policy variable and the supply elasticity of price in this sector is zero. Consequently, the supply schedule takes the form of a straight line parallel to the price axis.

The allocation of the labor force will be determined as follows. The total labor force is given exogenously. In the manufacturing and service sectors, the volume of employment in terms of the number of workers will be determined, once the amount of capital stock is given, on the basis of the aforementioned SFS production functions. The part of the labor force not absorbed by these sectors will be employed in sector 1. Consequently, the supply of products in agricultural sector in the short-run will be determined

Figure 6.2 THE SKELTON OF THE MODEL



independently from their price level since the amount of production is given automatically under the Cobb-Douglas type production function once the capital stock and labor force is predetermined with exogenous cultivated land.

It is postulated that the wage level in Sector 1 is determined at the level equivalent to the value marginal productivity in that sector. Therefore, manufacturing and service sectors would have to offer wages higher than the value marginal productivity in the agricultural sector for them to mobilize labor force from the agricultural sector. This question will be discussed in detail in Chapter 7, Section 4.

It must be clear by now that supplies of the four sectors are all interdependent with each other through the inter-sectorally related structure of wage determination and intermediate inputs.

The Second Block

The determination of sectoral factor incomes and incomes by types of economic actors will be dealt with in this block. To be more specific, distribution to compensation of employees, income from unincorporated enterprises, income from property, income from private corporation, and also government receipts such as indirect and direct taxes, and charges on private corporation as well as on households are described. Personal disposable incomes and saving of private corporations will be determined at the same time. Detailed discussion of the subjects of this block will be made in Chapter 8.

The Third Block

The determination of final demand items is the major subject of this block. A vector of such items as business consumption expenditure, private non-profit organization consumption expenditures, private housing investment, government's expenditures, and government's fixed capital formation, is given in terms of nominal values to each sector exogenously.

Personal consumption expenditures will be analyzed applying general equilibrium-type multi-item consumption functions to the data classifed into 5 major categories of expenditures. The analysis of personal consumption demand will be reported in detail in Chapter 10.

In analyzing private capital formation in the manufacturing and service sectors, investment good demand functions will be derived on the basis of firms' long-run profit maximization principle under the given conditions of production technology and expectations for future demand. In contrast, in the agricultural sector we presume that producers act on the basis of expected price of agricultural products in place of expected demand functions formulated for other sectors. Our analysis of investment demand will be discussed in detail in Chapter 11.

Inventory investment is analyzed by equations specified on the basis of the empirically estabished relationship between capital stock and the rate of changes of output in the previous period. This will be discussed in detail in Chapter 12. Imports are treated by giving import coefficients exogenously while exports are determined using export functions for 6 major categories of commodities.

The sectoral final demand will be determined by adding together these endogenous final demand items and the exogenous demand items mentioned earlier.

The Fourth Block

The total demand for goods and services by sector will be determined by imputing from the final demand vector using Leontief's inverse matrix coefficients. Also, by taking money balance into account in this block, the relative prices of real goods and services will now have their absolute level of price counterparts.

These blocks of our model are, as we stressed earlier, not mutually independent but rather closely interrelated. Therefore, the solutions of the equation system will be detemined simultaneously. Let us explain briefly how the logic of computation will proceed internally in arriving at the solutions of the simultaneous equations.

Given a certain amount of supply for each of the manufacturing and service sectors, supply prices for goods and services and sectoral wages will be determined accordingly within the first block. In the second block, factor incomes will be determined consistently with these supply prices, wages and exogenously given prices of products of sector 1. Of these incomes, personal income and corporate income will be taken into account in the third block as budgetary constraints of households and firms. The supply prices of goods and services, on the other hand, will be converted into supply prices classified by final demand items and will serve as constraints, together with income constraints determined in the second block, upon the demand of each economic actor. Consequently, the final demand vector derived in the third block will be consistent with computations in the first and second blocks. In the fourth block, the total demand vector classified by goods and services will be determined consistently with the final demand vector derived in the third block. In the agricultural sector the gap between the supply determined in the first block and the demand is treated as inventory. In other sectors, the supply given in the first block and the total demand obtained in the fourth block may not necessarily balance. The simultaneous solutions for the equation system by which the supply and demand for goods and services balance will eventually be reached by means of iterative computation. This process of iterative computation will be explained in Chapter 13.

The internal consistency between the sectoral classification of our model and the classification of final demand items is maintained by using price and quantity converters. The concept and estimation of such converters will be explained in Chapter 9.

Needless to say, it is imperative that component equations should be empirically valid as well as being theoretically consistent for the model itself to be valid for empirical analysis. Each of the following chapters, therefore, will be devoted to explanation of the theoretical derivation of the relevant equations and examination of their empirical validity. Estimation of parameters of equations is largely based on the method either of single equation estimation or partial structural estimation. However, equations derived from certain theories may not always be expressible in linear forms.

In the case of non-linear equations, one could still think of applying the least squares method by approximating them by linear forms. However, if this is done it often becomes difficult to distinguish structural parameters from estimated parameters. It is of course desirable that each of the parameters of structural equations is identifiable particularly because the theoretical requirements of internal consistency of the structural equation system themselves can serve as an effective test device, quite separately from the statistical test itself. From this point of view, we often employ non-linear estimation methods in estimating the structural equations of our model. The Newton method, used in estimation of short-run supply schedules, the complete determination method used in estimation of consumption functions, and the Pattern method used in estimation of investment good demand functions are such examples. Although there remain many problems to be qualified regarding the statistical properties of estimate obtained by non-linear estimation methods, we maintain that non-linear estimation methods are useful means to examine the validity of estimated parameters rigorously from a theoretical standpoint.

Before closing this chapter, let us present the system of structural equations of our model.

4 The Structural Equations

The subscripts 1, 2, 3 and 4 represent the number of industry sectors.

[The First Block]

§ The Determination of Labor Input

(1) $L_1 = \overline{L} - \sum_{i=2}^{4} L_i$

(2) $L_2 = c_2 K_2^{d_2}$

(3) $L_3 = c_3 K_3^{d_3}$

(4) $L_4 = c_4 K_4^{d_4}$

§ The Determination of Operation Hours

(5) $h_2 = (X_2/Q_2)^{\frac{1}{\alpha_2}}$

(6) $h_3 = (X_3/Q_3)^{\frac{1}{\alpha_3}}$

(7) $h_4 = (X_4/Q_4)^{\frac{1}{\alpha_4}}$

§ The Determination of Production Capacity

(8) $Q_2 = a_2 K_2^{b_2}$

(9) $Q_3 = a_3 K_3^{b_3}$

(10) $Q_4 = a_4 K_4^{b_4}$

§ The Determination of the Quantity of Supply of Sector 1

(11) $X_1/L_1 = a_1(A_1/L_1)^{b_1}(K_1 + K_{g1})^{e_1}$

§ The Simultaneous Determination of Wages and Supply Price

(12) $\partial V_1/\partial L_1 = (1-b_1)(p_1 - \sum_{i=1}^{4} p_i a_{i1})X_1/L_1 = w_1 h_1$

(13) $w_{23} = \varepsilon_{23} + \delta_{23} w_1$

(14) $w_4 = \varepsilon_4 + \delta_4 w_1$

(15) $p_2 = \frac{1}{\gamma_{s2}(a_{22} + t_{I2} - 1)}\left\{(X_2 - \gamma_{s2})\left(\frac{1}{\alpha_2} L_2 h_2 w_{23}/X_2 + \sum_{i=1\,(i \neq 2)}^{4} p_i a_{i2}\right)\right\}$

(16) $p_3 = \frac{1}{\gamma_{s3}(a_{33} + t_{I3} - 1)}\left\{(X_3 - \gamma_{s3})\left(\frac{1}{\alpha_3} L_3 h_3 w_{23}/X_3 + \sum_{i=1\,(i \neq 3)}^{4} p_i a_{i3}\right)\right\}$

(17) $p_4 = \frac{1}{\gamma_{s4}(a_{44} + t_{I4} - 1)}\left\{(X_4 - \gamma_{s4})\left(\frac{1}{\alpha_4} L_4 h_4 w_4/X_4 + \sum_{i=1\,(i \neq 4)}^{4} p_i a_{i4}\right)\right\}$

§ The Determination of Employment

(18) $E_{y1}=\beta_{01}+\beta_{11}L_1$

(19) $E_{y2}=\beta_{02}+\beta_{12}L_2$

(20) $E_{y3}=\beta_{03}+\beta_{13}L_3$

(21) $E_{y4}=\beta_{04}+\beta_{14}L_4$

[The Second Block]

§ The Determination of Indirect Taxes

(22) $T_{I1}=t_{I1}p_1X_1$

(23) $T_{I2}=t_{I2}p_2X_2$

(24) $T_{I3}=t_{I3}p_3X_3$

(25) $T_{I4}=t_{I4}p_4X_4$

(26) $T_I=\sum_{i=1}^{4}T_{Ii}$

§ The Determination of Value Added

(27) $V_1=(p_1-\sum_{i=1}^{4}p_i a_{i1})X_1$

(28) $V_2=(p_2-\sum_{i=1}^{4}p_i a_{i2})X_2$

(29) $V_3=(p_3-\sum_{i=1}^{4}p_i a_{i3})X_3$

(30) $V_4=(p_4-\sum_{i=1}^{4}p_i a_{i4})X_4$

§ The Determination of Compensation of Employees

(31) $E_{I1}=E_{y1}h_1w_1$

(32) $E_{I2}=E_{y2}h_2w_{23}$

(33) $E_{I3}=E_{y3}h_3w_{23}$

(34) $E_{I4}=E_{y4}h_4w_4$

(35) $E_I=\sum_{i=1}^{4}E_{Ii}$

§ The Determination of Provisions for the Consumption of Fixed Capital

(36) $D_{e1}/\eta_{Kp}=d_{e01}+d_{e11}K_1^t$

(37) $D_{e2\cdot 3}/\eta_{Kp}=d_{e023}+d_{e123}(K_2+K_3)^t$

(38) $D_{e4}/\eta_{Kp}=d_{e04}+d_{e14}K_4^t$

(39) $D_e=D_{e1}+D_{e23}+D_{e4}$

§ The Operating Surplus

(40) $B_{s1}=V_1-D_{e1}-B_{c1}-T_{I1}+S_{c1}-E_{I1}-TR_{IN1}$

(41) $B_{s23}=(V_2+V_3)-D_{e23}-(B_{c2}+B_{c3})-(T_{I2}+T_{I3})+(S_{c2}+S_{c3})-(E_{I2}+E_{I3})$
$-TR_{IN23}$

(42) $B_{s4}=V_4-D_{e4}-B_{c4}-T_{I4}+S_{c4}-E_{I4}-TR_{IN4}$

§ The Determination of Income from Unincorporated Enterprises

(43) $U_{cI1}=\kappa_{11}+\rho_{11}(B_{s1}+A_{p1}-Y_{G1}+D_{cG1})$

(44) $U_{cI23}=B_{s23}+(A_{p2}+A_{p3})-(Y_{G2}+Y_{G3})+(D_{cG2}+D_{cG3})-P_{I23}-C_{I23}$

(45) $U_{cI4}=B_{s4}+A_{p4}-Y_{G4}+D_{cG4}-P_{I4}-C_{I4}$

(46) $U_{cI}=U_{cI1}+U_{cI23}+U_{cI4}$

§ The Determination of Income from Property

(47) $P_{I1}=\kappa_{21}+\rho_{21}(B_{s1}+A_{p1}-Y_{G1}+D_{cG1})$

(48) $P_{I23}=\kappa_{223}+\rho_{223}(B_{s23}+A_{p2}+A_{p3}-Y_{G2}-Y_{G3}+D_{cG2}+D_{cG3})$

(49) $P_{I4}=\kappa_{2\cdot 4}+\rho_{2\cdot 4}(B_{s4}+A_{p4}-Y_{G4}+D_{cG4})$

(50) $P_I=P_{I1}+P_{I23}+P_{I4}$

§ The Determination of Income from Private Corporations

(51) $C_{I1}=B_{s1}+A_{p1}-Y_{G1}+D_{cG1}-U_{cI1}-P_{I1}$

(52) $C_{I23}=\kappa_{323}+\rho_{323}(B_{s23}+A_{p2}+A_{p3}-Y_{G2}-Y_{G3}+D_{cG2}+D_{cG3})$

(53) $C_{I4}=\kappa_{34}+\rho_{34}(B_{s4}+A_{p4}-Y_{G4}+D_{cG4})$

(54) $C_I=C_{I1}+C_{I23}+C_{I4}$

§ The Determination of Personal Income

(55) $Y_p=E_I+U_{cI}+P_I+\sum_{i=1}^{4}D_{Vi}$

(56) $T_p=t_pY_p$

(57) $Y_D=Y_p-T_p-TR_{pR}-TR_{pG}+TR_{Rp}+TR_{Gp}+TR_{cp}-TR_{sI}+I_{cp}$

(58) $E=\pi_1+\pi_2E^{t-1}+\pi_3Y_D$

(59) $S_p=Y_D-E$

§ The Determination of Saving of Private Corporations

(60) $C_{I2}=\{V_2/(V_2+V_3)\}C_{I23}$

(61) $C_{I3}=\{V_3/(V_2+V_3)\}C_{I23}$

(62) $T_{c1}=t_cC_{I1}$

(63) $T_{c2}=t_cC_{I2}$

(64) $T_{c3}=t_cC_{I3}$

(65) $T_{c4}=t_cC_{I4}$

(66) $T_c=\sum_{i=1}^{4}T_{ci}$

(67) $M_1=C_{I1}-T_{c1}-TR_{cp1}-I_{cc1}-D_{V1}$

(68) $M_2=C_{I2}-T_{c2}-TR_{cp2}-I_{cc2}-D_{V2}$

(69) $M_3=C_{I3}-T_{c3}-TR_{cp3}-I_{cc3}-D_{V3}$

(70) $M_4=C_{I4}-T_{c4}-TR_{cp4}-I_{cc4}-D_{V4}$

[The Third Block]

§ The Determination of Prices Classified by Final Demand Items

(71)～(83)

$$\begin{pmatrix} \eta_b \\ \eta_{c1} \\ \eta_{c2} \\ \eta_{c3} \\ \eta_{c4} \\ \eta_{c5} \\ \eta_{cp} \\ \eta_G \\ \eta_{Kp} \\ \eta_H \\ \eta_{KG} \\ \eta_{INV} \\ \eta_{EX} \end{pmatrix} = \begin{pmatrix} \text{Price-converters} \\ B^T \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

(84) $\eta_{EX1}=\gamma_{11}p_1+\gamma_{12}p_2+\gamma_{13}p_3+\gamma_{14}p_4$

(85) $\eta_{EX2}=\gamma_{21}p_1+\gamma_{22}p_2+\gamma_{23}p_3+\gamma_{24}p_4$

(86) $\eta_{EX3}=\gamma_{31}p_1+\gamma_{32}p_2+\gamma_{33}p_3+\gamma_{34}p_4$

(87) $\eta_{EX_4}=\gamma_{41}p_1+\gamma_{42}p_2+\gamma_{43}p_3+\gamma_{44}p_4$

(88) $\eta_{EX_5}=\gamma_{51}p_1+\gamma_{52}p_2+\gamma_{53}p_3+\gamma_{54}p_4$

(89) $\eta_{EX_6}=\gamma_{61}p_1+\gamma_{62}p_2+\gamma_{63}p_3+\gamma_{64}p_4$

§ The Determination of Private Consumption

(90)~(94)

$$\begin{pmatrix} \alpha_1^* & 0 & 0 & 0 & 0 & 1 \\ 0 & \alpha_2^* & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha_3^* & 0 & 0 & 1 \\ 0 & 0 & 0 & \alpha_4^* & 0 & 1 \\ 0 & 0 & 0 & 0 & \alpha_5^* & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{c1}q_1 \\ \eta_{c2}q_2 \\ \eta_{c3}q_3 \\ \eta_{c4}q_4 \\ \eta_{c5}q_5 \\ -\lambda^* \end{pmatrix} = \begin{pmatrix} \eta_{c1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{c2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{c3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{c4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_{c5} & 0 \\ 0 & 0 & 0 & 0 & 0 & E/M \end{pmatrix} \begin{pmatrix} -a_1^* \\ -a_2^* \\ -a_3^* \\ -a_4^* \\ -a_5^* \\ 1 \end{pmatrix}$$

$$\alpha_i^*=\frac{1}{\alpha_i},\quad a_i^*=\frac{1}{\alpha_i}(a_{io}+b_i m+c_i H_i)\quad (i=1\cdots5)\quad \lambda^*=\frac{1}{\lambda}$$

(95) $H_1=\sum_{t=0}^{t-1} q_1^t$

(96) $H_2=\sum_{t=0}^{t-1} q_2^t$

(97) $H_3=\sum_{t=0}^{t-1} q_3^t$

(98) $H_4=\sum_{t=0}^{t-1} q_4^t$

(99) $H_5=\sum_{t=0}^{t-1} q_5^t$

§ The Determination of Demand for Investment Goods

(100) $p_{A1}=a_{01}+a_{11}p_1+a_{21}(p_1-p_1^{t-1})$

(101) $\{p_{A1}(1-t_{I1}-a_{11})-\sum_{l=1\ (l\neq1)}^{4} p_l a_{l1}\}\cdot m\cdot lA_1^k L_1^{1-k}(K_1^{t+1}+K_{g1})^{l-1}$

$-K_1^{t+1}\eta_{Kp}^2\delta_1\cdot\varepsilon_1\{\eta_{Kp}(K_1^{t+1}-K_1^t)\}^{\varepsilon_1-1}-\eta_{Kp}[\delta_1\{\eta_{Kp}(K_1^{t+1}-K_1^t)^{\varepsilon_1}\}+i+d_{e1}]$

$=0$

(102) $YY_2=b_{21}+b_{22}GDP^{t-1}+b_{23}i_0+b_{24}(DUM_2)$

(103) $-\frac{(1-a_{22}-t_{I2})YY_2 a_2 \cdot b_2 K_2^{t+1 \cdot (b_2-1)} h_2^{*\alpha_2} \cdot \gamma_{L2}}{(a_2 K_2^{t+1 \cdot b2} h_2^{*\alpha 2}-\gamma_{L2})^2}$

$-a_2 b_2 K_2^{t+1 \cdot (b_2-1)} h_2^{*\alpha_2} \sum_{i=1\ (i \neq 2)}^{4} p_i a_{i2} - c_2 d_2 K_2^{t+1 \cdot (d_2-1)} h_2^{*} w_{23}$

$-\delta_2 \eta_{Kp} \{\eta_{Kp}(K_2^{t+1}-K_2^{t})+D_2+\rho_2\}^{\beta_2-1} M_2^{-\beta_2} \{(1+\beta_2)\eta_{Kp} K_2^{t+1}+D_2+\rho_2$

$-\eta_{Kp} K_2^{t}\}-(i+d_{e123})\eta_{Kp}=0$

(104) $YY_3=b_{31}+b_{32}GDP^{t-1}+b_{33}i_0+b_{34}(DUM_3)$

(105) $-\frac{(1-a_{33}-t_{I3})YY_3 a_3 b_3 K_3^{t+1 \cdot (b_3-1)} h_3^{*\alpha_3} \cdot \gamma_{L3}}{(a_3 K_3^{t+1 \cdot b_3} h_3^{*\alpha_3}-\gamma_{L3})}$

$-a_3 b_3 K_3^{t+1 \cdot (b_3-1)} h_3^{*\alpha_3} \sum_{i=1\ (i \neq 3)}^{4} p_i a_{i3} - c_3 d_3 K_3^{t+1 \cdot (d_3-1)} h_3^{*} w_{23}$

$-\delta_3 \eta_{Kp} \{\eta_{Kp}(K_3^{t+1}-K_3^{t})+D_3+\rho_3\}^{\beta_3-1} M_3^{-\beta_3} \{(1+\beta_3)\eta_{Kp} K_3^{t+1}+D_3+\rho_3$

$-\eta_{Kp} K_3^{t}\}-(i+d_{e123})\eta_{Kp}=0$

(106) $YY_4=b_{41}+b_{42}GDP^{t-1}+b_{43}i_0+b_{44}USINV_4+b_{45}(DUM_4)$

(107) $-\frac{(1-a_{44}-t_{I4})YY a_4 b_4 K_4^{t+1 \cdot (b_4-1)} h_4^{*\alpha_4} \gamma_{L4}}{(a_4 K_4^{t+1 \cdot b_4} h_4^{*\alpha_4}-\gamma_{L4})^2}$

$-a_4 b_4 K_4^{t+1 \cdot (b_4-1)} h_4^{*\alpha_4} \sum_{i=1\ (i \neq 4)}^{4} p_i a_{i4} - c_4 d_4 K_4^{t+1 \cdot (d_4-1)} h_4^{*} w_4$

$-\delta_4 \eta_{Kp} \{\eta_{KP}(K_4^{t+1}-K_4^{t})+D_4\}^{\beta_4-1}(M_4+\rho_4)^{-\beta_4}\{(1+\beta_4)\eta_{Kp} K_4^{t+1}$

$+D_4-\eta_{Kp} K_4^{t}\}-(i+d_{e14})\eta_{Kp}=0$

(108) $D_2=D_2^0+\sum_{t=1}^{t-1}\eta_{Kp}^{t}(K_2^{t+1}-K_2^{t})-\sum_{t=1}^{t-1}M_2^{t}, \quad D_2^0=0$

(109) $D_3=D_3^0+\sum_{t=1}^{t-1}\eta_{Kp}^{t}(K_3^{t+1}-K_3^{t})-\sum_{t=1}^{t-1}M_3^{t}, \quad D_3^0=0$

(110) $D_4=D_4^0+\sum_{t=1}^{t-1}\eta_{Kp}^{t}(K_4^{t+1}-K_4^{t})-\sum_{t=1}^{t-1}M_4^{t}, \quad D_4^0=0$

(111) $I_1=K_1^{t+1}-K_1^{t}$

(112) $I_2=K_2^{t+1}-K_2^{t}$

(113) $I_3=K_3^{t+1}-K_3^{t}$

(114) $I_4=K_4^{t+1}-K_4^{t}$

(115) $I_{NT}=\sum_{i=1}^{4} I_i$

(116) $D_T=d_{eT0}+d_{eT1}\sum_{i=1}^{4} K_i$

(117) $I_{GT}=D_T+I_{NT}$

§ The Determination of Exported Goods

(118) $\frac{\eta_{EX1}E_{X1}}{P_1}=e_{01}\left(\frac{W_1}{P_1}\right)^{e_{11}}\left(\frac{\eta_{EX1}}{P_1}\right)^{e_{21}}\left(\frac{1}{P_1}\right)^{e_{31}}$

(119) $\frac{\eta_{EX2}E_{X2}}{P_2}=e_{02}+e_{12}\left(\frac{W_2}{P_2}\right)+e_{22}\left(\frac{1}{P_2}\right)$

(120) $\frac{\eta_{EX3}E_{X3}}{P_3}=e_{03}\left(\frac{W_3}{P_3}\right)^{e_{13}}\left(\frac{\eta_{EX3}}{P_3}\right)^{e_{23}}\left(\frac{1}{P_3}\right)^{e_{33}}$

(121) $\frac{\eta_{EX4}E_{X4}}{P_4}=e_{04}\left(\frac{W_4}{P_4}\right)^{e_{14}}\left(\frac{\eta_{EX4}}{P_4}\right)^{e_{24}}$

(122) $\frac{\eta_{EX5}E_{X5}}{P_5}=e_{05}\left(\frac{W_5^{t-1}}{P_5^{t-1}}\right)^{e_{15}}\left(\frac{\eta_{EX5}}{P_5}\right)^{e_{25}}$

(123) $\frac{\eta_{EX6}E_{X6}}{P_6}=e_{06}+e_{16}\left(\frac{W_6}{P_6}\right)+e_{26}\left(\frac{\eta_{EX6}}{P_6}\right)+e_{36}\left(\frac{1}{P_6}\right)$

(124)~(129)

$EX_{Ni}=EX_i\cdot\eta_{EXi}\quad(i=1\cdots6)$

(130) $EX_T=\sum_{i=1}^{6} EX_{Ni}$

§ The Determination of Imported Goods

(131)~(134)

$IM_i=m_iX_i\quad(i=1\cdots4)$

(135) $IM_T=\sum_{i=1}^{4} IM_i p_i$

§ The Determination of Inventory Increases

(136) $SINV_2=\varepsilon_{02}+\varepsilon_{12}K_2^t+\varepsilon_{22}GW_2^{t-1}$

(137) $SINV_3=\varepsilon_{03}+\varepsilon_{13}K_3^t+\varepsilon_{23}GW_3^{t-1}$

(138) $SINV_4=\varepsilon_{04}+\varepsilon_{14}K_4^t+\varepsilon_{24}GW_4^{t-1}$

(139) $USINV_2=\varepsilon_{22}GW_2^{t-1}$

(140) $USINV_3=\varepsilon_{23}GW_3^{t-1}$

(141) $USINV_4=\varepsilon_{24}GW_4^{t-1}$

(142) $INV_1 = X_1{}^D - X_1{}^S$

(143) $INV_2 = SINV_2{}^t - SINV_2{}^{t-1}$

(144) $INV_3 = SINV_3{}^t - SINV_3{}^{t-1}$

(145) $INV_4 = SINV_4{}^t - SINV_4{}^{t-1}$

§ Vector of Final Demand

(146)~(149)

$$F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} = \begin{pmatrix} & & \\ & B & \\ & & \end{pmatrix} \begin{pmatrix} q_1 M \\ q_2 M \\ q_3 M \\ q_4 M \\ q_5 M \\ I_{GT} \\ EX \end{pmatrix} + \begin{pmatrix} B_{FC1}/p_1 \\ B_{FC2}/p_2 \\ B_{FC3}/p_3 \\ B_{FC4}/p_4 \end{pmatrix} + \begin{pmatrix} IH_1/p_1 \\ IH_2/p_2 \\ IH_3/p_3 \\ IH_4/p_4 \end{pmatrix} + \begin{pmatrix} G_1/p_1 \\ G_2/p_2 \\ G_3/p_3 \\ G_4/p_4 \end{pmatrix}$$

$$+ \begin{pmatrix} C_{N1}/p_1 \\ C_{N2}/p_2 \\ C_{N3}/p_3 \\ C_{N4}/p_4 \end{pmatrix} + \begin{pmatrix} I_{G1}/p_1 \\ I_{G2}/p_2 \\ I_{G3}/p_3 \\ I_{G4}/p_4 \end{pmatrix} + \begin{pmatrix} INV_1 \\ INV_2 \\ INV_3 \\ INV_4 \end{pmatrix}$$

[The Fourth Block]

§ The Determination of Money Demand

(150) $M_p = n_{p0} + n_{p1} Y_D$

(151) $M_c = n_{c0} (\sum_{i=1}^{4} p_i X_i)^{nc1} \cdot i^{nc2}$

(152) $M^D = M_p + M_c + M_G$

§ Saving and Investment

(153) $I_T = I_H + I_G + I_{GT} + \sum INV_i$

(154) $S_T = S_p + \sum_{i=1}^{4} M_i + D_1 + D_{23} + D_4 + GRE$

§ The Determination of Sectoral Demand

(155)~(158)

$$X^D = [I - A + m]^{-1} F$$

§ Recepts and Desposal of General Government

(159) $GR = T_P + T_C + T_I + \sum_{i=1}^{4} Y_{Gi} + TR_{sI} + TR_{pG} + TR_{RG} + D_G$

(160) $GRE = GR - \sum_{i=1}^{4} G_i + \sum_{i=1}^{4} S_{ci} - TR_{Gp} - TR_{GR}$

§ Gross Domestic Production and Economic Growth

(161) $NGDP = \sum V_i = \sum (F_i - I_{Mi}) p_i$

(162) $RGDP = NGDP/\mathbf{PP}$

(163) $\mathbf{PP} = (\sum p_i X_i)/\sum X_i$

(164)~(166)

$GW_i = \dot{X}^i / X^i \quad (i = 2 \cdots 4)$

Notes to Chapter 6

1) Galbraith (1967).
2) Frisch (1932).
3) Klein (1950).
4) Leontief (1941).
5) Stone and Others (1964).
6) See Arrow and Others (1961) for Constant Elasticity of Substitution Production Function (CES) and Christensen and Others (1971) for Constant Share Elasticity Production Function (CSE).
7) Johansen (1959).
8) Haavelmo (1944).
9) Frisch (1933a).
10) Cowles Commission (1953).
11) Frisch, *op.cit.*
12) Klein (1950).
13) United Nations (1958).
14) University of Cambridge (1962a), (1962b), (1963), (1964a), (1964b), (1965), (1966) and (1968).
15) Stone (1966), Ohlsson (1966) and Sigel (1962).

Chapter 7
The Short-run Supply Structure

Let us begin our empirical study of the general interdependency of the economy with analysis of the behavior of the firm. We will analyze, as we noted earlier, the behavior of an average representative firm for an industry on the basis of the equilibrium theory of the firm.[1]

The behavior of a firm may be analyzed in two aspects: one is its short-run supply behavior under a given capacity of productive equipment, and the other is its investment to increase the productive capacity of equipment for long term considerations. Although these two aspects are inseparable in determining the action of a firm, let us discuss the former aspect in this chapter and the latter in chapter 11.

The analysis of short-run supply behavior assuming the capacity of productive equipment as given is based on the analytical framework in which a firm is assumed to maximize its profits under technological constraints on production as well as the constraints of market conditions for factor inputs and output. The supply schedule of output and the demand schedule for factor inputs which will be derived from this analysis will serve as an integral part of the framework explaining the market equilibrium.

In Section 1 of this chapter, we will first try to examine empirically the technological conditions of production in the Japanese economy. In particular, the focus of our examination will be on the relationship between the economies of scale and the development of the Japanese economy. We will then explain why we decided to use in our model a non-homogeneous semi-factor-substitutable production function (we call this the Semi-Factor-Substitution production function or simply the SFS production function) in place of the familier Neo-Classical linear homogeneous production function to express the technological conditions of the manufacturing and service sectors.

In the second section of this chapter, we will discuss conditions of the product market for a firm. It would be self-contradictory to allow for economies of scale as a technological condition of production and at the same time to assume perfect competition in the product market. To reconcile this inconsistency, we have devised the concept of anticipated demand. The concept of anticipated demand represents the anticipation of a producer of the reactions of other competitive firms and of demand conditions in the market which would take place in response to his supply in the short-run. In other words, the producer will decide how much to produce and to supply to the market being guided by the market demand he expects. We will show that we can approximate the anticipated demand function quantitatively.

On the basis of the SFS production function and the anticipated demand function thus specified, we will in Section 3 estimate the supply schedules. Needless to say, the supply schedules of the four industry sectors are mutually interdependent being related through the prices of intermediate inputs and also through wages.

In Section 4, we will explain how the mechanism of determination of wages and allocation of labor force is formulated in our model. The inter-sectoral allocation of labor force depends upon the equilibrium between the level of wages in the modern industry sector (manufacturing and service sectors) and the marginal supply price of labor in the indigenous agricultural sector. The empirical validity of this specification will be examined.

In Section 5, we will summarize the analytical framework of simultaneous determination of supply prices and wages.

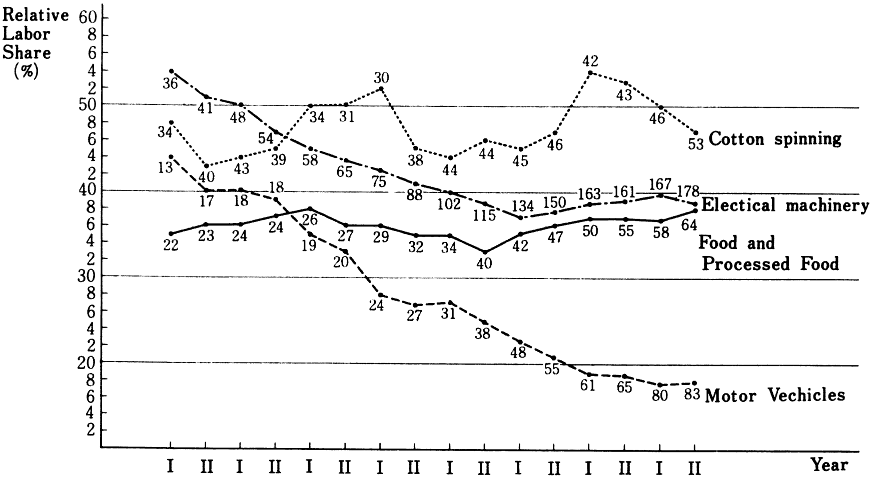
7.1 Economies of Scale and the Semi-Factor-Substitution Production Function[2]

During the post-war period, the Japanese economy has enjoyed a remarkable increase in labor productivity. The pace of increase has been particularly rapid since the mid-1950s. Let us first look at Figure 7.1 which shows movements in labor's relative share for selected major industries in Japan during the period 1956 to 1963.

In Figure 7.1, two types of industries are discernible: one consists of industries such as food and textiles, where the level of output increases sluggishly and labor's relative share remains stable and the other consists of industries such as automobiles and electrical machinery where the level of output increases rapidly and labor's relative share declines.

If we fit the well-known CES production function to this set of data, we will find that the elasticity of substitution for the former type of industry will

Figure 7.1 CHANGES IN RELATIVE LABOR-SHARE: 1956-1963



Notes: (1) I and II for each year denote the first half (April–September) and the second half (October–March) of the fiscal year.
(2) The number attached to each plotted point on the curve indicates the value added (in million yen) for each indusfry.
Source: The Bank of Japan, Shuyōkigyo Keiei-Bunseki (Survey of Management of Major Enterprise), relevant years.

be no greater than 1 or $\sigma < 1$, and that for the latter type of industries it will be greater than 1 or $\sigma > 1$. The findings associated with the latter type of industries, e.g. automobile and electric machinery manufacturing, may be interpreted as showing that, since the elasticity of substitution is greater than unity, the capital labor ratio has increased rapidly by rapid substitution of capital for labor in response to increases in wages relative to capital costs. Consequently, labor productivity has increased faster than wages and labor's relative share has declined inspite of sizeable increases in wages. This may be a plausible interpretation in explaining the development in manufacturing industries during the rapid growth period of the Japanese economy.[3]

However, the same phenomenon can be explained alternatively by a factor limitational type production function. Let us explain this alternative model, which we will use throughout in our analysis, in some detail.

Let us first assume a factor limitational input structure, for a certain level of production, and specify the model

[I]
$$L = a_L + b_L V,$$
$$K = a_K + b_K V,$$

or alternatively

[II]
$$L = a_L V^{b_L},$$
$$K = a_K V^{b_K},$$

where L is labor input, K is capital stock, V is output and a's and b's are relevant parameters.

On the basis of the latter form, Ozaki has specified the following input functions consisting of the three types of inputs using the cross-sectional data for manufacturing industries.[4]

(7.1) $L = \alpha_L X^{\beta_L}$ (labor input function),

(7.2) $K = \alpha_K X^{\beta_K}$ (capital input function),

(7.3) $M = \alpha_M X^{\beta_M}$ (raw material input function),

where X: the value of output for an establishment; L: number of workers during a year; K: tangible fixed assets at the beginning of the year; M: raw materials, power and fuel. The estimation was made for the data for 1963 and 1965.

Ozaki estimated the input functions of equations (7.1), (7.2) and (7.3) in the log linear form, using the cross-sectional data of 4 digit manufacturing industries, and found stable estimates of the following parameters:

$$0 < \beta_L < 1\,, \quad \beta_K \geq 1\,, \quad \text{and} \quad \beta_M \doteqdot 1\,.$$

This result implies that there are constant returns to scale with respect to raw material input, economies of scale with respect to labor input, and diseconomies of scale with respect to capital input. On the basis of this finding, we may explain an increase in labor productivity with an increase in output by assuming $\beta_L < 1$ for the following equation

$$\frac{X}{L} = \frac{1}{\alpha_L} X^{1-\beta_L}\,. \tag{7.4}$$

Likewise, an increase in capital labor ratio may be explained by assuming an increase in the level of output for the following equation where we assume $\beta_K > \beta_L$.

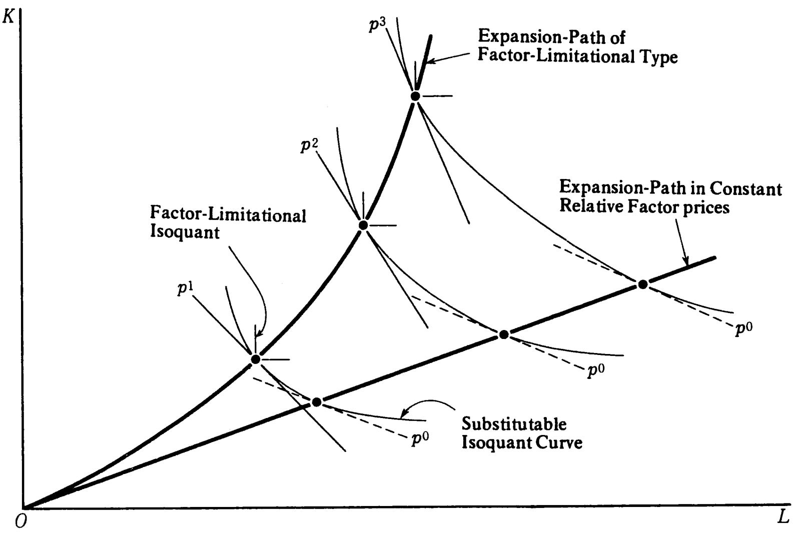
$$\frac{K}{L} = \frac{\alpha_K}{\alpha_L} X^{\beta_K - \beta_L}\,. \tag{7.5}$$

This formulation is also capable of explaining coherently the rapid increase in wages and even faster increase in labor productivity since the mid-1950s. That is to say, it is not surprising that an increase in wages and an increase in labor productivity and capital labor ratio which accompany an increase in output took place simultaneously in the period of rapid economic growth. While increases in labor productivity tend to vary from industry to industry, being governed by growth and technological factors of each industry, wages rise more or less uniformly in all industries because of the commonality associated with the labor market. Therefore, in industries with rapid productivity growth, the rate of increase of productivity may well surpass the rate of increase in wages and labor's relative share may decline consequently.

The observed facts in the course of Japanese economic development since the mid-1950s may be interpreted in two ways depending upon whether the specification of technological conditions is: (1) the case of allowing for factor substitution or (2) the factor limitational case. Let us illustrate these alternative explanations using Figure 7.2.

The simultaneous increase in wages and capital intensity K/L may be interpreted in the former way, namely, the shift of the equilibrium point up and to the right along the continuous isoquant curves in response to changes in the slope of the price line. It is also possible, however, to interpret the

Figure 7.2 ISOQUANT CURVES FACTOR-LIMITATIONAL TYPE AND FACTOR-SUBSTITUTABLE TYPE



Notes: (1) The vertical axis measures capital input, and the horizontal axis measures labor input.

(2) p^0, p^1, p^2 and p^3 represent relative factor prices.

observed phenomenon in the latter way, namely, the change in the ratio of factor combination for a certain level of output.

The method of introducing an element of technological change into the factor substitutable continuous production function implies in effect a relocation of the observed points onto the continuous isoquant surface by means of so modifying the measurement scale of the L and/or K axis.

In contrast, the simplistic models [I] and [II] mentioned above which allow for economies of scale are slightly modified versions of *Leontief's* original input function. From the point of view of the ideas behind this model, the observed points on the isoquant surface are regarded as elements of a set of activities which are represented by half-lines stemming from the origin. This set of activity rays is interpreted as indicating the technological conditions by which productivity increases with an increase in the level of output.

Taking the findings on economies of scale reported by Ozaki and others into account, we will try to formulate a production function model which has the virtues of both a factor substitutable function as the CES production function and a factor limitational function. We call this production function the Semi-Factor Substitution production function or simply the SFS production function. Let us explain the specification of this function in what follows.

Generally speaking, there are three kinds of technological relationships which need to be investigated. These are the relationships of output X with three kinds of inputs: labor, capital and raw materials. It has been ascertained by Ozaki and others that input of raw materials is proportional to the level of output. We assume here, therefore, that the assumption of a fixed input coefficient applies with respect to input of raw materials. On the other hand, the relationships of labor and capital inputs with output, will be specified as follows.

The relationship between capital equipment K and capacity output Q is specified as

$$Q = aK^b. \tag{7.6}$$

The capital equipment K and the number of workers attached to it L are assumed to be related as

$$L = cK^d \quad \text{or} \quad \frac{K}{L} = \left(\frac{1}{c}\right)K^{1-d}. \tag{7.7}$$

Note that in formulating our SFS production function we distinguish clearly between the number of workers and hours worked in the concept of labor input. We do so because we try to demonstrate explicitly the existence of economies of scale, which govern the relationship between the number of workers allocated and capital equipment.

Denoting the level of output during a year by X, we assume that the following relationship holds

$$X = Q'h^*(\frac{h}{h^*})^{\alpha} = Q'h^{*1-\alpha} \cdot h^{\alpha} = Qh^{\alpha}, \tag{7.8}$$

where h is actual hours of operation for a year, and Q is the capacity of hourly output, and h^* is normal operating hours planned at the stage of designing the equipment. Equation (7.8) indicates that even though the capacity of output is fixed in the short-run the amount of output does not necessarily vary proportionately with hours of operation if actual hours of operation h deviates from the normal hours of operation h^*. If we can regard the normal hours of operation h^* as being constant during the period of observation, then we would be able to express $Q'h^{*1-\alpha}$ simply as Q. This is the last part of equation (7.8). In analyzing observations for a relatively short period of time we will use this last portion of equation (7.8) unless otherwise stated.

Setting aside the cost of raw materials for a moment for the sake of simplicity, the remaining cost of production C can be approximated by a certain amount of fixed cost (for capital equipment) and a portion of variable cost (for labor). If we can regard that the hours of operation of productive equipment h and hours of labor input are approximately equal, then the variable cost may be given by hourly wage multiplied by man-hour labor input. The fixed cost may be expressed by $K \cdot r$, where r represents the unit capital cost. Thus we can write

$$C = L \cdot h \cdot w + K \cdot r. \tag{7.9}$$

Substituting equations (7.7) and (7.8) into (7.9) we will obtain

$$C = cK^d (\frac{X}{Q})^{\frac{1}{\alpha}} \cdot w + Kr. \tag{7.10}$$

When the amount of capital equipment is given in the short-run at the beginning of a period, the capacity of production Q will be determined by equation (7.6) and the number of workers by equation (7.7). The amount of output X can vary in the short-run with changes in hours worked (or hours of operation) h. However, the amount of labor input and the amount of capital equipment for a certain level of output X are fixed in the short-run, and therefore, the elasticity of substitution σ in this situation is 0.

The average cost function may be derived from equation (7.9) as

$$\frac{C}{X} = \frac{Lhw}{X} + \frac{Kr}{X} = \frac{Lhw}{Qh^{\alpha}} + \frac{Kr}{X} = (\frac{L}{Q})h^{1-\alpha}w + \frac{Kr}{X}. \tag{7.11}$$

The first term L/Q of the right hand side of equation (7.11) is the reciprocal of labor productivity. Therefore we may write

$$(7.12) \qquad \frac{L}{Q} = \frac{cK^d}{aK^b} = (\frac{c}{a})K^{d-b} .$$

Consequently, the portion of variable cost within the average cost may be interpreted as

$$\text{The Portion of Variable Cost within the Average Cost} = (h)^{1-\alpha} \cdot [\frac{\text{Wage Rate}}{\text{Productivity}}] .$$

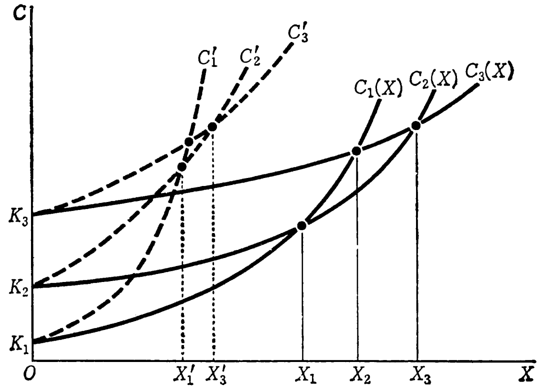
This formula implies that an average variable cost varies proportionately with the ratio of wage rate to productivity or equivalently, with the efficiency wage rate for a given level of hours of operation. If the amount of equipment increases assuming $b>d$, then the variable cost per unit output would decline for a given number of hours of operation and a given wage level while the fixed cost would increase since the labor productivity would increase with an increase in the amount of equipment as suggested by equation (7.12).

Assuming that the parameters are such that $\alpha<1$, $b<d$ and the amounts of capital equipment are such that K_1, K_2, K_3 ($K_1<K_2<K_3$) we may illustrate the cost curves as in Figure 7.3.

This diagram suggests that even if capital input K and attached employees L are perfectly complementary in the short-run, there still remains room for choice of the amount of K and thereby K/L even for the same level of output insofar as the cost function may be expressed as equation (7.10).

In terms of the cost curves represented by solid lines, K_1 will be chosen first for the range of $X<X_1$ according to the minimum cost principle, K_2 will be chosen next for the range of $X_1<X<X_3$, and finally K_3 for the range of $X>X_3$. However, when all the cost curves shift upward because of an increase in wage rate, as shown by the dotted lines, the capital equipment K_3 will become advantageous even before the level of output reaches X_1 since all intersection points shift to the left with the upward shifts of the cost curves. Thus, if wage rate w increases while the unit capital cost r remains constant, the more labor saving and capital intensive technology will be chosen according to the principle of cost minimization. In other words, although capital and labor are not substitutable ($\sigma = 0$) in this production function in the short-run, with the given capacity of equipment, there nevertheless exists the possibility of factor substitution at the stage when the producer chooses, in the face of a certain level of demand, the optimal amount of capital equipment according to the principle of cost minimization. It is in this sense that we call this production function the "Semi-Factor Substitution" production function.

Figure 7.3 SEMI-FACTOR SUBSTITUTION PRODUCTION FUNCTION AND THE SHORT-RUN COST CURVE



Notes: (1) The Vertical axis measures the total cost and the horizontal axis measures the level of output.

(2) $C_1(X)$, $C_2(X)$ and $C_3(X)$ represent the total cost curves for alternative amounts of equipment for given factor prices. C_1', C_2' and C_3' represent the total cost curves for alternative amounts of equipment for the case of increased warges.

Assuming that output X, and factor costs w and r are given exogenously, and then minimizing the cost of equation (7.9) by imposing the condition $\partial C/\partial K = 0$, we will obtain

$$K = \left\{\left(\frac{c}{a^{\frac{1}{\alpha}}}\right)\left(\frac{b}{\alpha}-d\right)\right\}^{\frac{\alpha}{\alpha+b-\alpha d}} X^{\frac{1}{\alpha+b-\alpha d}} \left(\frac{w}{r}\right)^{\frac{\alpha}{\alpha+b-\alpha d}}. \tag{7.13}$$

Substituting this into equation (7.7), we get

$$\frac{K}{L} = \left(\frac{1}{c}\right)\left\{\left(\frac{c}{a^{\frac{1}{\alpha}}}\right)\left(\frac{b}{\alpha}-d\right)\right\}^{\frac{\alpha(1-d)}{\alpha+b-\alpha d}} X^{\frac{1-d}{\alpha+b-\alpha d}} \left(\frac{w}{r}\right)^{\frac{\alpha(1-d)}{\alpha+b-\alpha d}}. \tag{7.14}$$

In this case, the elasticity of substitution σ will be defined for a given amount of output as

$$\sigma = \frac{\partial \log\left(\frac{K}{L}\right)}{\partial \log\left(\frac{w}{r}\right)} = \frac{\alpha(1-d)}{\alpha+b-\alpha d}. \tag{7.15}$$

Therefore, the elasticity of substituion σ can take any constant value.

On the other hand, the curvature of the cost curve for a certain capacity of capital equipment, as seen in Figure 7.3, is constrained by the value of parameter α. Since the capacity of production Q is determined uniquely once the size of equipment is given in the short-run, the different levels of output on the same cost curve in Figure 7.3 are generated by different lengths of hours of operation h. So long as the parameter α in equation (7.8) is less than 1, hours of operation cannot increase so far as to increase the capacity utilization ratio X/Q excessively. This is because an excessive increase in capacity utilization X/Q will increase the variable cost in equation (7.10) to a prohibitively high level since $1/\alpha>1$. In other words, the value of α thus constrains the extent to which the amount of output can vary with changes in hours of operation.

The isoquant curves in Figure 7.4 are useful in explaining the properties of the SFS production function.

Figure 7.4 consists of 4 quadrants. In each quadrant the relationship between the two variables defined by the respective two axes may be illustrated. In the first quadrant the relationship between Q and h may be depicted, in the second quadrant h and L, in the third quadrant L and K, and in the fourth quadrant K and Q.

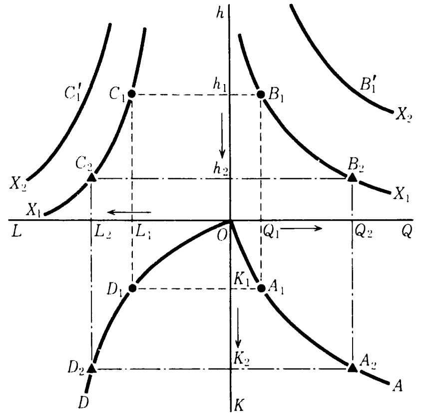
Equation (7.6) would be illustrated by the curve OA in the fourth quadrant if $b>1$. Equation (7.7) would be illustrated by the curve OD in the third quadrant if $d<1$. Equation (7.8) on the other hand will take a form of a hyperbola as shown in the first quadrant for a given level of X. The hyperbola will shift up and to the right with an increase in X. The curve illustrated in the second quadrant shows the relationship between L and h. Substituting equation (7.7) into equation (7.8) and rearranging, we will obtain

(7.16) $$X = a\left(\frac{1}{c}\right)^{\frac{b}{d}} L^{\frac{b}{d}} \cdot h^{\alpha},$$

which also represents a hyperbola for a given level of X. An increase in X in this case would shift the hyperbola up and to the left.

In the SFS production function, once the size of capital equipment K is determined at the level of K_1 then the capacity of production Q_1 will be determined corresponding to A_1 on the curve OA. At the same time, the number of employees allocated to the equipment will be determined at the level of L_1 corresponding to D_1 on the curve OD. On the other hand, although the level of output X can be set arbitrarily, changes in X will be totally dependent on changes in hours of operation h. When hours of operation h and the level of output X are determined, then man-hour labor input $L \cdot h$ (rectangular $h_1C_1L_1O$) will be determined simultaneously. Thus, when the size of capital equipment K is given, the combination of labor L and capital K will be determined uniquely for a given level of output X. It is in this sense that the elasticity of substituion of the SFS production function

Figure 7.4 AN ILLUSTRATION OF ISOQUANT CURVES OF SEMI-FACTOR SUBSTITUTION PRODUCTION FUNCTION



Notes: (1) the first (northeast) quadrant is defined by hours of operation (vertical axis) and the capacity of output (horizontal axis). The two isoquant curves are drawn on the basis of equation (7.8) for alternative cases.
The second (northwest) quadrant is defined by hours of operation (vertical axis) and the number of workers L (horizontal axis). The two isoquant curves show the relationships between h and L derived from

$$X = a\left(\frac{1}{c}\right)^{\frac{b}{d}} L^{\frac{b}{d}} h^{\alpha}$$

which is obtained by substituting equations (7.6) and (7.7) into equation (7.8), for alternative cases where $X=X_1$ and $X=X_2$.
The third (south west) quadrant is defined by capital stock K (vertical axis) and the number of workers L (horizontal axis). Curve D indicates the relationship derivable from equation (7.7).
The fourth (south east) quadrant is defined by capital sotck K (vertical axis) and the capacity of output (horizontal axis). Curve A illustrates the relationship represented by equation (7.6).
(2) Rectangle $A_2B_2C_2D_2$ corresponds to the amount of capital K_2, which is greater than K_1 to which rectangle $A_1B_1C_1D_1$ corresponds, and both rectangles correspond to the same level of output.

is zero in the short-run.

Now, let us consider the case in which a choice between alternative amounts of capital equipment is possible. Suppose the amount of capital equipment has increased from K_1 to K_2 in response to a change in relative prices of factor inputs. The capacity of production will increase from Q_1 to Q_2 accordingly. The level of output X_1 could be maintained in this situation by a reduction of hours of operation from h_1 to h_2. The labor input, on the other hand, has increased from L_1 to L_2. For the given level of output the capital intensity has obviously changed from K_1/L_1 to K_2/L_2. Thus, it seems as though labor L and capital K are mutually substitutable at the stage of choosing the appropriate size of capital equipment. If, on the other hand, hours of operation were kept constant in this situation in spite of an increase in the size of capital equipment, the level of output would have increased from X_1 to X_2 as illustrated by the higher level isoquant curve X_2.

The curvature of each curve presented in Figure 7.4 depends on the values of the parameters. The actual magnitude of elasticities of substituion mentioned above can be known only by means of empirical estimation. Let us show isoquant curves for different sectors which have been drawn using the estimates of the relevant parameters. Figures 7.5, 7.6 and 7.7 present the actually measured isoquant curves for the light manufacturing sector, heavy manufacturing sector, and commercial and service sector, respectively. The three isoquant curves in each diagram represent the curves for the observed level of output for 1955, 1960 and 1965, respectively.

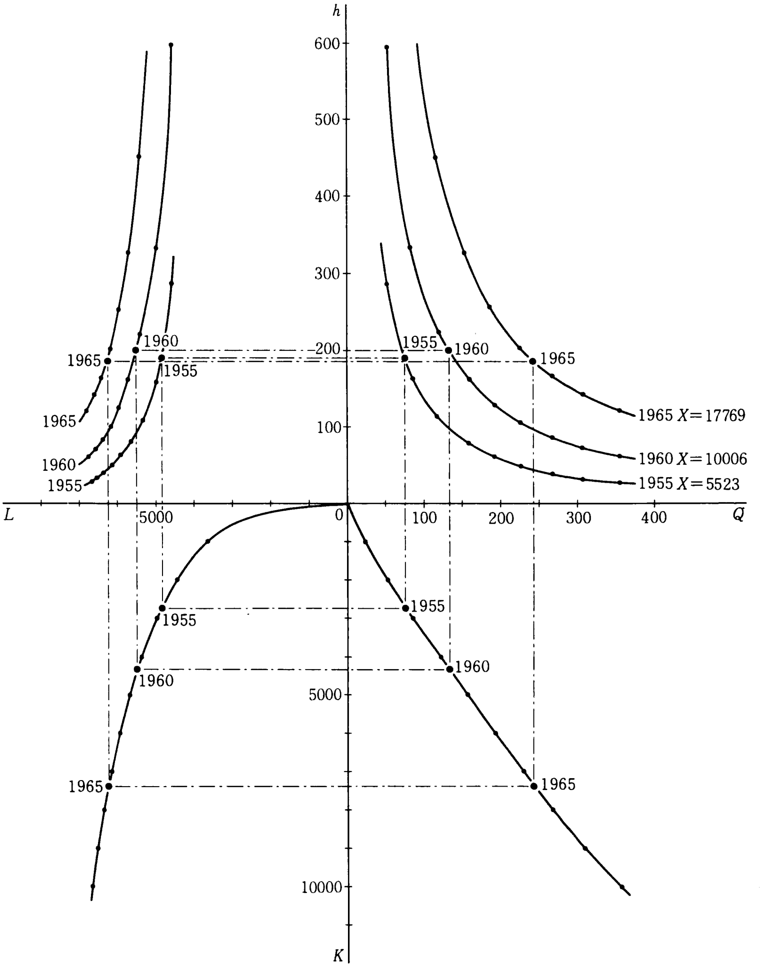
The shapes of the isoquant curves differ considerably depending upon the values of the parameters. The elasticity of substitution computed using equation (7.15) is 0.3635 for the light manufacturing sector, 0.2108 for the heavy manufacturing sector, and 0.2742 for commercial and service sector. Note that the heavy industry sector which contains highly capital intensive industries turned out to have a small elasticity of substituion which is what we would have expected.

Thus far, we have formulated a production function by explicitly incorporating the observed fact of the economies of scale. In contrast to the approach of introducing the element of technological change into the linear homogeneous production function, our approach is to represent technological conditions by the SFS production function focusing on the fact that technological innovation itself has been achieved solely in the process of pursuing the economies of scale, at least in the course of the post-war development of the Japanese economy.

7.2 Anticipated Demand Functions and Producers' Equilibrium

The firm should determine how much to produce considering how the conditions of demand for its products and how the competing firms are likely to react to its actions. The assumption itself that the market price is given to the firm may be interpreted as meaning that the firm operates under a specific presumption concerning the demand conditions in the market and reactions of competitors.

Figure 7.5 ACTUALLY MEASURED ISOQUANTS OF THE SFS PRODUCTION FUNCTION: LIGHT MANUFACTURING SECTOR



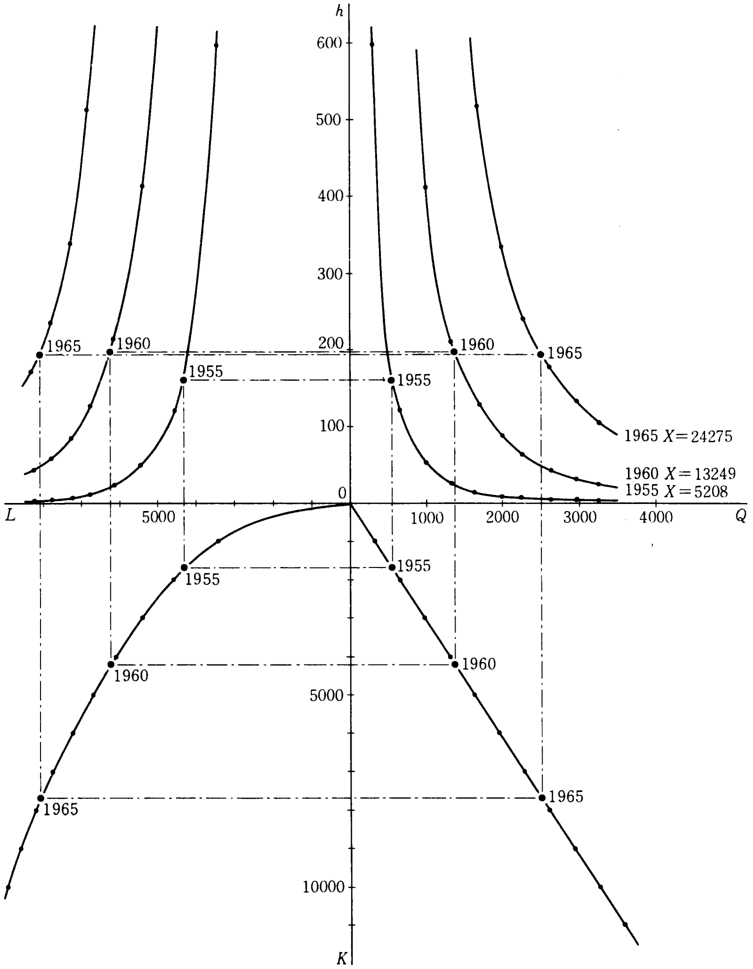
Notes: (1) The estimates of parameters of the SFS production function used in deriving the isoquants are as follows:

$\hat{a}$=0.000826, $\hat{b}$=1.1555011, $\hat{c}$=1275.96, $\hat{d}$=0.19265695, $\hat{\alpha}$=0.8173843

(2) For the procedudre of estimation, see Section of this chapter.

(3) The levels of isoquants for years 1955, 1960, and 1965 are respectively 5523, 10006 and 17769 billion yen at the 1965 constant prices.

Figure 7.6 ACTUALLY MEASURED ISOQUANTS OF THE SFS PRODUCTION FUNCTION: HEAVY MANUFACTURING SECTOR



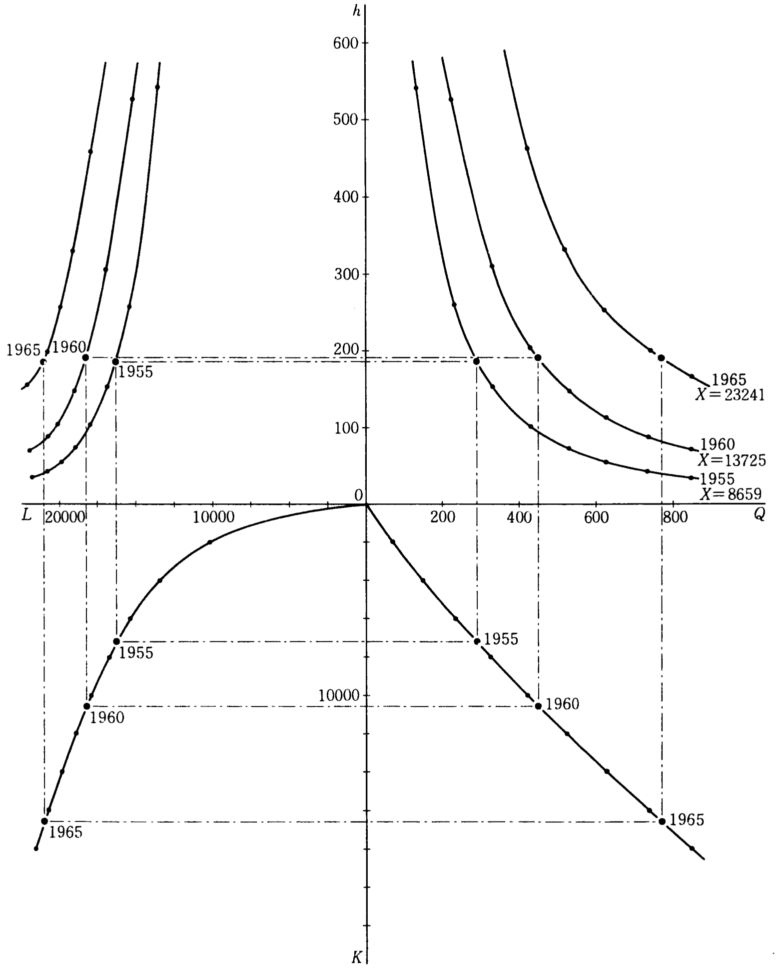
Notes: (1) The estimates of parameters of the SFS production function used in deriving the isoquants are as follows:

$\hat{a}$=0.33891 $\hat{b}$=0.99563189 $\hat{c}$=190.221 $\hat{d}$=0.41891948 $\hat{\alpha}$=0.43188414

(2) For the procedure of estimation, see Section of this chapter.

(3) The level of isoquants for years 1955, 1960 and 1965 are respectively 5208, 13249 and 24275 billion yen at the 1965 constant prices.

Figure 7.7 ACTUALLY MEASURED ISOQUANTS OF THE SFS PRODUCTION FUNCTION: COMMERCIAL AND SURVICE SECTOR



Notes: (1) The estimates of parameters of the SFS production function in deriving the isoquants are as follows:

$\hat{a} = 0.00723$ $\hat{b} = 1.190580$ $\hat{c} = 1012.06$ $\hat{d} = 0.31168829$

$\hat{\alpha} = 0.654125$

(2) For the procedure of estimation, see section of this chapter.

(3) Output levels of isoquants for years 1955, 1960 and 1965 are respectively 8659, 13725 and 23241 billion yen at the 1965 constant prices.

Suppose the market demand function for the j-th commodity is given by

$$p_j = f(X_j^T \mid Y, P)\,, \tag{7.17}$$

where X_j^T is the total demand for the j-th commodity, Y is nominal income, and P represents the level of prices in general. The so-called demand scheduled may be derived from the relationship between X_j^T and p_j while holding Y and P constant. Take consumption demand of households for example. This demand schedule may be regarded as the summation of demand schedules for the j-th commodity of individual households. Strictly speaking, the demand for the j-th commodity depends not only upon its price and incomes but also upon the prices of all other commodities. However, for the sake of simplicity here we let the general price level P represent the prices of all other goods.

Given the nominal income Y and the general price level P, we can express the impact of changes in the price of the j-th commodity upon its demand in the form of its price elasticity of demand η^* as

$$\eta^* = \frac{\partial \log X_j^T}{\partial \log p_j} = \frac{\partial X_j^T}{\partial p_j} \cdot \frac{p_j}{X_j^T}\,. \tag{7.18}$$

Confronting the demand as described above, there may exist more than one supplier of the j-th commodity in the market. Generally speaking, it would be more realistic to think that a number of firms are competing with each other in the market rather than to assume pure monopoly or a perfectly competitive situation where there are innumerable number of atomistic firms.

A firm which supplies the j-th commodity to the market will determine the amount of its supply taking into account its assessments of the prospective sales and the likely supplies of competing firms. As a result of such actions taken by individual firms, the total market supply X_j^T will be

$$X_j^T = X_j + \bar{X}\,, \tag{7.19}$$

where X_j is the j-th commodity supplied by the j-th firm and $\bar{X}$ is the total of the j-th commodity supplied by firms other than the j-th firm.

The j-th firm assesses prospective sales before deciding the amount of supply X_j. Let us postulate that this firm will presuppose the demand function with which it will be faced during the forthcoming period as

$$p_j = g(X_j \mid Y,\ P)\,. \tag{7.20}$$

The demand function (7.20) should be distinguished from the market demand function (7.17). Equation (7.20) expresses anticipated reactions in the market in response to the supply X_j of the j-th firm perceived by the j-th firm. Let us call this the anticipated demand function and distinguish it from the market demand function.

The j-th firm anticipates the amount of sales according to the assessment obtained from equation (7.20),

$$R = p_j X_j . \tag{7.21}$$

If the firm attempts to maximize profits, we may write, according to the theory of producer's equilibrium:

$$\Pi = p_j X_j - C_j , \tag{7.22}$$

and

$$\frac{\partial \Pi}{\partial X_j} = \frac{\partial p_j X_j}{\partial X_j} - \frac{\partial C_j}{\partial X_j} = 0 , \tag{7.23}$$

and consequently X_j, which will maximize the profit, will be determined.

In cases where p_j is given *a priori* exogenously we could set $\frac{\partial p_j}{\partial X_j} = 0$. In contrast, in our case, $\frac{\partial p_j}{\partial X_j}$ is not necessarily 0. Therefore equation (7.23) will be

$$\frac{\partial \Pi_j}{\partial X_j} = p_j + \frac{\partial p_j}{\partial X_j} X_j - \frac{\partial C_j}{\partial X_j} = 0 . \tag{7.24}$$

One of the components of the right hand of (7.24), which is the marginal revenue, may be rewritten as

$$\frac{\partial p_j X_j}{\partial X_j} = p_j + \frac{\partial p_j}{\partial X_j} X_j = p_j \left(1 + \frac{\partial p_j}{\partial X_j} \frac{X_j}{p_j}\right) . \tag{7.25}$$

Thus, we can write generally that the volume of equilibrium output is determined by the relationship,

Marginal revenue = marginal cost.

Let us consider further the meaning of the marginal revenue expressed by equation (7.25). In the anticipated demand function of the j-th firm, denoted by equation (7.20), the price elasticity η_j can be defined as

$$\eta_j = \frac{\partial \log X_j}{\partial \log p_j} = \frac{\partial X_j}{\partial p_j} \cdot \frac{p_j}{X_j} . \tag{7.26}$$

Substituting the price elasticity η_j in (7.26) into (7.25), marginal revenue MR may be rewritten as

$$MR = p_j \left(1 + \frac{1}{\eta_j}\right) . \tag{7.27}$$

On the other hand, we can rewrite equation (7.25) as

$$(7.28)\quad \frac{\partial p_j X_j}{\partial X_j} = p_j + \frac{\partial p_j}{\partial X_j^T} \cdot \frac{\partial X_j^T}{\partial X_j} X_j = p_j \left\{ 1 + \underbrace{\left(\frac{\partial p_j}{\partial X_j^T} \cdot \frac{X_j^T}{p_j}\right)}_{①} \underbrace{\left(\frac{\partial X_j^T}{\partial X_j} \cdot \frac{X_j}{X_j^T}\right)}_{②} \right\}.$$

The portion ① is merely a reciprocal of the price elasticity derived from the market demand function defined by equation (7.18).

$$① \quad \frac{\partial p_j}{\partial X_j^T} \cdot \frac{X_j^T}{p_j} = \frac{1}{\eta^*}.$$

The portion ② can be rewritten using equation (7.19) as

$$② \quad \frac{\partial X_j^T}{\partial X_j} \cdot \frac{X_j}{X_j^T} = \frac{\partial (X_j + \bar{X})}{\partial X_j} \cdot \frac{X_j}{X_j^T} = \left(1 + \frac{\partial \bar{X}}{\partial X_j}\right) \cdot \frac{X_j}{X_j^T}.$$

The term $\frac{\partial \bar{X}}{\partial X_j}$ indicates the reaction coefficient which represents the reactions of competing firms in response to the supply X_j of the j-th firm, and $\frac{X_j}{X_j^T}$ represents the market share of the j-th firm since it is the production of the supply of the j-th firm to the total supply. Thus, substituting ① and ② into (7.28) we will get

$$(7.29)\quad \frac{\partial p_j X_j}{\partial X_j} = p_j \left\{ 1 + \frac{1}{\eta^*} \cdot \left(1 + \frac{\partial \bar{X}}{\partial X_j}\right) \cdot \frac{X_j}{X_j^T} \right\}.$$

Comparing (7.26) and (7.29), the price elasticity η_j of the anticipated demand function will be expressed as

$$(7.30)\quad \eta_j = \frac{\eta^* \cdot \frac{X_j^T}{X_j}}{\left(1 + \frac{\partial \bar{X}}{\partial X_j}\right)}.$$

Equating (7.30) combines the price elasticity η^* of the j-th commodity in the market and the price elasticity η_j of the anticipated demand of the firm.

Suppose that the j-th firm enjoys a market share of 100 percent for the j-th commodity. In this case, since the supply of other firms in equation (7.19) is zero or $\bar{X}=0$, we will have the relationship $X_j^T = X$. Consequently, the reaction coefficient of other firms $\frac{\partial \bar{X}}{\partial X_j}$ will be zero, or

$$(7.31)\quad \frac{\partial X}{\partial X_j} = 0.$$

On the other hand, since the market share is 100 percent we have the relationship $\frac{X_j^T}{X_j} = 1$ Substituting (7.31) and $\frac{X_j^T}{X_j} = 1$ into equation (7.30) we obtain

$$(7.32) \qquad \eta_j = \frac{\eta^* \cdot \frac{X_j^T}{X_j}}{(1 + \frac{\partial \bar{X}}{\partial X_j})} = \frac{\eta^* \cdot 1}{1+0} = \eta^*.$$

Therefore, the price elasticity derived from the anticipated demand function will be equal to the price elasticity of demand for the j-th commodity in the market. Substituting $\eta_j = \eta^*$ into (7.27) we can write simply

$$(7.33) \qquad MR = p_j\,(1 + \frac{1}{\eta^*}).$$

This implies that the level of equilibrium output for a monopoly firm is determined at the point where marginal revenue equals marginal cost.

Let us suppose, on the contrary, a case in which the amount of supply of the j-th firm is negligibly small relative to the total supply in the market and thus that its market share is negligibly small. In this case, $\frac{X_j}{X_j^T}$ is very small and we may write approximately that

$$(7.34) \qquad \frac{X_j^T}{X_j} \doteqdot \infty.$$

The value of $\frac{\partial \bar{X}}{\partial X_j}$, on the other hand, may not always be constant depending upon the shares of other firms. However, regardless of its value, η_j in equation (7.30) will be

$$(7.35) \qquad \eta_j = \frac{\eta^* \cdot \frac{X_j^T}{X_j}}{(1 + \frac{\partial \bar{X}}{\partial X_j})} = \frac{\eta^* \cdot \infty}{1 + \frac{\partial \bar{X}}{\partial X_j}} = \infty.$$

Substituting (7.35) into (7.27), we get

$$(7.36) \qquad MR = p\,(1 + \frac{1}{\eta_j}) = p(1 + \frac{1}{\infty}) = p,$$

and the equilibrium output will be determined by the equality,

price = marginal cost

The assumption that the price is given exogenously may be interpreted as representing this case.

Monopoly and atomistic competition both represent an extreme case of market competition. Generally speaking, one should have an intermediate situation between the two extreme cases in mind. As is clear from equation (7.30), the price elasticity η_j derived from the anticipated demand function contains the element of conjecture in the j-th firm $\frac{\partial \bar{X}}{\partial X_j}$. Therefore it is not sound analytically to assume *a prior* that the market is either "monopolistic" or of "atomistic competition." This analytical pitfall can be avoided by introducing a general anticipated demand function into the model of producer's equilibrium. From equations (7.23) and (7.27), the level of equilibrium output will be determined by the relationship

$$\text{(7.37)} \qquad \frac{\partial \Pi}{\partial X_j} = p_j(1+\frac{1}{\eta_j}) - \frac{\partial C_j}{\partial X_j} = 0 \; .$$

In other words, the amount of equilibrium output may be thought to be determined by the equality:

Marginal revenue derived from
the anticipated demand function = marginal cost.

We specify the anticipated demand function (7.20) of the j-th firm for the j-th commodity as

$$\text{(7.38)} \qquad \frac{p_j X_j}{P} = \alpha_{sj} Y + \beta_{sj} W + \gamma_{sj} \frac{p_j}{P} + \eta_{sj}$$

or

$$p_j = \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})} ,$$

where Y is real *GDP*, W is the real amount of world trade. Both of these variables influence exogenously the determination of the volume of demand. This specific form is called a linear expenditure system. In the anticipated demand function, it is presumed generally that $\alpha_{sj}, \beta_{sj} > 0$ since the demand increases with increases in Y and W, and also that $\gamma_{sj} < 0$ since the relative increase in the price of the j-th commodity will decrease the demand.

Based on the anticipated demand function (7.38), we can rewrite the amount of sales (7.21) of the j-th firm as

$$\text{(7.39)} \qquad R = p_j X_j = \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})} X_j ,$$

and the equation for marginal revenue (7.25) is now rewritten as

$$\text{(7.40)} \quad MR_j = \frac{\partial p_j X_j}{\partial X_j} = - \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})^2} X_j + p_j = - \frac{\gamma_{sj} P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})^2} .$$

Since $\gamma_{sj}<0$ generally, MR_j is positive.

Differentiating equation (7.40) once again with respect to X_j we get

$$\frac{\partial^2 p_j X_j}{\partial X_j^2} = \frac{2\gamma_{sj} P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj})}{(X_j-\gamma_{sj})^3} < 0, \tag{7.41}$$

which means that the sales curve will be of a convex shape toward above passing through the origin. Further, we have viewed from the

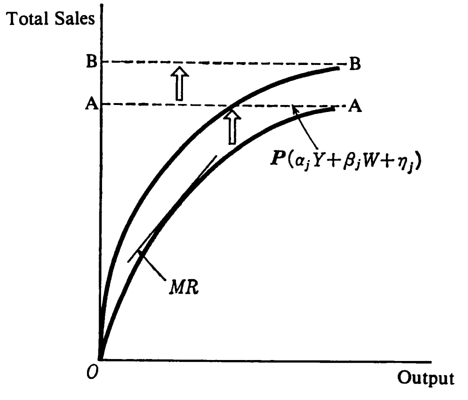
$$\lim_{x_j\to\infty} p_j X_j = \lim_{x_j\to\infty} \frac{P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj})}{(X_j-\gamma_{sj})} X_j = P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj}), \tag{7.42}$$

which implies that the sales will converge to $P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj})$ as X_j increases, and will shift upward as Y, W and P increase.

Figure 7.8 indicates this relationship. If we add a cost curve as shown by Figure 7.9, we can obtain the equilibrium amount of output from the equilibrium condition, namely, marginal revenue = marginal cost.

The slope of the half-line which combines A and the origin in Figure 7.9 represents price p_j. If the sales curve shifted from R^1 to R^2 under the given capital equipment K_j in the short-run, the equilibrium output would increase from X_j^1 to X_j^2 and price would increase from p_j^1 to p_j^2. If the sales curve shifts continuously, then we could draw a price-output schedule.

Figure 7.8 AN ILLUSTRATION OF A SHIFT IN THE ANTICIPATED DEMAND FUNCTION

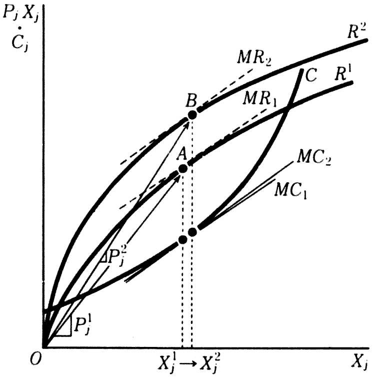


Notes: (1) Curves OA and OB represent sales curves corresponding to different levels of the anticipated demand.

(2) The levels of anticipated demand are expressed by liens AA and BB, represented by the term $P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj})$ in equation (7.38), to which sales curves approach asymptotically.

(3) The marginal revenue is represented by the slope of a tangent, for example line Mr, to the sales curve.

Figure 7.9 ANTICIPATED DEMAND FUNCTIONS AND PROFIT MAXIMIZATION



Notes: (1) The vertical axis measures the total sales or total cost, and the horizontal axis measures the level of output.
(2) OR_1 and OR_2 stand for the sales curves corresponding to the different levels of anticipated demand.
(3) Curve C represents the total cost curve.
(4) The level of output X_j^1 or X_j^2 is determined at the point of profit maximization where the marginal revenue, expressed by the slope of the tangent to the total sales curve, equals the marginal cost, expressed by the slope of the tangent to the total cost curve.
(5) The levels of prices p_j^1 and p_j^2 are represented by the slope of lines OA and OB, respectively.

7.3 The Supply Functions in Manufacturing and Service Industries

In this section we analyze the production structure of three of the four sectors classified in our model, namely: light industries (sector 2), heavy industries (sector 3), and commercial and service industries (sector 4).

For reasons stated in Chapter 6, in our analysis using the spectorally integrated data, we regard the data as reflecting the actions of an average firm for each industry sector. In treating the conditions of market competition, we posit that the firm acts according to the assumption that the market price could change in response to changes in the volume of its own supply. We feel that this is more realistic than the assumption that the firm is a pure price taker. In other words, we will employ here the idea of anticipated demand function discussed the previous section

The production function is specified in the form of the Semi-Factor Substituion production function which was formulated in Section 1 of this Chapter. The supply schedule, when the capital stock is fixed in the short-run, will be derived from the anticipated demand function which represents the firm's assessments of the reaction of competitors and the SFS production function which represents the technological constraints on production.

Needless to say, the empirical validity of the supply schedule that will be partially examined in this section will have to be examined again in connection with the entire system of our model.

1. The Formulation of Supply Functions

Let us formulate the production structure for sectors 1, 2 and 3 using the SFS production function. By subscript denotes the j-th number attached to each sector.

$$Q_j = a_j K_j^{b_j} \quad , \tag{7.43}$$

$$L_j = c_j K_j^{d_j} \quad , \tag{7.44}$$

$$X_j = Q_j' h_{j*} \left(\frac{h_j}{h_{j*}}\right)^{\alpha_j} = Q_j' h_j^{*1-\alpha_j} h_j^{\alpha_j} = Q h_j^{\alpha_j} , \tag{7.45}$$

where Q_j is the production capacity of the j-th sector for each period, K_j is capital stock of the j-th sector in constant prices, L_j is the number of workers employed in the j-th sector, X_j is output of the j-th sector for a unit period, and h_j is hours of operation of the j-th sector for a unit period. h_j^* is the normal hours of operation planned at the stage of designing the production equipment with the capacity Q_j'. Parameters a_j, b_j, c_j, d_j and α_j represent technological properties of production. Equation (7.45) approximates the relationship that when the output exceeds the normal capacity of production the output can not increase proportionately with an increase in hours of operation.

Let us now define the cost of production

$$C_j = L_j h_j w_j + K_j \eta_{Kp} (r_j + d_{ej}) + \sum_{i=1}^{4} p_i a_{ij} X_j + t_{Ij} p_j X_j \tag{7.46}$$

where C_j is production costs of the j-th sector, $L_j h_j w_j$ is labor cost, $K_j \eta_{kp}(r_j + d_{ej})$ is capital cost, $\sum_{i=1}^{4} p_i a_{ij} X_j$ is raw material cost. η_{Kp} is the investment good deflator, r_j is unit capital cost, and d_{ej} is the rate of depreciation. The last term t_{Ij} is the rate of indirect tax for the j-th sector. The amount of tax is estimated here by multiplying the tax rate by the value of sales $p_j X_j$. The indirect tax rate is given exogenously for each sector. The treatment of capital cost will be described in detail in Chapter 11.

The raw material cost is the sum of intermediate inputs of the input-output table. Therefore, specifying the cost of production as (7.46), the profit will then be defined as

$$\Pi_j = p_j X_j - C_j . \tag{7.47}$$

The necessary condition for equilibrium in this formulation is equality between marginal revenue and marginal cost. With the capital stock fixed in the short-run, production capacity Q_j will be determined by equation (7.43) and employment L_j by (7.44). Consequently, once the level of output X_j is determined by the profit maximization principle, then hours of operation h_j will be determined from equation (7.45).

Now, marginal revenue may be written generally as

$$MR_j = \frac{\partial p_j X_j}{\partial X_j} = p_j + \frac{\partial p_j}{\partial X_j} X_j . \tag{7.48}$$

In the case of a perfectly competitive market we have $\frac{\partial p_j}{\partial X_j} = 0$, while in the case of an imperfectly competitive market $\frac{\partial p_j}{\partial X_j}$ is generally not zero. Since we do not presume for our market analysis such extreme cases as a perfectly competitive market or monopoly, we let p_j vary depending upon the specific conditions of market competition.

Marginal cost, on the other hand, also contains prices $p_i (i = 1,\dots,4)$ of raw materials. We assume here that $\frac{\partial p_i}{\partial X_j} = 0$ for the prices of other goods p_i $(i \neq j)$. We treat prices of other goods in this way for the sake of simplicity and also for the reason that we do not think it is necessary to incorporate into our model an unrealistic assumption that the firm changes the level of its production according to changes in prices of commodities of other sectors induced by changes in its own production.

As an approximation of the anticipated demand function, we use here the demand function analogous to a linear expenditure system shown in previous section.

$$\frac{p_j X_j}{P} = \alpha_{sj} Y + \beta_{sj} W + \gamma_{sj} \frac{p_j}{P} + \eta_{sj} \quad \text{or} \quad p_j = \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})}, \tag{7.49}$$

where P is the level of prices in general, Y is real GDP, and W is the real value of world trade.

Using equation (7.49) we can write firms' sales as

$$p_j X_j = \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})} \cdot X_j . \tag{7.50}$$

As noted earlier, the sales curve (7.50) is concave passing through the origin which converges to $P(\alpha_{sj}Y + \beta_{sj}W + \eta_{sj})$ asymptotically as X_j increases. Generally the parameters are regarded to have the character $\alpha_{sj} > 0$, $\beta_{sj} > 0$, $\gamma_{sj} < 0$.

The marginal revenue is derived from equation (7.50) as

$$(7.51) \qquad MR_j = \frac{\partial p_j}{\partial X_j} X_j + p_j = -\frac{P(\alpha_j Y + \beta_j W + \eta_j)}{(X_j - \gamma_{sj})^2} X_j + p_j$$

$$= -\frac{\gamma_{sj} P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})^2} = -p_j\left(\frac{\gamma_{sj}}{X_j - \gamma_{sj}}\right),$$

in which eventually only the parameter of the anticipated demand function γ_{sj} is included.

The marginal cost on the other hand will be given from equation (7.46) if $w_j, \eta_{kp}, r_j, d_{ej}, p_i\,(i \neq j), t_{ij}$ are given exogenously,

$$(7.52) \qquad MC_j = \frac{\partial C_j}{\partial X_j} = \frac{\partial\,(L_j h_j w_j + K_j \eta_{Kp}\,(r_j + d_{ej}) + \Sigma p_i a_{ij} X_j + t_{Ij} p_j X_j)}{\partial X_j}$$

$$= L_j w_j \frac{\partial h_j}{\partial X_j} + \sum_{i=1}^{4} p_i a_{ij} + \frac{\partial p_j}{\partial X_j} a_{jj} X_j + t_{Ij} \frac{\partial p_j}{\partial X_j} X_j + t_{Ij} p_j$$

$$= \left(\frac{1}{\alpha_j}\right) L_j h_j w_j / X_j + \sum_{i=1}^{4} p_i a_{ij} + t_{Ij} p_j - (a_{jj} + t_{Ij}) X_j \frac{P(\alpha_{sj} Y + \beta_{sj} W + \eta_{sj})}{(X_j - \gamma_{sj})^2}$$

$$= \left(\frac{1}{\alpha_j}\right) \frac{L_j h_j w_j}{X_j} + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i a_{ij} - (a_{jj} + t_{Ij}) \frac{\gamma_{sj}}{(X_j - \gamma_{sj})} \cdot p_j .$$

The last equation was obtained by substituting $\frac{\partial h_j}{\partial X_j} = \left(\frac{1}{\alpha_j}\right) \frac{h_j}{X_j}$ derived from equation (7.45) and also equation (7.51) into equation (7.52).

Deriving the equilibrium condition $MR_j = MC_j$ from equations (7.51) and (7.52) and rearranging it with respect to p_j, we get

$$(7.53) \qquad p_j = \frac{(X_j - \gamma_{sj})}{\gamma_{sj}(a_{jj} + t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \frac{L_j h_j w_j}{X_j} + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i a_{ij} \right\}$$

$$= \frac{(X_j - \gamma_{sj})}{\gamma_{sj}(a_{jj} + t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \frac{c_j K_j^{d_j} h_j w_j}{X_j} + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i a_{ij} \right\},$$

which is the short-run supply equation of the j-th sector with fixed capital equipment.

Obviously, equation (7.53) contains the condition of equal marginal productivities. In the SFS production function, the marginal productivity with respect to L_j is meaningless since the number of workers employed L_j is determined automatically once the capital stock K_j is given in the short-run. However, we can define the marginal productivity with respect to man-hour labor input L_jh_j. From equation (7.45) we may write

$$\frac{\partial X_j}{\partial L_j h_j} = \frac{\partial X_j}{L_j \partial h_j} = \alpha_j \frac{X_j}{L_j h_j}$$

On the other hand, differentiating the profit equation (7.47) with respect to L_jh_j and equating this with zero, we get

$$\text{(7.54)} \quad \frac{\partial \Pi_j}{\partial L_j h_j} = (1-t_{Ij})\left\{\frac{\partial p_j}{\partial X_j} \cdot \frac{\partial X_j}{\partial L_j h_j} X_j + p_j \frac{\partial X_j}{\partial L_j h_j}\right\} - w_j - \frac{\partial p_j}{\partial X_j} \cdot \frac{\partial X_j}{\partial L_j h_j} a_{jj} X_j - \Sigma p_i a_{ij} \frac{\partial X_j}{\partial L_j h_j} = 0.$$

Substituting $\frac{\partial p_j}{\partial X_j}$ and $\frac{\partial X_j}{\partial L_j h_j}$ into equation (7.54) and rearranging it with respect to p_j, we will again obtain equation (7.53). Rearranging equation (7.54) with respect to $\frac{\partial X_j}{\partial L_j h_j}$ we obtain

$$\frac{\partial X_j}{\partial L_j h_j} = \frac{w_j}{(1-t_{Ij})\left\{\frac{\partial p_j}{\partial X_j} X_j + p_j\right\} - \left\{\frac{\partial p_j}{\partial X_j} a_{jj} X_j + \Sigma p_i a_{ij}\right\}},$$

which is the equation of equal marginal productivities.

The supply equation (7.53) relies on parameters c_j, d_j, α_j of the SFS production function and parameter γ_{sj} of the anticipated demand function. Therefore, it is necessary to estimate them empirically. The equilibrium condition, i.e. marginal revenue = marginal cost, which has been qualified so far is the necessary condition for profit maximization. To see whether the estimated parameters satisfy the sufficient condition for profit maximization, we need to know the sign conditions of the second order derivative. The sufficient condition in this case may be written as

$$\text{(7.55)} \quad \frac{\partial^2 \Pi}{\partial X_j^2} = \frac{2\gamma_{sj}(1-a_{ij}-t_{Ij})}{(X_j-\gamma_{sj})^2} p_j - \left(\frac{1}{\alpha_j}\right)\left(\frac{1-\alpha_j}{\alpha_j}\right) L_j Q_j^{-\frac{1}{\alpha_j}} X_j^{\frac{1}{\alpha_j}-2} w_j < 0\,.$$

For the estimated parameters to be meaningful in terms of economic theory, it has to be ascertained that they satisfy the sign conditions of equation (7.55).

2. Estimation

Direct Estimation of Single Equation

One possible procedure for estimate the parameters of the supply equations would be to estimate the parameters of the SFS production function and of the anticipated demand function separately by means of the direct estimation method and to select the estimates which are mutually logically compatible.

Estimates for parameters a_j, b_j, c_j, d_j, and α_j of the production functions (7.43), (7.44) and (7.45) are not directly obtainable since production capacity Q_j is not necessarily observable. To circumvent this difficulty, let us substitute equation (7.43) into (7.45) to get

$$X_j = Q_j h_j^{\alpha_j} u_j = a_j K_j^{b_j} h_j^{\alpha_j} u_j \tag{7.56}$$

and

$$L_j = c_j K^{d_j} v_j \tag{7.57}$$

and obtain estimates by fitting log-linear forms of the above equations to the data. u_j and v_j represent disturbance terms.

The data used for estimation are the time-series data from 1955 to 1965. The data for K_j are capital stock at constant prices estimated by the Economic Planning Agency, L_j are the number of persons employed by sectors based on the Labor Force Survey, and X_j are annual domestic production at constant prices based on the Input-Output Table as a bench mark. The data for h_j are sectoral hours of operation estimated from the data of monthly hours worked obtained from the *Monthly Labour Survey*.

The results of the least squares estimation of the log-linear form of equation (7.56) are:

Sector 2

$$\log X_2 = -8.4709086 + 1.1833682 \log K_2 + 1.467950 \log h_2 . \tag{7.58}$$

$$(5.5580) \qquad (0.05535) \qquad (0.9824)$$

$$\bar{R} = 0.9949 \quad d.w. = 1.607, \quad d.f. = 8$$

Sector 3

$$\log X_3 = -5.5250264 + 0.9365409 \log K_3 + 1.3678992 \log h_3 . \tag{7.59}$$

$$(2.8806) \qquad (0.07695) \qquad (0.5969)$$

$$\bar{R} = 0.9797 \quad d.w. = 1.898, \quad d.f. = 8$$

Sector 4

(7.60) $$\log X_4 = -18.735953 + 1.2037573 \log K_4 + 3.2677651 \log h_4 .$$
$$(4.1752) \quad (0.02571) \quad (0.7891)$$
$$\bar{R} = 0.9977 \quad d.w. = 1.969, \quad d.f. = 8$$

The figures in parentheses are standard deviations for parameters, $\bar{R}$ is the multiple correlation coefficient adjusted for the degree of freedom, *d.w.* is the Durbin-Watson ratio and *d.f.* is the degree of freedom for estimation.

All the parameters are statistically significant and the overall fit is also good. The formulation of $X_j = Q_j h^{\alpha j}$ in the SFS production function means that the variable cost increases at an increasing rate once production exceeds the normal production capacity. In this situation, the value of the parameter α_j is theoretically expected to fall in the range $0 < \alpha j < 1$. However, the estimated value of α_j turned out to be greater than unity for every sector. From the viewpoint of the methodology of statistical estimation, while K_j is a predetermined endogenous variable, h_j is an endogenous variable to be determined in the present period. Then h_j is not independent from u_j of equation (7.56). Since independent variables and a disturbance term should be mutually independent for the least squares method to be unbiased, it is possible that the obtained estimates contain upward biases because this methodological presumption was violated.

The results of log-linear estimation of equation (7.57) for the different sectors are:

(7.61) $$\log L_2 = 7.1514541 + 0.19265695 \log K_2 .$$
$$(0.1409) \quad (0.01679)$$
$$\bar{r} = 0.9638 \quad d.w. = 1.38, \quad d.f. = 9$$

(7.62) $$\log L_3 = 5.2481878 + 0.41891948 \log K_3 .$$
$$(0.2832) \quad (0.03424)$$
$$\bar{r} = 0.9680 \quad d.w. = 0.486, \quad d.f. = 9$$

(7.63) $$\log L_4 = 6.9197492 + 0.31168829 \log K_4 .$$
$$(0.2087) \quad (0.02249)$$
$$\bar{r} = 0.9748 \quad d.w. = 0.728, \quad d.f. = 9$$

The results of estimation are found to be statistically significant both in terms of intercept and of regression coefficient. However, the Durbin-Watson ratios indicate that there exists some degree of serial correlation for each of the three sectors analyzed.

Let us now estimate the parameters of anticipated demand equation (7.49) for each sector. The anticipated demand function has to be distinguished from the demand function in the actual market since the former is supposed to approximate the firm's anticipation of market response to its supply. The data for firms' anticipated demand are therefore not directly observable. However, we may obtain some information about parameters of anticipated demand function by means of fitting equation (7.64) to relevant observed data in the actual market.

$$\frac{p_j X_j}{P} = \alpha_{sj} Y + \beta_{sj} W + \gamma_{sj} \left(\frac{p_j}{P}\right) + \eta_{sj} + u_j \qquad (7.64)$$

where u_j is a random disturbance term. Approximate estimates for the parameters of the anticipated demand function can be obtained by fitting equation (7.64) to the data by the least squares method. The notations are: P: prices in general, Y: real *GDP*, W: quantity index of world trade, p_j: output deflator in the j-th sector (1965=100), and X_j: output of the j-th sector.

The parameters are theoretically expected to have values in the ranges $\alpha_{sj} > 0$, $\beta_{sj} > 0$, $\gamma_{sj} < 0$. The results of the estimation are:

$$\frac{p_2 X_2}{P} = \underset{(18.7756)}{19.0082Y} - \underset{(0.0396)}{0.0082W} + \underset{(21.743)}{81.251}\left(\frac{p_2}{P}\right) - \underset{(2002.62)}{138.48}\,. \qquad (7.65)$$

$\bar{R} = 0.9980 \qquad d.f. = 7$

$$\frac{p_3 X_3}{P} = -\underset{(30.628)}{37.4240Y} + \underset{(0.0738)}{0.1644W} + \underset{(10.009)}{97.3614}\left(\frac{p_3}{P}\right) - \underset{(2059.78)}{6230.152}\,. \qquad (7.66)$$

$\bar{R} = 0.991 \qquad d.f. = 7$

$$\frac{p_4 X_4}{P} = \underset{(112.96)}{371.790Y} + \underset{(0.2467)}{0.3272W} - \underset{(19.700)}{0.7191}\left(\frac{p_4}{P}\right) - \underset{(9756.4)}{21822.48}\,. \qquad (7.67)$$

$\bar{R} = 0.9984 \qquad d.f. = 7$

The correlation coefficient is significant at the 1% level for every sector. However, the parameters β_{s2}, γ_{s2} for Sector 2 and α_{s3}, γ_{s3} for Sector 3 do not satisfy the theoretically expected sign conditions. In Sector 4, γ_{s4} is not statistically significant. This suggests that these estimates of the parameters suffer from multi-collinearity, probably caused by the high correlation between real *GDP*, *Y* and the quantity index of world trade *W*. This result illustrates the difficulty associated with the application of the simple direct method for estimating the parameters of the anticipated demand function.

For the purpose of deriving stable supply schedules, we need to find stable values of the parameters α_j and γ_{sj}. The results obtained above showed that the direct estimates of both of these parameters are either statistically insignificant or contradictory to theoretical sign conditions. Therefore, it is necessary to take an alternative approach, namely the method of structural equation estimation.

The Structural Equation Estimation

Let us consider once again, with the help of a diagram, the nature of data of sectoral output deflators (price indices) and outputs.

Figure 7.10 illustrates hypothetical shifts of the supply and demand schedules of the j-th sector. The supply schedule can shift with changes in output capacity of production equipment, wage rates, and prices in raw materials. The demand schedule, on the other hand, can shift with changes in the level of incomes and relative prices. If we can regard that changes in inventories are also contained in the market demand then we may interpret the observable data of output deflator p_j and output X_j as reflecting the locus of shifts in demand-supply equilibrium from year to year. If for example, the supply schedule shifts from S_{1965} to S_{1968} and the demand schedule from D_{1965} to D_{1968} during the period 1965 to 1968, as shown by Figure 7.10, then the corresponding values of p_j and X_j are interpreted to indicate positions A, B, C and D on the locus of shifts of demand-supply equilbrium.

The anticipated demand curve, on the other hand, may not necessarily conform with the actual demand curve D. However, equation (7.49) implies a hyperbola on the plane of p_j and X_j, and the term $P\,(\alpha_{sj}Y + \beta_{sj}W + \eta_{sj})$ can be interpreted as the shift variable of the hyperbola. If a certain value is given to γ_{sj}, then the value of $\frac{\partial p_j}{\partial X_j}$ will be determined for a certain combination of X_j will be determined for a certain combination of X_j and p_j uniquely, regardless of the level of $P(\alpha_{sj}Y + \beta_{sj}W + \eta_{sj})$, according to the following relationship,

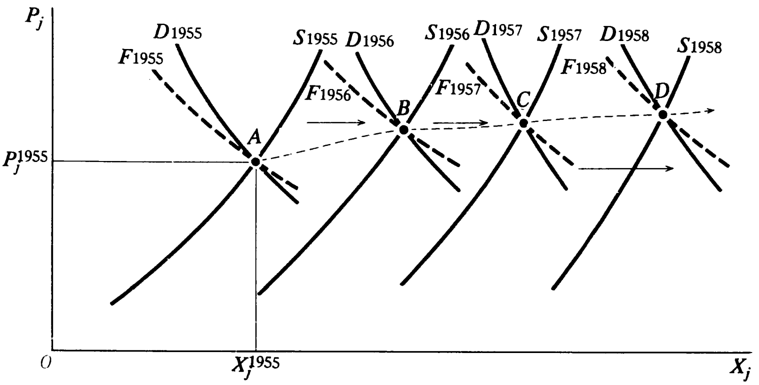
$$(7.68) \qquad \frac{\partial p_j}{\partial X_j} = \frac{P(\alpha_{sj}Y+\beta_{sj}W+\eta_{sj})}{(X_j-\gamma_{sj})^2} = -\frac{p_j}{(X_j-\gamma_{sj})} \ .$$

Similarly regardless of the level of $P(\alpha_{sj}Y + \beta_{sj}W + \eta_{sj})$, marginal revenue will also be determined by

$$(7.69) \qquad MR_j = \frac{\partial p_j X_j}{\partial X_j} = \frac{\partial p_j}{\partial X_j}X_j + p_j = -\frac{p_j X_j}{(X_j-\gamma_{sj})} + p_j = -p_j\left(\frac{\gamma_{sj}}{X_j-\gamma_{sj}}\right).$$

Therefore although we can not observe the true level of anticipated demand, the anticipated demand function may also be regarded as passing through the intersections of the actual demand and supply curves such as A, B, C and D in Figure 7.10. Consequently, parameter γ_{sj} may be estimated by fitting the

Figure 7.10 A HYPOTHETICAL ILLUSTRATION OF A SHIFT OF A DEMAND-SUPPLY EQUILIBRIUM POINT



Notes: (1) The vertical and horizontal axes measure price and the level of output, respectively.

(2) Solid curves denoted by D_t and S_t represent demand and supply schedules, respectively. Dotted curves F_t represent anticipated demand schedules. The subscript t denotes year of observation, e.g.; t = 1955, 1956, 1957 or 1958.

(3) A, B, C and D stand for the points of demand-supply equilibrium.

supply equation to the time-series data of p_j and X_j. Curves F_{1965}, ..., F_{1968} illustrate anticipated demand functions drawn on the basis of an assumption that they pass through demand-supply equilibrium points.

Reforming the supply function (7.53), we may obtain

(7.70)

$$p_j + \frac{\sum\limits_{(i \neq j)}^{i} p_i a_{ij}}{(a_{jj} + t_{Ij} - 1)} = \frac{1}{\gamma_{sj}\alpha_j}\left(\frac{L_j h_j w_j}{a_{jj} + t_{Ij} - 1}\right) + \frac{1}{\gamma_{sj}}\left(\frac{X_j \sum\limits_{(i \neq j)}^{i} p_i a_{ij}}{a_{jj} + t_{Ij} - 1}\right) - \frac{1}{\alpha_j}\left\{\frac{L_j h_j w_j}{(a_{jj} + t_{Ij} - 1)X_j}\right\},$$

where a_{ij} is the vector of intermediate input coefficients of the j-th sector, which will be estimated using the time-series data originally compiled for the purpose of estimating the converters which will be explained later in chapter 9. t_{Ij} is the rate of indirect taxes for the j-th sector. Supplying in addition to them the data for p_j: output deflator, X_j: output, h_j: hours of operation, L_j: the number of workers and w_j wage rate per man-hour for the period 1955 to 1965, we can estimate parameters γ_{sj} for the anticipated demand function and α_j for the production function.

Rewriting equation (7.70) we get

(7.71) $$y_j = A_{1j}x_{1j} + A_{2j}x_{2j} + A_{3j}x_{3j},$$

where

$$y_j = p_j + \frac{\sum_{(i\neq j)}^{i} p_i a_{ij}}{(a_{jj} + t_{Ij} - 1)},$$

$$x_{1j} = \frac{L_j h_j w_j}{(a_{jj} + t_{Ij} - 1)},$$

$$x_{2j} = \frac{X_j \sum_{(i\neq j)}^{i} p_i a_{ij}}{(a_{jj} + t_{Ij} - 1)},$$

$$x_{3j} = \frac{L_j h_j w_j}{(a_{jj} + t_{Ij} - 1) X_j},$$

$$A_{1j} = \frac{1}{\gamma_{sj}\alpha_j}, \qquad A_{2j} = \frac{1}{\gamma_{sj}}, \qquad A_{3j} = -\frac{1}{\alpha_j}.$$

And if we take the disturbance term u_j into account, equation (7.71) would be written as

(7.72) $$y_j = A_{1j}x_{1j} + A_{2j}x_{2j} + A_{3j}x_{3j} + u_j,$$

which may be regarded as the linear regression equation of y_j on independent variables x_{1j}, x_{2j} and x_{3j}. The parameters A_{1j}, A_{2j}, A_{3j} have to satisfy theoretically the following conditions.

(7.73) $$A_{1j} = -A_{2j} \cdot A_{3j}, \text{ namely } \frac{1}{\gamma_{sj}\alpha_j} = -(\frac{1}{\gamma_{sj}}) \cdot (-\frac{1}{\alpha_j}).$$

To comply with this condition, we have to find such values of parameters A_{1j}, A_{2j}, A_{3j} that the sum of squares of residuals of equation (7.72)

$$\sum_{t=1}^{11} u^{t^2} = \sum_{t=1}^{11} (y_j^t - A_{1j}x_{1j}^t - A_{2j}x_{2j}^t - A_{3j}x_{3j}^t)^2$$

may be minimized under the constraint of equation (7.73) Denoting the Lagrange multiplier by λ, the objective function may be expressed as

(7.74) $$\phi_j = \Sigma u_j^2 = \Sigma(y_j - A_{1j}x_{1j} - A_{2j}x_{2j} - A_{3j}x_{3j})^2 - \lambda(A_{1j} + A_{2j}A_{3j}).$$

From the conditions for minimizing the objective function we will get the following system of normal equations:

$$\frac{\partial \phi_j}{\partial A_{1j}} = -2\Sigma x_{1j}(y_j - A_{1j}x_{1j} - A_{2j}x_{2j} - A_{3j}x_{3j}) - \lambda = 0,$$

$$\frac{\partial \phi_j}{\partial A_{2j}} = -2\Sigma x_{2j}(y_j - A_{1j}x_{1j} - A_{2j}x_{2j} - A_{3j}x_{3j}) - \lambda A_{3j} = 0,$$

$$\frac{\partial \phi_j}{\partial A_{3j}} = -2\Sigma x_{3j}(y_j - A_{1j}x_{1j} - A_{2j}x_{2j} - A_{3j}x_{3j}) - \lambda A_{2j} = 0,$$

$$\frac{\partial \phi_j}{\partial \lambda} = A_{1j} + A_{2j}A_{3j} = 0\,.$$

The last equation of the normal equations represents the constraint itself. We can rewrite the first three equations in terms of matrix notation as:

(7.75)

$$\begin{bmatrix} \Sigma x_{1j}^2 & \Sigma x_{1j}x_{2j} & \Sigma x_{1j}x_{3j} \\ \Sigma x_{2j}x_{1j} & \Sigma x_{2j}^2 & \Sigma x_{2j}x_{3j} - \frac{1}{2}\lambda \\ \Sigma x_{3j}x_{1j} & \Sigma x_{3j}x_{2j} - \frac{1}{2}\lambda & \Sigma x_{3j}^2 \end{bmatrix} \begin{bmatrix} A_{1j} \\ A_{2j} \\ A_{3j} \end{bmatrix} = \begin{bmatrix} \Sigma x_{1j}y_j + \frac{1}{2}\lambda \\ \Sigma x_{2j}y_j \\ \Sigma x_{3j}y_j \end{bmatrix}$$

Since equation (7.75) is the system of normal equations of regression equations about the origin, parameters $\hat{A}_{1j}$, $\hat{A}_{2j}$, $\hat{A}_{3j}$ can be estimated once a certain value is given for λ in equation (7.75). The value of λ needs to be such that the estimated parameters satisfy the conditions $\hat{A}_{1j} + \hat{A}_{2j} \times \hat{A}_{3j} = 0$. In other words, A_{1j}, A_{2j}, A_{3j} are functions of λ, respectively, through equation (7.75). Thus the constraint equation is also the nonlinear function of λ such as

(7.76) $$\theta(\lambda) = A_{1j}(\lambda) + A_{2j}(\lambda) \cdot A_{3j}(\lambda) = 0.$$

Solving equation (7.76) with respect to λ, we may obtain estimates of parameters A_{1j}, A_{2j}, A_{3j} simultaneously.

Because it is non-linear, it has been necessary to rely on the convergence computation method to get the solution for λ of equation (7.76). We used the Newton method here. The process of convergence was quite quick, and the solution was obtained within five steps.

Table 7.1 presents converged values together with the initial values classified by sectors. For each sector, the initial value was chosen with $\lambda = 0$, representing, the case in which the constraint $A_{1j}+A_{2j}\cdot A_{3j}=0$ is not imposed.

In the case of Sector 2, the initial values are $A_{12} = .10387 \times 10^{-4}$, $A_{22} = -.19086 \times 10^{-4}$ and $A_{32} = -.11722 \times 10$. In this case $A_{12}+A_{22}\cdot A_{32} = \theta(0)$ takes the values of $.32761 \times 10^{-4}$, which is quite small. Since, theoretically,

$$A_{12} = \frac{1}{\gamma_{s2}\alpha_2}, \quad A_{22} = \frac{1}{\gamma_{s2}}, \quad A_{32} = -\frac{1}{\alpha_2}$$

the values of the parameters should be $A_{12} < 0$, $A_{22} < 0$, $A_{32} < 0$ if $\gamma_{s2} < 0$ and

Table 7.1 RESULTS OF NON-LINEAR ESTIMATION (THE NEWTON METHOD) OF PARAMETERS OF THE SECTORAL SUPPLY SCHEDULES

	Sector 2		Sector 3		Sector 4	
	Initial	Final	Initial	Final	Initial	Final
Steps	0	3	0	2	0	3
λ	0.0	3.9117	0.0	−0.5358	0.0	8.1123
A_{1j}	$.10387\times10^{-4}$	$-.10517\times10^{-4}$	$-.10154\times10^{-5}$	$-.38192\times10^{-6}$	$.17344\times10^{-4}$	$.10043\times10^{-5}$
A_{2j}	$-.19086\times10^{-4}$	$-.85809\times10^{-5}$	$.18727\times10^{-6}$	$-.16405\times10^{-6}$	$-.44999\times10^{-4}$	$.66024\times10^{-6}$
A_{3j}	$-.11722\times10$	$-.12234\times10$	$-.23144\times10$	$-.23136\times10$	$-.16312\times10$	$-.15596\times10$
$\theta(\lambda)$	$.32761\times10^{-4}$	$-.191096\times10^{-7}$	$-.14488\times10^{-5}$	$-.23751\times10^{-8}$	$.90744\times10^{-4}$	$-.25396\times10^{-7}$
γ_{sj}		$-.11653\times10^{6}$		$-.60958\times10^{7}$	$-.22222\times10^{5}$	
α_j		.81735392		0.43222479	0.61306199	

Notes: (1) Columns named "Initial" and "Final" list, respectively, initial values and final converged values of the parameters.
(2) Notations are:
Steps: the number of steps of iterative computation,
λ: Lagrangian multiplier,
A_{ij}: i-th parameter (i = 1, 2 and 3) of equation (7.71) for the j-th sector (j = 2, 3 and 4),
$\theta(\lambda)$: the value of the objective function,
α_j, γ_{sj}: parameters of the supply schedules for the j-th sector (j = 2, 3 and 4).

$\alpha_2 > 0$. However, the sign conditions of the parameters are not satisfied since $A_{12} > 0$, according to the result of this estimation. By applying the Newton method, $\lambda = 3.9117$ was obtained by three interative computations. The constraint $\theta(3.9117)$ in this case has become $-.191096 \times 10^{-7}$ which is approximately 1/2000 of the initial value $\theta(0)$. In this situation, the values of the parameters turned out to be $A_{12} = -.10517 \times 10^{-5}$, $A_{32} = -.12234 \times 10$ which satisfy the theoretical sign conditions. Based on this result we can compute values for the parameters such that $\gamma_{s2} = -.11653 \times 10^{-6}$ and $\alpha_2 = 0.81735392$.

Similarly for Sector 3, by the convergence computation, the value of objective function has converged to take a value which is 1/2000 of the initial value, and the parameters satisfied the sign conditions with respect to the convergence values. The parameters turned out to be $\gamma_{s3} = -.60958 \times 10^{7}$ and $\alpha_3 = 0.43222479$.

For Sector 4, on the other hand, parameters A_{14}, A_{24} turned out to have theoretically incorrect signs although the constraint of $\theta(\lambda)$ has converged to 1/3000 of the initial value.

Of the various possible reasons for this result, the following two problem deserve attention. The first problem relates to a technical aspect of the convergence computation. The Newton method adopted here has the deficiency that the convergence point depends largely on the initial value when the objective function has a complex form although the method is advantageous in that convergence may be quickly obtained. Therefore, if the arbitrarily chosen initial value happened to be in the neighborhood of the point of true minimum (or maximum) then the value converges quickly to this point. However, if the initial value happened to be far away from the point of true minimum (or maximum) then it may well happen that a local point of minimum (or maximum) is mistakenly chosen in place of the point of true minimum (or maximum).

The second point is a statistical question of choosing directions of errors. The Newton method was applied to the objective function (7.74). The random disturbance term u_i of the sum of squares of residuals of equation (7.74) is measured along the direction of the dependent variable y_j in equation (7.71). In this case, the direction of errors is chosen for convenience of estimation by formulating the dependent variable y_j as

$$y_j = p_j + \frac{\sum\limits_{(i \neq j)}^{i} p_i a_{ij}}{(a_{jj} + t_{Ij} - 1)}.$$

However, in the theoretically derived supply equation (7.53), the dependent variable is p_j. Therefore, if the estimation is to be made consistently with equation (7.53), it would be preferrable to choose the direction of errors in the direction of p_j.

For these two reasons, we attempted another convergence computation based on the values of convergence obtained by the Newton method now by altering the direction of errors to the direction of p_j using the Pattern method. The random disturbance term u_j in this case will be derived from equation (7.53) as

$$(7.77) \qquad u_j = p_j - \frac{(X_j - \gamma_{sj})}{\gamma_{sj}(a_{jj} + t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \frac{L_j h_j w_j}{X_j} + \sum_{i=1 (i \neq j)}^{4} p_i a_{ij} \right\}.$$

The objective function is the sum of squares of the disturbance term as

$$(7.78) \qquad \varphi = \Sigma u_i^2 = \Sigma \left[p_j - \frac{(X_j - \gamma_{sj})}{\gamma_{sj}(a_{jj} - t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \frac{L_j h_j w_j}{X_j} + \sum_{i=1 (i \neq j)}^{4} p_i a_{ij} \right\} \right]^2$$

As for the initial values, the convergence values obtained by the Newton method were used for Sectors 2 and 3. For Sector 4, the same convergence value did not satisfy the sign condition. Therefore we used the values, $\gamma_{s4} = -.22222 \times 10^5$ and $\alpha_4 = 0.61306199$, for the initial values which were obtained approximately from A_{24} and A_{34} by setting $\lambda = 0$.

Table 7.2 presents the result of computation by the Pattern method.

The Theil's U listed in Table 7.2 is an indicator of the goodness of fit which is expressed with respect to the observed values p_j and theoretically predicted values $\hat{p}_j$. The theoretical value (1) is the value of price p_j predicted using the values of parameters γ_{sj} and α_j at the initial stage of computation and by giving the actual amount of output X_j. The theoretical value (2) is the

Table 7.2 RESULTS OF NON-LINEAR ESTIMATION (THE PATTERN METHOD) OF PARAMETERS OF THE SECTORAL SUPPLY SCHEDULES

	Sector 2		Sector 3		Sector 4	
	Initial	Final	Initial	Final	Initial	Final
γ_{sj}	−116538. 55	−116428. 63	−6095807. 3	−6092035. 3	−22222. 808	−2305380. 0
α_j	0. 81735392	0. 8173843	0. 43222479	0. 43188414	0. 61306199	0. 6541250
SS	0. 006459	0. 006459	0. 007013	0. 007011	5. 1369	0. 003573
r	0. 9997	0. 9997	0. 9997	0. 9997	0. 9904	0. 9997
U	0. 01266	0. 01266	0. 01319	0. 01318	0. 3001	0. 01123

year	Price			Price			Price		
	OB	ES(1)	ES(2)	OB	ES(1)	ES(2)	OB	ES(1)	ES(2)
1955	0. 9453	0. 9508	0. 9508	0. 8619	0. 8917	0. 8921	0. 6395	0. 9588	0. 6533
1956	0. 9429	0. 9722	0. 9723	0. 9612	0. 9566	0. 9569	0. 6783	0. 9953	0. 6650
1957	0. 9590	0. 9650	0. 9650	1. 0180	0. 9969	0. 9973	0. 7210	1. 1073	0. 7230
1958	0. 9160	0. 9016	0. 9017	0. 9340	0. 9716	0. 9719	0. 7200	1. 1506	0. 7343
1959	0. 9300	0. 8944	0. 8945	0. 9410	0. 9196	0. 9200	0. 7580	1. 2212	0. 7610
1960	0. 9480	0. 9011	0. 9011	0. 9570	0. 9093	0. 9097	0. 7520	1. 2458	0. 7324
1961	0. 9700	0. 9966	0. 9968	0. 9740	0. 9625	0. 9629	0. 8010	1. 3774	0. 7697
1962	0. 9700	1. 0024	1. 0024	0. 9650	0. 9828	0. 9832	0. 8510	1. 5599	0. 8349
1963	0. 9940	0. 9884	0. 9885	0. 9720	0. 9788	0. 9793	0. 9070	1. 7693	0. 8984
1964	0. 9910	0. 9828	0. 9829	0. 9880	0. 9699	0. 9704	0. 9520	1. 9958	0. 9631
1965	1. 0000	1. 0011	1. 0012	1. 0000	1. 0263	1. 0267	1. 0000	1. 2137	1. 0341

Notes: (1) Columns named "Initial" and "Final" in the upper tier present, respectively, initial values and final converged values of the parameters.

(2) Notations are:

γ_{sj}, α_j : parameters of the supply schedule for the j-th sector,
SS : the sum of squared residuals,
r : correlation coefficient,
U : Theil's U,
OB : actually observed values,
ES(1): theoretical values obtained from the single supply equation for the j-th sector, and
ES(2): theoretical values obtained from the system of simultaneous equations.

theoretically predicted value computed using the convergence values of the parameters.

For Sectors 2 and 3, the obtained convergence values are not significantly different from the convergence values computed by the Newton method. This result, therefore, may be considered as the result of only partial revision in terms of direction of errors. For Sector 4, the errors involved in the initial value were quite large. The sum of squares of residuals was 5.1369 and Theil's U was 0.3001. By the convergence computation the fit has been improved considerably to make the sum of squares of residuals 0.003573 and Theil's U 0.01123. The improvement can be seen clearly by comparing the theoretical values (1) and (2).

Thus far, we have tried to estimate parameters γ_{sj} and α_j of the short-run supply function for each sector. We regard the convergence values listed in Table 7.2 as the parameters of the short-run supply function.

3. The Shifts of Measured Supply Curves

Using the results of estimation, we can derive the supply schedule for each sector. According to the formulation discussed earlier, the price flexibility will be

$$\frac{\partial p_j}{\partial X_j} \cdot \frac{X_j}{p_j} = -\frac{P(\alpha_{sj}Y + \beta_{sj}W + \eta_{sj})}{(X_j - \gamma_{sj})^2} \cdot \frac{X_j}{p_j} = -\frac{X_j}{X_j - \gamma_{sj}}. \tag{7.79}$$

The results of our estimation revealed that the absolute value of γ_{sj} is quite large for each sector, implying therefore that the price flexibility is small.

Table 7.3 presents the price flexibility for each sector computed at the actual level of output. The flexibility turned out to be very small for each sector, which implies that the price elasticity of demand for the anticipated demand function is very large.

The supply function for each sector has thus been obtained empirically as

(7.80)

$$p_2 = \frac{(X_2 + 116428.63)}{-116428.63\,(a_{22} + t_{I2} - 1)} \left\{ \frac{1}{.8173843} L_2 w_2 \left(\frac{X_2}{Q_2}\right)^{\frac{1}{.8173483}} / X_2 - \sum_{(i \neq j)}^{i} p_i a_{i2} \right\},$$

(7.81)

$$p_3 = \frac{(X_3 + 6092035.3)}{-6092035.3(a_{33} + t_{I3} - 1)} \left\{ \frac{1}{.43188414} L_3 w_3 \left(\frac{X_3}{Q_3}\right)^{\frac{1}{.43188414}} / X_3 - \sum_{(i \neq j)}^{i} p_i a_{i3} \right\},$$

(7.82)

$$p_4 = \frac{(X_4 + 2305380.0)}{-2305380.0\,(a_{44} + t_{I4} - 1)} \left\{ \frac{1}{.6541250} L_4 w_4 \left(\frac{X_4}{Q_4}\right)^{\frac{1}{.6541250}} / X_4 - \sum_{(i \neq j)}^{i} p_i a_{i4} \right\}.$$

Table 7.3 ESTIMATES OF PRICE FLEXIBILITIES OBTAINED FROM THE ANTICIPATED DEMAND FUNCTIONS

year	Sector 2 Output (billion yen)	Sector 2 Price Flexibility	Sector 3 Output (billion yen)	Sector 3 Price Flexibility	Sector 4 Output (billion yen)	Sector 4 Price Flexibility
1955	5523. 3	−0. 04529	5207. 8	−0. 000854	8659. 3	−0. 003742
1956	6524. 4	−0. 05306	6674. 6	−0. 001094	9265. 8	−0. 004002
1957	7409. 1	−0. 05983	7849. 2	−0. 001287	10048. 1	−0. 004340
1958	7609. 1	−0. 06135	7849. 9	−0. 001287	10812. 7	−0. 004668
1959	8468. 8	−0. 06781	9994. 8	−0. 001638	11656. 4	−0. 005031
1960	10006. 7	−0. 07914	13249. 9	−0. 002170	13725. 4	−0. 005918
1961	11513. 5	−0. 08998	16698. 6	−0. 002733	15652. 9	−0. 006744
1962	12924. 2	−0. 09991	17937. 3	−0. 002935	17369. 8	−0. 007478
1963	14879. 7	−0. 11332	19657. 3	−0. 003216	19548. 1	−0. 008408
1964	16577. 2	−0. 12464	23256. 0	−0. 003803	21810. 9	−0. 009372
1965	17769. 4	−0. 13241	24274. 6	−0. 003969	23241. 0	−0. 009981

The supply schedule for each sector may be plotted against p_j and X_j for a given level of Q_j in each of the above equations. Figures 7.11, 7.12 and 7.13 show the plotted supply schedules for given levels of output capacity corresponding to different years during the period from 1955 to 1965.

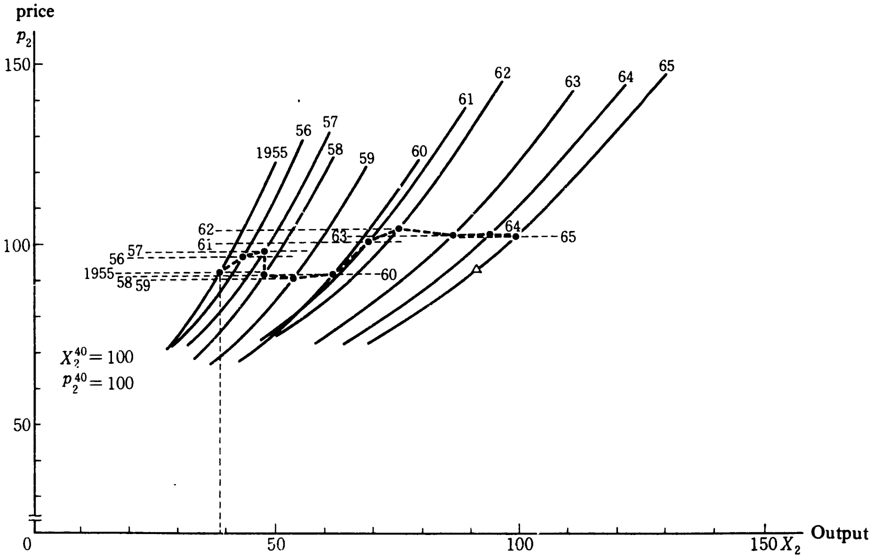
The fact that the estimate of parameter α_j is relatively small implies that the elasticity of production with respect to hours of operation for each sector is small. In other words, it indicates that output X does not vary proportionately with changes in hours of operation when the actual hours of operation h deviate considerably from the normal hours of operation h^*. In particular, the fact that α_3 of Sector 3 is small suggests that Sector 3 consists largely of heavy manufacturing industries which are highly capital intensive and thus that there is little scope for adjusting the amount of output by changing actual hours of operation from the normal hours of operation.

On the other hand, the fact that the estimate of parameter γ_{sj} is large suggests that each firm acts, in the short-run, in the anticipation of a fairly competitive markets.

We attempted to re-estimate parameters a_j and b_j by inserting into equation (7.43) the data of Q_j which were imputed from the estimate of α_j using a modified form of (7.45) as

$$Q_j = \left(\frac{X_j}{h_j}\right)^{\frac{1}{\alpha_j}}$$

Figure 7.11 ACTUALLY MEASURED SUPPLY SCHEDULES: LIGHT MANUFACTURING SECTOR



Notes: (1) The vertical axis measures the price index of the output of the light manufacturing sector standardized (1965 price=100), and the horizontal axis measures the level of output in billions of yen at 1965 constant prices.

(2) The estimates of parameters of the SFS production function used in deriving the schedules are as follows: $\hat{a}=0.00826$, $\hat{b}=1.1555011$, $\hat{c}=1275.96$, $\hat{d}=0.19265695$, $\hat{\alpha}=0.8173843$, $\hat{\gamma}_s=-116428.63$.

(3) The notation •-•-• represents changes in price corresponding to changes in the observed level of output, and ---- represents the schedule of the anticipated demand function for each year.

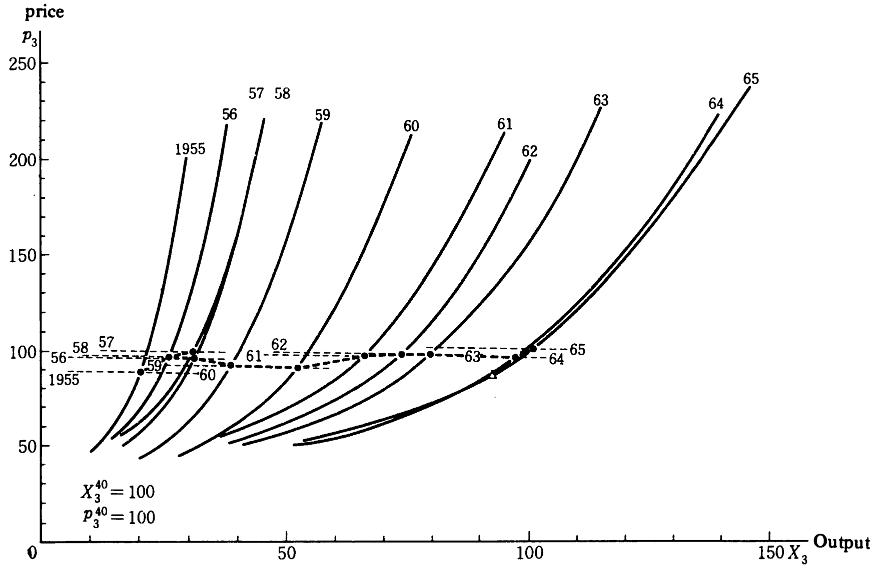
(4) The mark Δ represents the level of output in 1965 assuming that the prices are held unchanged at the 1955 price level.

The results are:

(7.83) $$\log Q_2 = -4.7953674 + 1.1555011 \log K_2.$$
$$(0.2939) \quad (0.03501)$$
$$\bar{r} = 0.9954 \quad d.w. = 1.63, \quad d.f. = 9$$

(7.84) $$\log Q_3 = -1.0820038 + 0.99563189 \log K_3.$$
$$(0.5908) \quad (0.7141)$$
$$\bar{r} = 0.9751 \quad d.w. = 1.21, \quad d.f. = 9$$

Figure 7.12 ACTUALLY MEASURED SUPPLY SCHEDULES: HEAVY MANUFACTURING SECTOR



Notes: (1) The vertical axis measures the price index of the output of the heavy manufacturing sector standardized (1965 price = 100), and holizontal axis measures the level of output in billions of yen at 1965 constant prices.

(2) The estimates of parameters of the SFS production function used in deriving the schedules are as follows:

$\hat{a}=0.33891$ $\hat{b}=0.99563189$ $\hat{c}=190.221$
$\hat{d}=0.41891948$ $\hat{\alpha}=0.43188414$ $\hat{\gamma}_t=-6092035.3$

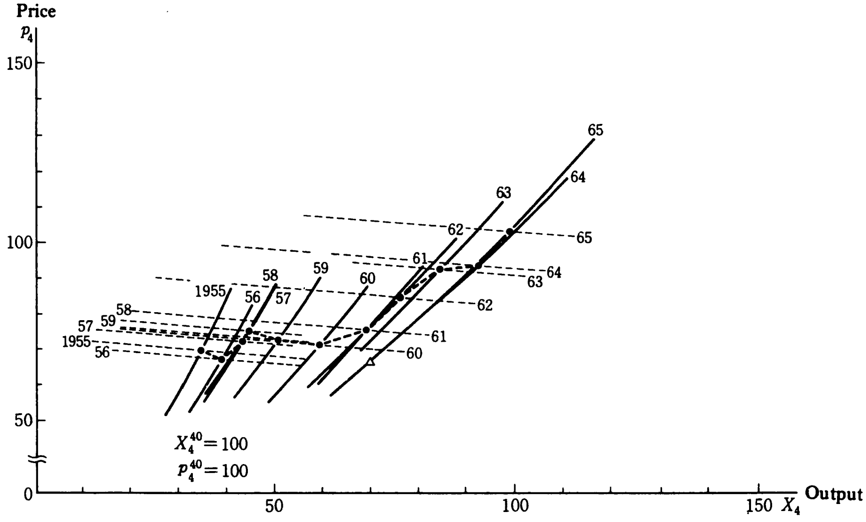
(3) The notation •‥•‥• represents changes in the price corresponding to changes in the observed level of output, and ---- represents the schedule of the anticipated demand function for each year.

(4) The mark △ represents the level of output in 1965 assuming that pries are held unchanged at the 1955 price level.

$$\text{(7.85)} \quad \log Q_4 = -4.929043 + 1.19058 \log K_4 .$$
$$(0.3436) \quad (0.03702)$$
$$\bar{r} = 0.9952 \quad d.w. = 1.21, \quad d.f. = 9$$

In Figures 7.11, 7.12 and 7.13, the shifts of the supply curves year after year indicate expansion of output capacities. By tracing time-series movements of demand-supply equilibrium points we can learn the relationship between the shifts of output capacity and increases in prices.

Figure 7.13 ACTUALLY MEASURED SUPPLY SCHEDULES: COMMERCIAL AND SERVICE SECTOR



Notes: (1) The vertical axis measures the price index of the output of the commercial and services sector standardized (1965 price=100), and the horizontal axis measures the level of output in billions of yen at 1965 constant prices.

(2) The estimates of parameters of the SFS Production function used in deriving the schedules are as follows:

$\hat{a}$ =0.000723, $\hat{b}$=1.190580, $\hat{c}$=1012.06, $\hat{d}$=0.31168829, $\hat{\alpha}$=0.6541250, γ_s=−2305380.0.

(3) The notation •--•--• represents changes in the observed level of output, and ---- represents the schedule of the anticipated demand function for each year.

(4) The mark △ represents the level of output is 1965 assuming that prices are held unchanged at the 1955 price level.

Although there still remain some estimation errors, as suggested from the disparities between theoretical values and actually observed values in Table 7.2, the actual time-series changes in prices are fairly closely approximated in these Figures. The time-series movement of • mark linked by dotted line on the supply curve may be taken to represent fairly closely the actual changes in prices in response to actual amounts of supply. To visualize the relationship between quantities and prices clearly, both of them are measured on the index scale at 1965 (1965 = 100).

The actual demand function has not yet been estimated so far. The actually observable demand function would be a composite function consisting of functions which determine such final demand categories as consumers' demand, demand for investment goods and exports, and demand for intermediate goods. This actual demand function should be a downward sloping curve with a smaller price elasticity than that of the above mentioned anticipated demand function. If it is drawn diagramatically, it would look like the ones in Figure 7.10. The equilibrium point for each year in Figures 7.11 to 7.13 should be interpreted as the point of intersection of the supply curve with the actual demand curve. In other words, while the supply curve for each sector shifted right-ward from 1955 to 1965 the demand curve also shifted to the right. The intersection of the two curves for each year, therefore, is interpreted as determining the combination of price and quantity for the year in question. Since we can not plot the actual demand curves as yet, we have drawn the schedules of the anticipated demand functions as dotted lines which pass through the intersections of actual price and output.

The fact that the demand-supply equilibrium point shifted to the right from 1955 to 1965 was due to the right-ward shift of the demand curve caused by increases in income during the period of rapid economic growth and to the right-ward shift of the supply curve generated by investments in productive equipment. If the shift of the demand function was relatively larger then the equilibrium price would increase, and if the shift of the supply function was greater then the equilibrium price would decline.

During this period, prices of products of light manufacturing industries increased moderately, prices of products of heavy manufacturiang industries stayed more or less intact, and prices of services began to increase rapidly since 1960. Let us consider these actual changes in terms of our model.

When we measure the extent of shift of the supply curve by examining the amount of supply in 1965 at a price level equal to the equilibrium price of 1955, we will find that the amount for the light industry sector (Sector 2) increased from 37 to 90 or approximately 2.4 times, for the heavy industry sector (Sector 3) from 20 to 92 or approximately 4.6 times, and for the service sector (Sector 4) from 34 to 70 or approximately 2.0 times. This finding suggests that there exists an inverse relationship between the extent of shifts of the supply curve and the rate of increases in prices. The actual amount of quantity supplied during this period increased from 37 to 100 or 2.7 times for the light industry sector, from 20 to 100 or 5.0 times for the heavy industry sector, and from 34 to 100 or 3.0 times for the service sector. Since the rate of increase of the actual quantities supplied reflects increases in demand in the process of economic growth, the case in which this rate differs little from the shift in the supply schedule would suggest that both the demand and supply move in parallel, while the case in which these differ greatly would imply that changes in the demand and supply are uneven.

This fact suggests that prices increased only moderately in the light manufacturing sector because increases in demand were modest even though increases in supply capacity were much smaller than those seen in the heavy industry sector. In the sector of heavy industries, on the other hand, although demand increased greatly prices did not rise because of large increases in supply capacity. In contrast, the service sector suffered from a sharp rise in prices because demand increased more than it did in the light industry sector, while supply capacity increased less than it did in the light industry. Increases in supply capacity were much less than even the relatively mild increases in demand.

Increases in supply capacity generated by the investment resulted not only in shifts of the supply curves but also in changes in the slope of the curves. The slope of the supply curve tended to be flatter from 1955 to 1965 for each sector. This implies that changes in supply prices in response to changes in the quantities supplied at each point in time tend to be smaller with increases in capital equipment. In other words, even a slight increase in demand would have caused a rapid increase in prices around 1955, while the similar change in demand would not induce the similar increases in prices around 1965.

The supply function is formulated as

$$p_j = -\frac{X_j - \gamma_{sj}}{\gamma_{sj}(a_{jj} + t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \frac{c_j K_j^{d_j} h_j w_j}{X_j} + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i a_{ij} \right\} . \tag{7.86}$$

This may be rewritten by using equation (7.45) of the SFS production function as

$$p_j = \frac{X_j - \gamma_{sj}}{\gamma_{sj}(a_{jj} + t_{Ij} - 1)} \left\{ \left(\frac{1}{\alpha_j}\right) \left(\frac{c_j}{a_j}\right) K_j^{d_j - b_j} h_j^{1-\alpha_j} w_j + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i a_{ij} \right\} . \tag{7.87}$$

Suppose that hours of operation h_j are fixed at the level of normal hours of operation h_j^* which was planned at the stage of designing the productive equipment (for example, 200 hours a month or 8 hours × 25 working days). If in this case the relationship $d_j - b_j < 0$ holds in equation (7.87) then the supply curve would shift down (and right ward) further the greater the amount of capital equipment K_j. Consequently, the price at which a certain amount of output X_j can be supplied would decrease.

Rewriting equation (7.45) of the SFS production function, it is also possible to obtain the equation of labor productivity

$$\frac{X_j}{L_j h_j} = \frac{a_j K_j^{b_j} h_j^{\alpha_j}}{L_j h_j} = \left(\frac{a_j}{c_j}\right) K_j^{b_j - d_j} h_j^{\alpha_j - 1} . \tag{7.88}$$

Here again we can see at the point where actual hours of operation are equal to normal hours of operation that the labor productivity increases as the

Table 7.4 ESTIMATES OF STRUCTURAL PARAMETERS IN THE SHORT-RUN SUPPLY FUNCTIONS

	Sector 2	Sector 3	Sector 4
$\log a_j$	−4.7953674	−1.0820038	−4.929043
b_j	1.1555011	0.99563189	1.190580
$\log c_j$	7.1514541	5.2481878	6.9197492
d_j	0.19265695	0.41891948	0.31168829
α_j	0.8173843	0.43188414	0.6541250
γ_{sj}	−116428.63	−6092035.3	−2305380.0

Note: Parameters a_j and b_j are the parameters of equation (7.6), C_j and d_j are of equation (7.7), α_j is of equation (7.8) and γ_{sj} is of equation (7.38). Subscript j (j = 2, 3 and 4) denotes the sector number.

amount of capital equipment increases, if $b_j-d_j>0$. The equations (7.87) and (7.88) jointly seem to imply that an increase in the volume of capital equipment will have the effect of reducing the supply price through an accompanying increase in labor productivity.

Table 7.4 summarizes the estimates of the parameters we have been discussing so far. The table shows that the relationship $b_j>d_j$ hold in all the sectors. This result endorses our assessment above. It must be added quickly that this effect of reducing supply prices would be offset, at least partially, if wages w_j or prices of raw materials p_j were to increase, as can clearly be seen from equation (7.87).

7.4 Agricultural Production and Wages

We have emphasized so far that sectoral supplies are interdependent through the channels of raw material and labor markets. Similarly, sectoral demands are also mutually dependent. In view of these general interdependent relationships of the economy, we need to incorporate here the sector of agriculture, forestry and fisheries which has been set aside until now.

The structure of agricultural production has long been an important area of economic analysis since the days of Classical economics. The major propositions for empirical analysis of agricultural production may be summarized in the following three points: (1) the law of diminishing returns of land, (2) the homogeneity of first degree with respect to returns to labor and capital inputs into land, and (3) shifts in the level of labor productivity due to capital input.

Based on these basic and conventional propositions, we have formulated the production function for the sector of agriculture, forestry and fisheries (Sector 1) which is a variation of the Cobb-Douglas type function,

$$X_1 = a_1 A_1{}^{b_1} L_1{}^{1-b_1} (K_1 + K_{g1})^{c_1}, \tag{7.89}$$

where X_1 is the annual output of Sector 1, A_1 is the area of cultivated land, L_1 is the number of persons employed, K_1 is private capital stock, and K_{g1} is public capital stock invested in Sector 1.[5]

Equation (7.89) indicates that the output is of homogeneous to the first degree with respect to input of land and labor. The term K_1+K_{g1} is the sum of private and public capital stocks and is meant to express the fact that labor productivity increases with increases in capital equipment. We may rewrite equation (7.89) in the form of labor productivity as:

$$\frac{X_1}{L_1} = a_1 \left(\frac{A_1}{L_1}\right)^{b_1} (K_1 + K_{g1})^{c_1}. \tag{7.90}$$

Since all the parameters are of a positive value, it is implied from this equation that labor productivity will increase when capital stock increases even though the degree of land intensity, A_1/L_1 remains unchanged.

The public capital stock K_{g1} and land A_1 are treated as exogenous variables. The parameters of equation (7.89) are estimated by the least squares method using the time-series data from 1955 to 1965. The result of estimation is:

$$\log\left(\frac{X_1}{L_1}\right) = \underset{(0.5011)}{-8.3004598} + \underset{(0.00057)}{0.3036} \log\left(\frac{A_1}{L_1}\right) + \underset{(0.05602)}{0.83086476} \log (K_1 + K_{g1}). \tag{7.91}$$

$$\bar{R} = 0.9779, \quad d.w. = 1.56, \quad d.f. = 9$$

All the parameters are statistically significant.

Let us review briefly the reallocation of the labor force from the agricultural sector to other sectors which took place during a decade around 1960. The number of employed persons in manufacturing and service industries in the urban sector increased from 26.37 millions in 1955 to 35.90 millions in 1965, or an increase of 9.5 million persons. In contrast, the employment in agriculture, forestry and fisheries declined during the same period from 16.04 millions to 11.54 millions, or a decrease of 4.5 millions. In other words, nearly a half of the increase in non-agricultural employment was supplied from the outflow of labor from the agricultural sector.

The inter-sectoral labor mobility from agricultural to non-agricultural sectors is itself an important phenomenon accompanying the process of economic development, as aptly theorized in studies of economic development since the provocative work of W. Arthur Lewis.[6] If we can replace the concept of indigenous and modern sectors commonly used in the conventional literature of the economics of development by our classification of the sector of agriculture, forestry and fisheries (Sector 1) and the non-agricultural sector (Sectors 2, 3 and 4), then we could analyze the observed inter-sectoral movement of the labor force in terms of our model.

The gross value added or income V_1 of Sector 1 is given by the product of output X_1 and the value added per unit of output, $(p_1 - \Sigma p_i a_{i1})$. This value V_1 may be interpreted as the income that self-employed households in Sector 1 obtain by inputing labor force L_1, including non-paid family workers, into their productive activities. The loss of income due to the outflow of one worker from the labor force L_1 may be regarded as being equal to the marginal value added productivity of this marginal worker. From equation (7.89), the marginal value added productivity is given as

$$\frac{\partial V_1}{\partial L_1} = \frac{(1-b_1)(p_1 - \Sigma p_i a_{i1})}{L_1} X_1 \tag{7.92}$$

$$= (1-b_1)(p_1 - \Sigma p_i a_{i1}) \cdot a_1 A_1{}^{b_1} L_1^{-b_1} (K_1 + K_{g1})^{c_1} \ .$$

Since the worker leaves Sector 1 seeking employment opportunities in non-agricultural sectors, he would not leave unless his expected wages in non-agricultural sectors are greater than the loss of income due to his migration out of the agricultural sector. For this reason, wages in non-agricultural sectors are compared with the marginal value added in Sector 1, as expressed by equation (7.92). This may be called "marginal supply wage" at which the labor force is supplied from Sector 1 to other sectors. Substituting the estimates of parameters of equation (7.91) into equation (7.92), we get

$$\frac{\partial V_1}{\partial L_1} = 0.6964 \cdot (p_1 - \Sigma p_i a_{i1}) \cdot \frac{X_1}{L_1} \ . \tag{7.93}$$

This suggests that the marginal value added productivity is approximately 70 percent of the average value added productivity.

Columns (1), (2) and (3) of Table 7.5 respectively present the marginal value added productivity computed from equation (7.93), physical productivity and value added per unit of output for the period from 1955 to 1965. During this period the marginal value added productivity increased by 2.5 times. This increase was attained in part by the increase in physical labor productivity of 1.6 times due to increased capital equipment, and in part by the increase in value added per unit of output of 1.6 times which was caused largely by the significant increase in the level of prices of Sector 1.

If annual working hours per worker of Sector 1, as listed in column (5) of

Table 7.5 MARGINAL VALUE-ADDED PRODUCTIVITY IN AGRICULTURE AND NON-AGRICULTURAL WAGES

	Marginal Value-added Productivity in Sector 1					Hourly Rate of Starting Wages in Non-Agricultural Sector	
	(1) *M.V.A.*	(2) *X/L*	(3) *V.A./X*	(4) *MV.A./LH*	(5) *H*	W_M	W_F
1955	0.076477851	0.26637	0.41227	19.6	3899.1	19.75	17.96
1956	0.075713240	0.26976	0.40373	19.2	3926.4	21.30	19.81
1957	0.085502617	0.28372	0.43374	21.9	3903.0	23.00	21.37
1958	0.087685589	0.30163	0.41745	26.7	3926.0	25.19	23.44
1959	0.094856016	0.32705	0.41647	29.9	3926.0	29.30	23.24
1960	0.104709510	0.33343	0.45075	26.7	3921.0	28.61	26.74
1961	0.117689140	0.34304	0.49265	31.6	3724.0	35.77	32.89
1962	0.132000480	0.35946	0.52732	36.8	3582.0	45.95	43.83
1963	0.152661080	0.37034	0.59192	46.6	3274.0	50.36	47.97
1964	0.168145550	0.41145	0.58592	53.0	3171.0	59.73	55.24
1965	0.193037320	0.42645	0.65000	62.0	3109.0	68.74	66.61

Notes: Notations are:

M.V.A.: marginal value-added productivity (yen) per worker,
X/L: physical labor productivity,
V.A./X: value-added per unit of output,
M.V.A./LH: hourly marginal value-added productivity (yen) per worker,
H: hours worked per year, and
W_m, W_F: hourly rates of starting wages (in yen) in small firms (with 30 to 99 employees) in non-agricultural industries for male and female middle school leavers, respectively. These wage rates are listed here to represent wage rates for typical unskilled laborers in the Japanese labor market.

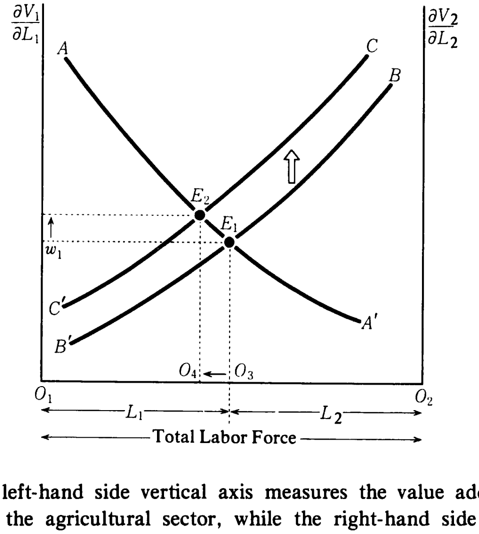
Table 7.5, are given exogenously, we can compute hourly marginal value added productivity, as shown in column (4). The hourly marginal value added productivity is found to have increased from 19.6 yen in 1955 to 62.0 yen in 1965.[7]

Now let us compare the marginal supply price (or wage) of labor force in Sector 1 with wages in the non-agricultural sectors. Since there exists a broad range of wage differentials in the non-agricultural sectors by age, occupation, size of firm etc., it would be more reasonable to compare the marginal supply price of agricultural labor with the wage rate in the lowest part of the differential structure rather than the average non-agricultural wage rate. To satisfy this criterion we chose the series of hourly rates of starting wages of small firms for male and female middle school leavers. The wage rates are listed in columns (6) and (7) for males and females respectively. We can see that these wage rates compare quite well with the hourly marginal value added productivity of Sector 1 of column (4).

Figure 7.14 illustrates diagramatically the fact that the marginal value added productivity in Sector 1 determines the marginal supply price of labor in that sector and that this roughly corresponds to the lowest wage rate of the non-agricultural sector.

Suppose that the total size of the labor force is fixed and is represented by the horizontal distance between O_1 and O_2. We may depict the down (and right) ward sloping marginal value added productivity curve AA' of Sector 1 from the left-hand end, and similarly the downward (and left ward) sloping marginal value productivity curve BB' of workers of the non-agricultural sector as a whole (the sum of Sectors 2, 3 and 4) from the right-hand end. The inter-sectoral allocation of labor will be determined at the equilibrium point E_1 or the intersection of AA' and BB', with the share of the agricultural labor force being O_1O_3 and of the non-agricultural labor force

Figure 7.14 AN ILLUSTRATION OF THE ALLOCATION OF LABOR FORCE BETWEEN AGRICULTURAL AND MANUFACTURING SECTORS

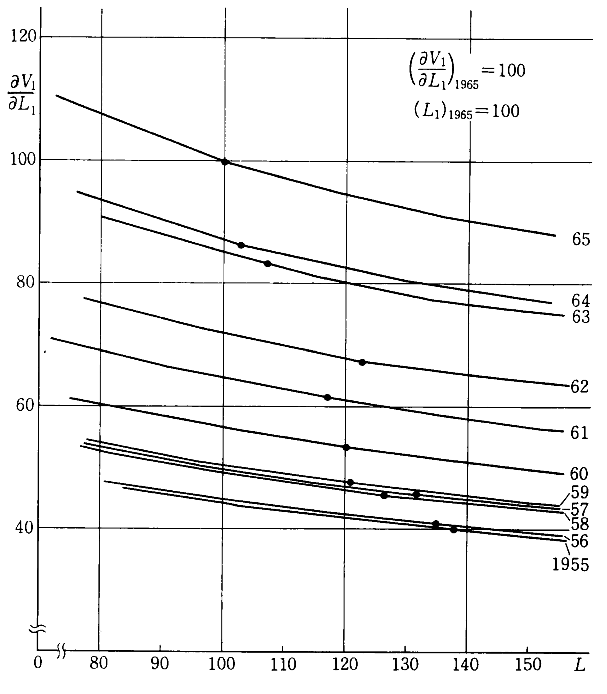


Notes: (1) The left-hand side vertical axis measures the value added marginal productivity in the agricultural sector, while the right-hand side vertical axis measures the value added marginal productivity in the manufacturing sector.
(2) The notation L_1 represents the number of workers in the agricultural sector, and L_2 the number of workers in the manufacturing sector. Curves AA', BB' and CC' represent the value added marginal productivity curves of the agricultural and manufacturing sectors, respectively.

being O_3O_2. The marginal supply wage in this situation will be at the level of w_1. If the marginal productivity curve of the non-agricultural sector shifts from BB' to CC' while AA' of Sector 1 remains unchanged, the equilibrium point would shift from E_1 to E_2 and the labor force as many as O_3O_4 would move from Sector 1 to the non-agricultural sector. In reality, however, the marginal value added productivity curve of Sector 1 itself shifted during 1955 to 1965 due to increases in the level of prices of Sector 1 and expansions of capital equipment, and accordingly gave rise to increases in the marginal supply price of labor force.

Figure 7.15 describe the marginal value added productivity curve of Sector 1 for different years during the period 1955 to 1965, which are plotted by using the estimated parameters of th production function. Both the marginal value added productivity and employment are measured against the scale standardized as 1965 = 100. The • mark on the diagram indicates

Figure 7.15 THE ACTUALLY MEASURED VALUE ADDED MARGINAL PRODUCTIVITY CURVES OF SECTOR 1



Note: The vertical axis represents the index of value added marginal productivity (the value of 1965 = 100), and the horizontal axis measures the index of the number of workers in Sector 1 (the level of 1965 = 100).

the level of marginal value added productivity for each year at a given level of employment of labor in Sector 1. Although the curve shifts upward with the passage of time, the equilibrium point (•) shifts left and upward. This shift of the equilibrium point in this diagram reflects the fact that the shift of the marginal productivity of the non-agricultural sector was much greater than the corresponding shift in the agricultural sector. The mechanism behind the massive reallocation of labor force from the agricultural to non-agricultural sectors during the period of rapid industrialization from 1955 to 1965 is clearly illustrated by this exposition.

The wage rate for unskilled workers such as new recruits from middle schools tends to be considerably lower than the average wage rate in each sector. However, changes of these two wage series over time are not unrelated. Figure 7.16 exhibits the relationship between earned income ($w_j h_j$) per employee and the marginal value added productivity of Sector 1 both computed using the data from 1955 to 1965.

The figure indicates that the movement of marginal value added productivity of Sector 1 is correlated closely with the movements of wages in the non-agricultural sectors. If we approximate these relationships by fitting linear equations, we get

$$(7.94) \quad w_{23} = \underset{(0.000057)}{0.0002327764} + \underset{(1.5938)}{34.558113} w_1 .$$

$\bar{r} = 0.9895 \quad d.w. = 0.9245 \quad d.f. = 9$

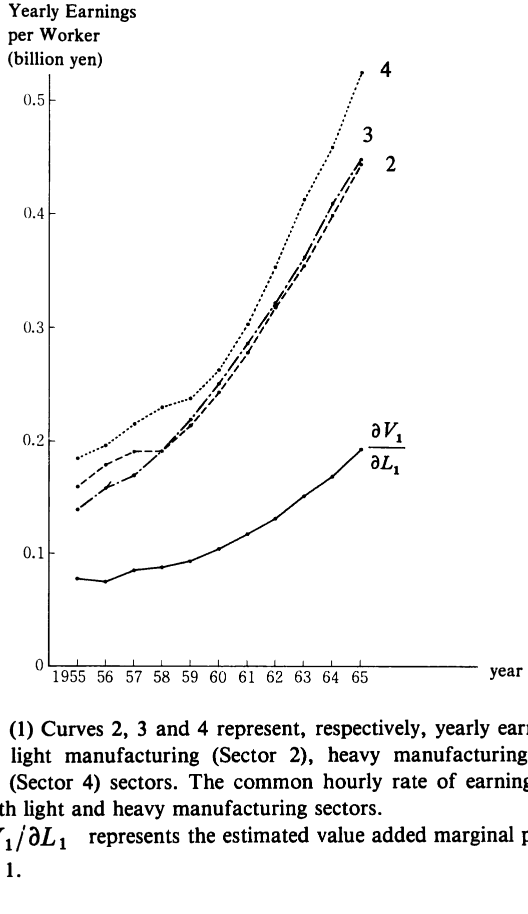
$$(7.95) \quad w_4 = \underset{(0.000036)}{0.0002537835} + \underset{(1.0053)}{41.936762} w_1 .$$

$\bar{r} = 0.9971 \quad d.w. = 0.9341 \quad d.f. = 9$

If the marginal value added productivity of Sector 1 is equal to the lowest rate of wage distribution in the non-agricultural sectors, then we may regard equations (7.94) and (7.95) as empirical equations representing the inter-sectoral wage differentials.

In both of these equations the dependent variable is the hourly wage rate w_j (j = 23,4) which was computed by dividing the earned income per employee $w_j h_j$ by the monthly hours worked h_j obtained from the *Monthly Labour Survey*. The corresponding independent variable is the marginal supply price of Sector 1 $\partial V_1 / \partial L_1$ divided by the annual hours worked in agriculture h_1 obtained from the *Agricultural Households Survey*. Since the wage data for Sectors 2 and 3 are indistinguishable, wages in these sectors are dealt with as a single data series w_{23}. Although further improvements in terms of precision are desired, equations (7.94) and (7.95) are taken at this

Figure 7.16 THE TIME-SERIES MOVEMENTS OF SECTORAL WAGE RATES YEARLY EARNINGS PER WORKER BY SECTORS



Notes: (1) Curves 2, 3 and 4 represent, respectively, yearly earnings per worker in the light manufacturing (Sector 2), heavy manufacturing (Sector 3) and service (Sector 4) sectors. The common hourly rate of earnings has been used for both light and heavy manufacturing sectors.

(2) $\partial V_1/\partial L_1$ represents the estimated value added marginal productivity in Sector 1.

stage as being satisfactorily reliable empirical equations describing changes in inter-sectoral wage differentials.

As we have discussed earlier, the equations which we call "supply equations of non-agricultural sectors" are simply the equalities of marginal productivities with respect to man-hour labor inputs $L_j h_j$ for respective sectors. The diagramatical analysis of Figure 7.14 of an equilibrium between the marginal supply wage of Sector 1 and the unskilled wage rate in the non-agricultural sector, therefore, may be interpreted in terms of our model as the simultaneous system of supply equations of non-agricultural sectors and the agricultural marginal productivity equation in which wage rates and supply prices of all sectors are detemined simultaneously. However, we also need to take the aforementioned inter-sectoral wage differentials into account in analyzing the simultaneous determination of wages and prices. We will explain our analytical system of simultaneous determination of these variables in the next section.

7.5 The Simultaneous Determination of Supply Prices and Wages

We have explained in the previous section the theoretical notion in which wages are determined and labor is allocated at a point of equilibrium between wages in the modern sectors and the marginal supply price of the indigeneous sector.

The equilibrium conditions between the modern and indigenous sectors are expressed, in this case, in terms of the marginal value added productivity on the basis of the wage differential equations. The marginal value added productivities of various sectors are not only related mutually through wages but also related to prices of products of other sectors through the interrelated network of intermediate inputs.

Let us digress briefly to emphasize two of the basic features of our model before explaining the system of simultaneous equations. Clarification of those two points will facilitate to have a letter understanding of our formulations.

The first point is the fact that, in our model, once the amount of capital stock is given in the short-run, the number of workers employed L_j is determined accordingly through the SFS production function. Therefore, the equilibrium model between indigeneous and modern sectors as mentioned above is interpreted to mean, in terms of our model, the mechanism by which wages and prices are adjusted so as to make themselves compatible with the inter-sectoral allocation of the labor force which is determined mechanically by the allocation of capital stock K_j at the beginning of each period. However, it should be added that the wages, prices and outputs determined within the t-th period will influence the productive capacity of the $(t+1)$th period by influencing the determination of investment demand during the t-th period. In this sense, the allocation of labor force which depends primarily on the allocation of capital stock is not really unrelated to wages and prices, but is subject to the feedback effect from wages and prices.

The second point is that the price of Sector 1 (agriculture, forestry and fisheries) is given as an exogenous variable. When the capital stock of Sector 1 is fixed in the short-run at the level of the beginning of the period, then the amount of output will be determined automatically once the size of labor force left for Sector 1, L_1, is determined, since the land A_1 and public capital stock K_{g1} are both given exogenously as noted earlier. Therefore, the output X_1 of Sector 1 is independently determined from the price p_1 within the period. This formulation was made by taking into account institutional elements associated with the determination of prices of agricultural produce, notably rice. The price p_1 of Sector 1 is thus treated as an exogenous variable.

Now, having these two points in mind, let us now explain the mechanism of simultaneous determination of supply prices p_j (j=2,3,4) and wages w_j (j=2,3,4) at certain levels of production X_j (j=2,3,4). The system of equations developed so far may be summarized as follows:

Sector 1 (Agriculture, Forestry and Fisheries)

a. The Size of Labor Force

$$L_1 = \bar{L} - \sum_{j=2}^{4} L_j . \tag{7.96}$$

b. The Production Function

(7.97)

$$\log\left(\frac{X_1}{L_1}\right) = -8.3004598 + 0.3036\log\left(\frac{A_1}{L_1}\right) + 0.83086476\log(K_1 + K_{g1}).$$

c. The Value Added Marginal Productivity

$$\frac{\partial V_1}{\partial L_1} = w_1 h_1 = \frac{0.6964\times(p_1 - \Sigma p_i a_{i1})X_1}{L_1} \tag{7.98}$$

Sector 2 (Light Manufacturing Industries)

a. The Production Function

$$\log L_2 = 7.1514541 + 0.19265695\log K_2 . \tag{7.99}$$

$$h_2 = \left(\frac{X_2}{Q_2}\right)^{0.8173843} \tag{7.100}$$

$$\log Q_2 = -4.7953674 + 1.15550111\log K_2 . \tag{7.101}$$

b. The Supply Function

(7.102)

$$p_2 = \frac{(X_2 + 116428.63)}{-116428.63(a_{22} + t_{I2} - 1)} \times \left\{ \left(\frac{1}{.8173843}\right)\cdot\frac{L_2 h_2 w_2}{X_2} - \sum_{(i\neq 2)}^{i} p_i a_{i2} \right\} .$$

Sector 3 (Heavy Manufacturing Industries)

a. The Production Function

$$\log L_3 = 5.2481878 + 0.41891948\log K_3 . \tag{7.103}$$

(7.104) $$h_3 = \left(\frac{X_3}{Q_3}\right)^{0.43188414}.$$

(7.105) $$\log Q_3 = -1.0820038 + 0.99563189 \log K_3 .$$

b. The Supply Function

(7.106)

$$p_3 = \frac{(X_3 + 6092035.3)}{-6092035.3(a_{33} + t_{I3} - 1)} \times \left\{ \left(\frac{1}{.43188414}\right) \cdot \frac{L_3 h_3 w_3}{X_3} - \sum_{(i \neq 3)}^{i} p_i a_{i3} \right\} .$$

c. The Wage Determination Equation for Sectors 2 and 3

(7.107) $$w_{23} = 0.0002327764 + 34.558113 w_1 .$$

Sector 4 (Service Industries)

a. The Production Function

(7.108) $$\log L_4 = 6.9197492 + 0.31168829 \log K_4 .$$

(7.109) $$h_4 = \left(\frac{X_4}{Q_4}\right)^{0.654125}.$$

(7.110) $$\log Q_4 = -4.9290430 + 1.1905800 \log K_4 .$$

b. The Supply Function

(7.111)

$$p_4 = \frac{(X_4 + 2305380.0)}{-2305380.0(a_{44} + t_{I4} - 1)} \times \left\{ \left(\frac{1}{.654125}\right) \cdot \frac{L_4 h_4 w_4}{X_4} - \sum_{(i \neq 4)}^{i} p_i a_{i4} \right\} .$$

c. The Wage Determination Equation

(7.112) $$w_4 = 0.0002537835 + 41.936762 w_1 .$$

The system consists of 17 equations. The variable K_j is a predetermined endogenous variable which is determined by the beginning of the current period as a result of investment during the preceding period. The price p_1 of Sector 1 is given as an exogenous variable as mentioned earlier. The variables of land A_1, public capital stock K_{g1}, working hours h_1 of Sector 1, and the total labor force $\bar{L}$ are all exogenous variables. Therefore, the above equations contain altogether 20 endogenous variables: quantity supplied X_j ($j = 1, \ldots, 4$), supply prices p_j ($j = 2, \ldots, 4$), wages w_j ($j = 1, 23, 4$), hours of operation h_j ($j = 2, \ldots, 4$), the number of persons employed L_j ($j = 1, \ldots, 4$) and output capacity Q_j ($j = 2, 3, 4$).

When K_j and the exogenous variables are given, L_j ($j = 2, \ldots, 4$) will be determined from equations (7.99), (7.103) and (7.108), and hence L_1 will be determined from equation (7.96). When L_1 is determined, then X_1 will be determined from equation (7.97) since K_1, K_{g1}, and A_1 are already given. Q_j

($j = 2, \ldots, 4$) will also be determinned from equations (7.101), (7.105) and (7.110).

Suppose now that firms in Sectors 2, 3 and 4 decided to supply certain amounts of outputs X_j ($j = 2, \ldots, 4$). The hours of operation h_j ($j = 2, \ldots, 4$) will be determined from already determined Q_j and X_j using equations (7.100), (7.104) and (7.109). The 6 endogenous variables p_j ($j = 2, \ldots, 4$) and w_j ($j = 1, 23, 4$) will be determined, therefore, simultaneously, using the remaining 6 equations (7.98), (7.102), (7.106), (7.107), (7.111) and (7.112). The p_j ($j = 2, \ldots, 4$), thus obtained imply supply prices corresponding to the amounts of arbitrarily decided supply X_j ($j = 2, \ldots, 4$) and w_j($j = 1, 23, 4$) on the other hand imply equilibrium wage rates corresponding to supply prices. It should be born in mind, however, that the arbitrarily decided amounts of supply X_j and the corresponding supply prices p_j are not necessarily the actual sectoral quantities supplied and the corresponding prices at the equilibrium points. Whether these two sets of quantities and prices conform with each other would depend upon how the final demand would be determined responding to the supply prices obtained here and the earned incomes corresponding to them. In this sense, the detemination of equilibrium quantities and prices will depend upon how the demand behavior is formulated.

The 6 simultaneous equations may be rewritten in a matrix form. This system of simultaneous equations will represent in a condensed form the inter-sectoral dependence through transaction of intermediate goods and also through the labor market. The system may be expressed as (7.113)

(7.113)

$$\begin{bmatrix} 0 & \frac{1}{a_2}L_2h_2(1-\frac{\gamma_{s2}}{X_2}) & 0 & \gamma_{s2}(1-a_{22}-t_{I2}) & a_{32}(X_2-\gamma_{s2}) & a_{42}(X_2-\gamma_{s2}) \\ 0 & \frac{1}{a_3}L_3h_3(1-\frac{\gamma_{s3}}{X_3}) & 0 & a_{23}(X_3-\gamma_{s3}) & \gamma_{s3}(1-a_{33}-t_{I3}) & a_{43}(X_3-\gamma_{s3}) \\ 0 & 0 & \frac{1}{a_4}L_4h_4(1-\frac{\gamma_{s4}}{X_4}) & a_{24}(X_4-\gamma_{s4}) & a_{34}(X_4-\gamma_{s4}) & \gamma_{s4}(1-a_{44}-t_{I4}) \\ -\eta_{23} & 1 & 0 & 0 & 0 & 0 \\ -\eta_4 & 0 & 1 & 0 & 0 & 0 \\ -L_1h_1 & 0 & 0 & -(1-b_1)a_{21}X_1 & -(1-b_1)a_{31}X_1 & -(1-b_1)a_{41}X_1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_{23} \\ w_4 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} -(X_2-\gamma_{s2})p_1a_{12} \\ -(X_3-\gamma_{s3})p_1a_{13} \\ -(X_4-\gamma_{s4})p_1a_{14} \\ \epsilon_{23} \\ \epsilon_4 \\ (1-b_1)p_1X_1(a_{11}-1) \end{bmatrix}$$

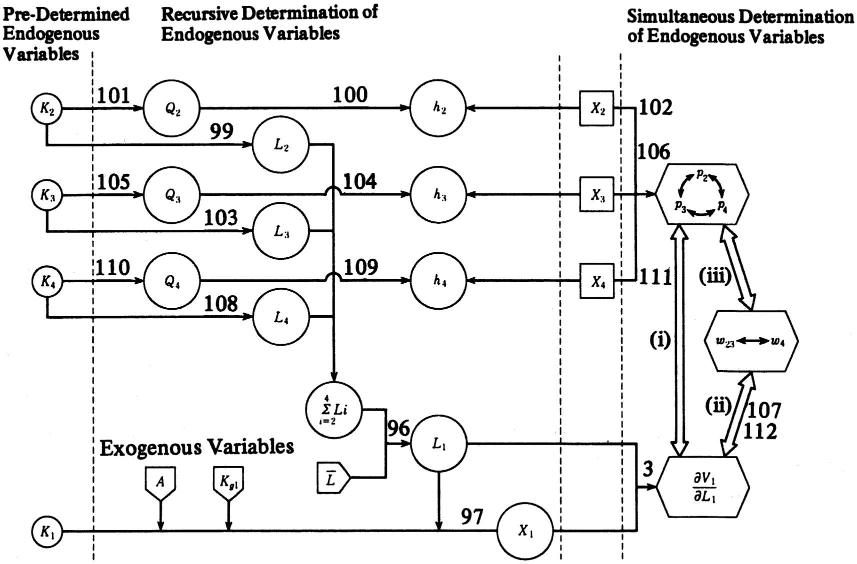
Denoting this simply as

$$A\Gamma = B$$

we can get the solutions for the 6 endogenous variables w_j ($j = 1, 23, 4$) and p_j ($j = 2, \ldots, 4$) in the form of $\Gamma = A^{-1}B$ since usually $|A| \neq 0$.

Figure 7.17 summarizes the interdependent relationships of the endogenous variables contained in the 17 equations. The numbers in the Figure are the numbers attached to equations of the system.

Figure 7.17 THE STRUCTURE OF INTER-SECTORAL DEPENDENCE IN THE SHORT-RUN SUPPLY



Notes: (1) Notations are:

K_j capital stock at the beginning of each period for the j-th sector (j = 1, 2, 3 and 4),

Q_j : the capacity of output per period for the j-th sector (j = 2, 3 and 4)

L_j : the number of employed workers for the j-th sector (j = 1, 2, 3 and 4)

h_j : hours of operation for the j-th sector (j = 2, 3 and 4),

X_j : the level of output for the j-th sector (j = 1, 2, 3 and 4).

A : the area of cultivated land,

K_{g1} : public capital stock invested in Sector 1,

$\bar{L}$: total labor force

p_j : the price of the commodity of the j-th sector (j = 1, 2, 3 and 4)

w_j : wage rate for the j-th sector (j = 1, 2, 3 and 4) and

$\frac{\partial V_1}{\partial L_1}$: value added marginal productivity in Sector 1.

(2) The numbers (96, ..., 112) are the numbers attached to equations in this chapter.

(3) Arrows (i), (ii) and (iii) represent the directions of the interdependence by which the values of the endogeneous variables are determined simultaneously.

Given the capital stock K_j (j = 1, ..., 4) for each sector at the beginning of each period, Q_j and L_j of the non-agricultural sectors will be determined from equations (7.101), (7.105), (7.110) and (7.99), (7.103) and (7.108).

Since the total labor force $\bar{L}$ is given exogenously, the labor force left for the agricultural sector L_1 will be determined as the residual remaining after subtracting the sum of non-agricultural employment $\sum_{j=2}^{4} L_j$ from the total labor force $\bar{L}$ through equation (7.96). The output X_1 of the agricultural sector will be determined from given the values of L_1 and exogenously given land A_1 and public capital stock K_{g1} using the production function (7.97). Even though output X_1 was determined, the marginal value added productivity of Sector 1 and prices of other sectors p_j ($j = 2, \ldots, 4$) are still interdependent through intermediate inputs from the non-agricultural sectors. This interdependence is represented by the thick arrow (i) in the system of simultaneous equations in Figure 7.17.

The levels of wages of the non-agricultural sectors change as the marginal value added productivity in agriculture changes as seen from equations (7.107) and (7.112) with certain proportional differentials. The thick arrow (ii) indicates this linkage.

The non-agricultural sectors will have their supply prices p_j ($j = 2, \ldots, 4$) in correspondence to certain quantities supplied X_j ($j = 2, \ldots, 4$) through equations (7.102), (7.106) and (7.111). In this case, p_j's are also interdependent with wages w_j and prices of intermediate inputs from other sectors. The thick arrows (i) and (iii) indicate the interdependence.

In other words, out of the 6 simultaneous equations, the supply equations (7.102), (7.106) and (7.111) imply the equality between marginal productivity and wage rate for each sector, and equations (7.107) and (7.112) are empirical equations indicating inter-sectoral wage differentials. Therefore, the solutions of the simultaneous equations, which are obtained by giving arbitrarily certain amounts of outputs X_j, can be regarded as suitable supply prices p_j and wages w_j which will allocate the employment L_j, the level of which has already been determined by the allocation of capital stock K_j, compatibly with optimal choices of actors in various sectors.

We can predict the theoretical values of supply prices and wages by using the observed levels of output as the data for output X_j for Sectors 2, 3 and 4 for the period of 1955 to 1965. Table 7.6 shows the estimated result.

Since the observed values are used for the levels of output, the theoretical values thus derived should be equal to the actual equilibrium prices. However, the theoretical values obtained here are different from the simultaneous solutions of the entire model. What has been obtained is imply the result of a partial test in the sense that the observed data are used for the levels of output.

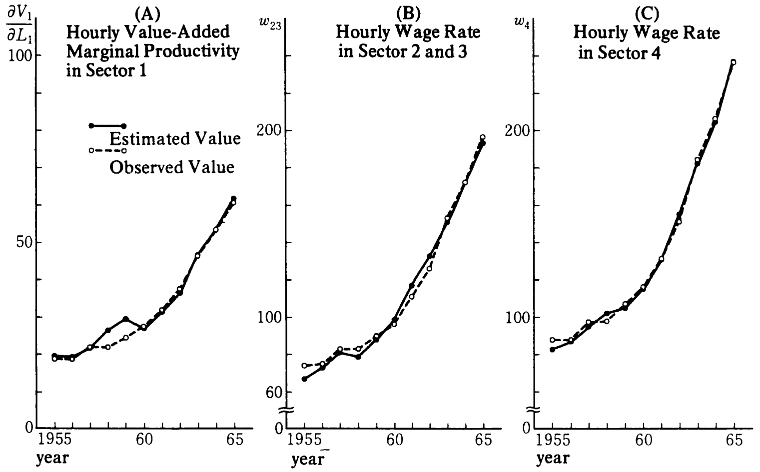
The results of this partial test are reasonably satisfactory as can be seen from Table 7.6, Figure 7.18 and 7.19.

Table 7.6 ESTIMATES OF WAGES AND SUPPLY PRICES OBTAINED FROM THE SIMULTANEOUS DETERMINATION

	Wages (Yen/man-hour)						Prices (1965=1.00)					
	Sector 1	Value Marginal Productivity	Sector 2.3	Wage	Sector 4	Wage	Sector 2	Output Deflator	Sector 3	Output Deflator	Sector 4	Output Deflator
	*OB**	*ES*	*OB*	*ES*	*OB*	*ES*	*OB*	*ES*	*OB*	*ES*	*OB*	*ES*
1955	19.6	19.2	67.4	74.6	83.3	88.2	0.9453	1.0044	0.8619	0.9697	0.6395	0.6935
1956	19.2	19.2	73.9	74.9	86.9	88.3	0.9429	0.9756	0.9612	0.9698	0.6783	0.6759
1957	21.9	21.8	80.3	82.3	95.6	97.5	0.9590	0.9762	1.0180	1.0185	0.7210	0.7364
1958	26.7	22.2	79.6	83.3	101.3	98.7	0.9160	0.9211	0.9340	1.0015	0.7200	0.7199
1959	29.9	24.3	88.3	89.4	105.3	106.0	0.9300	0.8966	0.9410	0.9206	0.7580	0.7625
1960	26.7	27.2	99.9	97.6	115.5	116.1	0.9480	0.8801	0.9570	0.8775	0.7520	0.7282
1961	31.6	31.8	116.3	111.1	132.6	132.5	0.9700	0.9684	0.9740	0.9310	0.8010	0.7668
1962	36.8	37.1	135.2	126.3	155.4	150.8	0.9700	0.9677	0.9650	0.9383	0.8510	0.8123
1963	46.6	46.6	151.3	153.5	182.8	183.9	0.9940	0.9947	0.9720	0.9865	0.9070	0.9038
1964	53.0	53.1	172.2	172.2	204.4	206.6	0.9910	0.9845	0.9880	0.9729	0.9520	0.9700
1965	62.0	61.4	193.0	196.1	236.4	235.6	1.0000	1.0202	1.0000	1.0482	1.0000	1.0362

*OB**: marginal value added productivity estimated from the production function in the agricultural sector (Sector 1),
OB : actually observed values,
ES : estimates obtained from the simultaneous determination.

Figure 7.18 VALUE ADDED MARGINAL PRODUCTIVITY AND WAGES: ESTIMATES AND OBSERVED VALUES

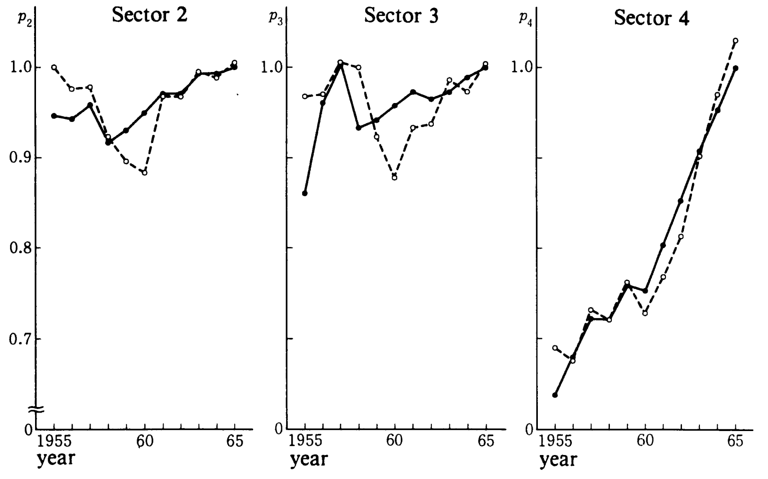


Notes: (1) $\frac{\partial V_1}{\partial L_1}$ denotes hourly value added marginal productivity in Sector 1 expressed in terms of yen, and w_j represents hourly wage rate for the j-th sector (j = 2, 3 and 4. The notation w_{23} indicates that the common wage rate is used for both Sectors 2 and 3.

(2) In Panel (A), the solid line represents the estimates of hourly value added marginal productivity in Sector 1 derived directly from the production function specified in this chapter, while the dotted line indicates the alternative estimates of the same thing derived from the system of simultaneous determination of supply prices and quantities.

In Panels (B) and (C), the solid lines represent the movements of observed values, while the dotted lines represent the estimates derived from the system of simultaneous determination mentioned above.

Figure 7.19 ESTIMATES AND OBSERVED VALUES OF SUPPLY PRICES



Notes: (1) The notation p_j denotes the price index (the price in 1965 = 1.0) for the j-th sector (j = 2, 3 and 4).
(2) The dotted lines represent the estimates and the solid lines represent the observed values.

Notes to Chapter 7

1) See the discussion in the section entitled "The Design of Experiments" in Chapter 6.
2) There exist many reviews on production functions which are useful for our purposes. Among them are, for example, reviews by Walters (1963), Solow (1967), Nerlove (1967) and Johansen (1972).
3) It is well known that an epock-making contribution in the field of production function analysis has been made by Arrow and others (1961). Their ingeneous formulation, known by the name of the Constant Elasticity of Substitution (CES) production function, was made in an attempt to explain international cross sectional observations, in which wages and relative labor shares were positively correlated by applying a hypothesis which allows for factor substitutions in the form of a general type homogeneous production function of degree one. Along the lines of this class of CES production function, many contributions have been made in estimating production functions using cross sectional data. For such developments, see Fuchs (1963), Minasian (1961), Minhas (1963), Leontief (1964), Solow (1964) and Dhrymes (1965). There also exists a rich stock of research findings which are based on time-series data. See, for example, Kravis (1959), Kendrick and Sato (1963), Brawn and De Cani (1963) and Ferguson (1965).

A criticism has been put forward, most notably by Leontief (1964) on the ground that the estimated elasticity of substitution lacks the internal consistency. The Leontief's criticism was made using the same data used by Minhas (1963) in estimating the CES production function.

We have also conducted similar experiments on the CES production function in an attempt to examine the internal consistency of the estimated parameters using time-series data of the Japanese economy. We have not found either in our case the internal consistency on the elasticity of substitution.

However, one of the remarkable findings obtained from analysis of Japanese data was that in industries where output increased rapidly such as automobile and electrical machinery industries the elasticity of substitution σ was greater than unity while in industries where output increases sluggishly and labor's relative share remained stable such as textiles it was less than unity. This result is quite different from what was found by Arrow and others in their aforementioned study of international cross-sectional data in which the elasticity of substitution was less than unity in almost all industries. As for detailed information of our analysis of Japanese data, see Kuroda (1974).

4) Ozaki (1966) estimated input functions specified as

$$M_i = \alpha_{mi} X^{\beta_{mi}},$$

using the cross-sectional data of four-digit manufacturing industries reported in the Census of Manufactures 1964 and 1965. In his experiment, materials are sub-divided into three categories: M_1 = raw materials, M_2 = energy and M_3 = fuel. It was found that β_{m1} was not significantly different from unity while β_{m2} and β_{m3} were different from unity. In our model, we assumed, following the Ozaki's finding, the constancy in input coefficients of material inputs which include energy and fuel. This assumption is not unreasonable since the relative weight of energy and fuel is minor in the total value of material inputs.

5) Informulating the agricultural production function in our model, we took advantage of a stock of ample research findings which have been accumulated since the pioneering work of Wicksell (1916). For subsequent developments, see, for example, Tolley and Others (1924), Tintner (1944), Heady (1946) and Johnson (1944).

6) Lewis (1954).

7) The levels of marginal productivity of male and female workers in Japanese agriculture for 1959 and 1961, controlling for the family size of farm household, for 10 regions, were measured by Torii. See Torii (1965a) and (1965b).

Chapter 8
The Determination of Value Added and Factor Incomes

In chapter 7 we described the system of simultaneous determination of sectoral supply prices and wages. Each of these simultaneous equations is naturally related, in the framework of general interdependence of the economy, to the determination of factor incomes and volumes of final demand items. While the gross value added will comprise on the one hand incomes to factor inputs such as labor and capital, it also constitutes on the other hand the sources of incomes for such economic actors as individuals (households), firms, and the government. It is the distribution of incomes classified by these economic actors which serve as budget constraints imposed upon these actors in determining the volume of final demand in the economy. In this context, we shall describe in this chapter the determination of factor incomes and budget constraints on various economic actors of our model.

8.1 The Determination of Factor Incomes

The gross value added for each sector is defined as:

(8.1) $$V_j = (p_j - \Sigma p_i a_{ij}) X_j \quad (j = 1, \ldots, 4).$$

The gross value added for each sector V_j as defined above will be broken down as:

1. Business consumption expenditure	B_{cj}	exogenous
2. Compensation of employees	E_{Ij}	endogenous
3. Income from unicorporated enterprises	U_{cIj}	endogenous
4. Income from property	P_{Ij}	endogenous
5. Income from private corporations	C_{Ij}	endogenous
6. General government income from property and entrepreneurship	Y_{Gj}	exogenous
7. (Less) interest on public debt 8. (Less) Interest on consumers' debt	D_{cGj}	exogenous
9. (Less) Stock valuation adjustment	A_{pj}	exogenous
10. (Less) Imputed service charges	I_{cj}	exogenous
11. Operating surplus (= 3. + 4. + ... + 10.)	B_{sj}	endogenous
12. Provisions for the consumption of fixed capital	D_{ej}	endogenous
13. Indirect taxes	T_{Ij}	endogenous
14. (Less) Current subsidies	S_{cj}	exogenous
15. Net factor income from abroad	TR_{INj}	exogenous
Total value added	V_j	

In addition to these items, we take into account direct taxes (personal income taxes, and corporation income taxes) and other transfer incomes. These other items are, however, included in incomes of various actors in the above classification scheme such as compensation of employees, income from unincorporated enterprises, income from property and income from private corporations. Let us now explain in turn some of the major components of value added.

Business Consumption Expenditure

The ratio R_{Bcj} of business consumption expenditure to the gross value added is given by

$$R_{Bcj} = B_{cj}/V_j \quad (j=1,\ldots,4). \tag{8.2}$$

The value R_{Bcj} is given exogenously each year in our model for each sector.

COMPENSATION OF EMPLOYEES (endogenous)

The number of employees E_y is defined as the number of workers employed L minus the number of self-employees and unpaid family workers. The number of employees for each sector can be approximated fairly well by the following linear regression equations:

$$(8.3) \qquad E_{y1} = \underset{(5.6667)}{42.581300} + \underset{(0.000424)}{0.086442681}\, L_1 . \qquad \bar{r} = 0.9999$$

$$(8.4) \qquad E_{y2} = \underset{(145.1758)}{-56.703900} + \underset{(0.02251)}{0.87011164} L_2 . \qquad \bar{r} = 0.9970$$

$$(8.5) \qquad E_{y3} = \underset{(16.7069)}{-178.14270} + \underset{(0.00264)}{0.96767723} L_3 . \qquad \bar{r} = 0.9999$$

$$(8.6) \qquad E_{y4} = \underset{(86.3294)}{-1288.1030} + \underset{(0.004697)}{0.72751133} L_4 . \qquad \bar{r} = 0.9999$$

Compensation of employees in each sector may be obtained consequently by

$$(8.7) \qquad E_{I1} = Ey_1 h_1 w_1 .$$

$$(8.8) \qquad E_{I2} = Ey_2 h_2 w_{23} .$$

$$(8.9) \qquad E_{I3} = Ey_3 h_3 w_{23} .$$

$$(8.10) \qquad E_{I4} = Ey_4 h_4 w_4 .$$

PROVISIONS FOR THE CONSUMPTION OF FIXED CAPITAL (endogenous)

Provisions for the consumption of fixed capital can be subdivided into damage of fixed capital by accidents and depreciation allowances. The depreciation of equipment corresponds theoretically to supplementary investment. However, the item of depreciation used here includes also a kind of retained earnings such as accelerated depreciation allowances which are admitted by certain tax systems.

According to the "renewal theory" of the theory of supplementary investment, the amount of supplementary investment approaches asymptotically a certain ratio of accumulated capital stock regardless of the shapes of distribution of supplementary investment specific to individual pieces of equipment so long as the assumption of either constant capital stock or a constant rate of growth of capital holds.[1] Assuming that the actually observed provisions for the consumption of fixed capital are used entirely for supplementary investment, then theoretically we have the relationship

$$D_{ej}/\eta_{Kp} = \delta_j K_j \quad (j = 1, \ldots, 4),$$

where η_{Kp} is the implicit deflator of investment goods. However, it would probably be more appropriate to formulate the relationship as

$$D_{ej}/\eta_{Kp} = \delta_j K_j + \epsilon_j \quad (j = 1, \ldots, 4),$$

so that an institutional element of raising the rate of depreciation such as an accelerated depreciaton system may be taken into account explicitly by $\epsilon_j < 0$. The least squares estimates for the three sectors (Sectors 2 and 3 are lumped together because of the limited availability of data) are:

(8.11) $$D_{e1}/\eta_{Kp} = -317.87463 + 0.086547698K_1.$$
$$(50.4905) \quad (0.008018)$$
$$\bar{r} = 0.9593 \quad d.w. = 0.5871 \quad d.f. = 9$$

(8.12) $$D_{e23}/\eta_{Kp} = -303.9837 + 0.12170884\,(K_2 + K_3).$$
$$(32.5158) \quad (0.003338)$$
$$\bar{r} = 0.9962 \quad d.w. = 1.93 \quad d.f. = 9$$

(8.13) $$D_{e4}/\eta_{Kp} = -691.54620 + 0.15879786K_4.$$
$$(65.8944) \quad (0.005691)$$
$$\bar{r} = 0.9936 \quad d.w. = 0.5939 \quad d.f. = 9$$

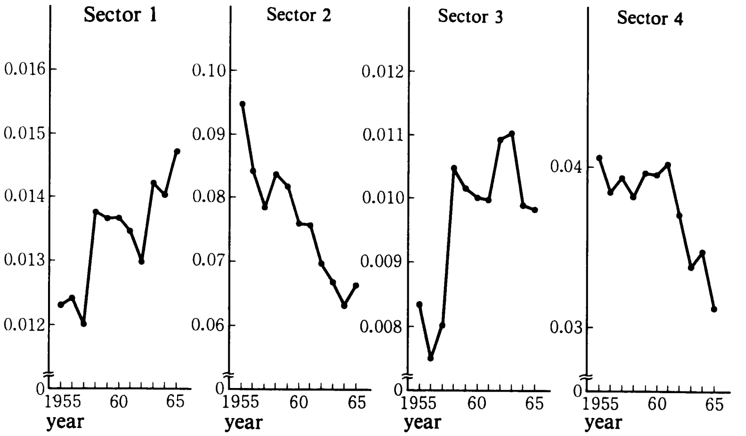
Both intercepts and regression coefficients are statistically significant. The negative intercepts, as theoretically expected, suggest that the average rates of depreciation have been rising over time. Although the results deserve further examinations both theoretically and in data, the obtained estimates indicate that the rates of marginal depreciation are 8.6, 12.2 and 15.9 percents respectively for the above three sectors, and accordingly the periods of depreciation are 11.5, 8 and 6 years.

Indirect Taxes and Charges (endogenous)

In our fairly aggregate model, it is quite difficult to relate the legally specified indirect tax rate for each commodity to the average rate of indirect tax for each sector. We therefore used an average rate of indirect tax for each sector on the assumption that all indirect taxes are levied according to the values of commodities, using the following formula,

(8.14) $$T_{Ij} = t_{Ij}\,(p_j X_j).$$

Figure 8.1 OBSERVED MOVEMENTS OF INDIRECT TAX RATES



Note: The indirect tax rate, measured along the vertical axis, is defined here as the value of indirect taxes divided by the value of sales.

The average rate thus obtained is now given exogenously each year for each sector. The average indirect tax rate for each of the four sectors is shown in Figure 8.1.

NET FACTOR INCOME FROM ABROAD (exogenous)

This is an item created to adjust the data of National Income Statistics to the classification scheme of Input-Output Tables.[2] This variable is given exogeneous for each sector.

Subtracting from the gross value added V_j the above listed endogenous and exogenous elements such as business consumption expenditure, compensation of employees, provisions for the consumption of fixed capital, indirect taxes and charges, current subsidies and net factor income from abroad, we can obtain operating surplus B_s for each sector as follow:

(8.15) $$B_{s1} = V_1 - D_{e1} - B_{c1} - T_{I1} + S_{c1} - E_{I1} - TR_{IN1}.$$

(8.16) $$B_{s23} = (V_2 + V_3) - D_{e23} - (B_{c2} + B_{c3}).$$

$$- (T_{I2} + T_{I3}) + (S_{c2} + S_{c3}) - (E_{I2} + E_{I3}) - TR_{IN23}.$$

(8.17) $$B_{s4} = V_4 - D_{e4} - B_{c4} - T_{I4} + S_{c4} - E_{I4} - TR_{IN4}.$$

Note that because of the limited availability of data, Sectors 2 and 3 are lumped together.

For components of operating surplus such as general government income from property and entreprenuership Y_{Gj}, interests on public and consumers' debt D_{cGj} and stock valuation adjustment A_{pj}, we computed their ratios to the total operating surplus respectively as

(8.18) $$R_{YGj} = Y_{Gj}/B_{sj}. \quad (j = 1,\ 23,\ 4).$$

(8.19) $$R_{DcGj} = D_{cGj}/B_{sj} \quad (j = 1,\ 23,\ 4).$$

(8.20) $$R_{Apj} = A_{pj}/B_{sj} \quad (j = 1,\ 23,\ 4).$$

Given these ratios exogenously, we can compute the values of items Y_{Gj}, D_{cGj} and A_{pj} corresponding to the volumes of operating surplus from equations (8.15), (8.16) and (8.17).

For income from private corporations, income from unincorporated enterprises and income from property, we estimated empirical equations by which sub-divide the residual which remains after subtracting Y_{Gj}, D_{cGj}, A_{pj} from the operating surplus into these three types of incomes using the time-series data of the National Income Statistics.

Income from Unincorporated Enterprises

(8.21) $$U_{cI1} = \underset{(72.1878)}{67.9815} + \underset{(0.04151)}{0.91253561}\,(B_{s1} + A_{p1} - Y_{G1} + D_{cG1}).$$

$\bar{r} = 0.9897 \quad d.w. = 2.19 \quad d.f. = 9$

(8.22) $$U_{cI23} = \underset{(54.2374)}{-1.02739} + \underset{(0.01431)}{0.27572046}\,\{B_{s23} + (A_{p2} + A_{p3}) - (Y_{G2} + Y_{G3}) + (D_{cG2} + D_{cG3})\}.$$

$\bar{r} = 0.9855 \quad d.w. = 0.92 \quad d.f. = 9$

(8.23) $$U_{cI4} = \underset{(68.3632)}{313.5087} + \underset{(0.01696)}{0.35588237}\,(B_{s4} + A_{p4} - Y_{G4} + D_{cG4}).$$

$\bar{r} = 0.9888 \quad d.w. = 0.76 \quad d.f. = 9$

Income from Property

(8.24) $$P_{I1} = -83.058720 + 0.078812881\,(B_{s1} + A_{p1} - Y_{G1} + D_{cG1}).$$
$$(11.3025) \quad (0.006499)$$
$$\bar{r} = 0.9674 \quad d.w. = 2.54 \quad d.f. = 9$$

(8.25) $$P_{I23} = -149.23602 + 0.20106021\,\{B_{s23} + (A_{p2} + A_{p3})$$
$$(41.3580) \quad (0.01092)$$
$$-(Y_{G2} + Y_{G3}) + (D_{cG2} + D_{cG3})\}$$
$$\bar{r} = 0.9841 \quad d.w. = 1.25 \quad d.f. = 9$$

(8.26) $$P_{I4} = -166.5641 + 0.31455447\,(B_{s1} + A_{p4} - Y_{G4} + D_{cG4}).$$
$$(27.2653) \quad (0.006764)$$
$$\bar{r} = 0.9977 \quad d.w. = 1.43 \quad d.f. = 9$$

Income from Private Corporations

(8.27) $$C_{I1} = -20.759425 + 0.021670483\,(B_{s1} + A_{p1} - Y_{G1} + D_{cG1}).$$
$$(8.2755) \quad (0.004758)$$
$$r = 0.8146 \quad d.w. = 3.12 \quad d.f. = 9$$

(8.28) $$C_{I23} = 150.2626 + 0.52321958\,\{B_{s23} + (A_{p2} + A_{p3})$$
$$(93.2235) \quad (0.02460)$$
$$-(Y_{G2} + Y_{G3}) + (D_{cG2} + D_{cG3})\}$$
$$\bar{r} = 0.9880 \quad d.w. = 1.01 \quad d.f. = 9$$

(8.29) $$C_{I4} = -116.4137 + 0.32371964\,(B_{s4} + A_{p4} - Y_{G4} + D_{cG4}).$$
$$(51.7530) \quad (0.01284)$$
$$\bar{r} = 0.9922 \quad d.w. = 0.95 \quad d.f. = 9$$

For each sector, either one of U_{cIj}, P_{Ij} or C_{Ij} can be obtained eventually as a residual. In the estimation procedure of our model, we obtained income from private corporations as a residual for Sector 1 and income from unincorporated enterprises as a residual for Sectors 2 3 and 4. Using the above results, we can obtain the sum of each type of income as

(8.30) $$U_{cI} = \sum_{j=1}^{4} U_{cIj}.$$

(8.31) $$P_I = \sum_{j=1}^{4} P_{Ij}.$$

(8.32) $$C_I = \sum_{j=1}^{4} C_{Ij}.$$

The sum of personal incomes, on the other hand, is expressed as

(8.33) $$Y_p = \sum_{j=1}^{4} E_{Ij} + U_{cI} + P_I.$$

Since we are using the National Income Statistics, we have to subtract imputed service charges respectively from the sum of corporate incomes and from the sum of personal incomes.

This completes the description of the procedures to estimate the value added components presented earlier.

8.2 Budget Constraints on Economic Actors

Let us now classify the income items described so far by economic actors, namely households and corporations. Account of households and private non-profit institutions is given by Table 8.1.

Personal Income tax T_p can be obtained by

(8.34) $$T_p = t_p\ (Y_p - I_{cp}),$$

Table 8.1 ACCOUNTS OF HOUSEHOLD

Outgoings		Incomings	
Personal Consumption	E	Compensation of Employees	E_I
Personal Direct Taxes and Charges	T_p	Income from Unincorporated Enterprises	U_{cI}
Social Insurance Contributions	TR_{sI}	Income from Property	P_I
Transfers from Households and Private Non-Profit Institutions to Government	TR_{pG}	Transfers from Private Corporations to Households	TR_{cp}
Transfers from Households and Private Non-Profit Institutions to the Rest of the World	TR_{pR}	Transfers from Government to Households	TR_{GP}
Personal Saving	S_p	Transfers from the Rest of the World to Households	TR_{RP}
		(Less) Imputed Service Charges by Persons	I_{cp}

where the rate of personal income tax t_p is given exogenously. Personal disposable income Y_D can be obtained by giving transfer incomes TR_{cp}, TR_{Gp}, TR_{pG}, TR_{pR} exogenously by the relation of

(8.35) $$Y_D = Y_p - T_p - TR_{sI} - TR_{pG} - TR_{pR} + TR_{cp} + TR_{Gp} + TR_{Rp} + I_{cp} .$$

Personal disposable income can be sub-divided betwen personal consumption expenditure and personal saving. The latter may be estimated by the following equation as the residual which remains after subtracting total personal consumption expenditure from personal disposable income.

(8.36) $$S_p = Y_D - E.$$

(8.37) $$E = 119.2105 + 0.48699E_{-1} + 0.4302\,Y_D.$$

$$(145.5659)\quad (0.08624)\quad (0.1338)$$

$$\bar{r} = 0.9996 \quad d.w. = 1.76 \quad d.f. = 8$$

This equation gives a more stable result than by estimating a saving function directly. The total personal expenditure E serves as a budget contraint upon personal consumption expenditure which is an important element of the final demand. On the other hand, retained earnings of private corporation for each sector can be obtained from the following definitional relationship:

Retained Earnings = Income from Private Corporations
− Corporation Income Taxes and Charges
− Transfers from Private Corporations to Households
− Dividend Payments
− Imputed Service Charges for Private Corporations

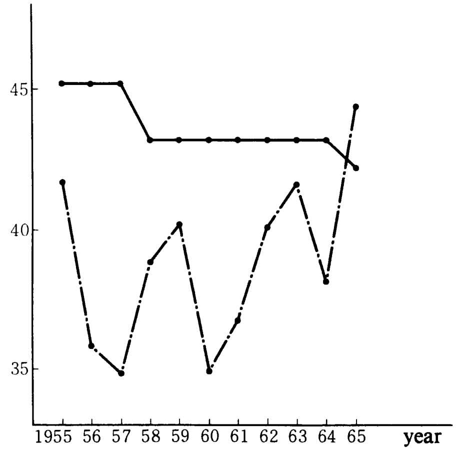
Corporation income tax and charges are obtained by the relation

(8.38) $$T_{cj} = t_c \cdot C_{Ij} \ (j = 1, \ \ldots, \ 4),$$

where the rate of corporation income tax t_c is given exogenously each year. It should be noted, that the patterns of movement of the average rate of corporation income tax computed by the National Income Statistics does not necessarily conform to the movement of the legally specified tax rates.

As shown in Figure 8.2, the average rate of corporation income tax used in our model has changed quite differently from legal tax rates. Further improvements are yet to be seen in adjusting time lags and other differences associated with the data.

Figure 8.2 OBSERVED MOVEMENTS OF CORPORATION TAX RATES



Note: The solid line indicates the average rate of corporation income tax derived from our model, while the broken line represents the legally specified rate of corporation income tax.

Retained earnings of private corporations M_j can be obtained from the following definitional equation by giving exogenously transfers from private corporations to households TR_{cpj}, dividend payments D_{vj}, and imputed service charges for private corporations.

$$M_j = C_{Ij} - T_{cj} - TR_{cpj} + I_{ccj} - D_{vj}. \tag{8.39}$$

The retained earnings M_j will also play an important role as a constraint upon firms' demand for investment goods in determining the final demand.

Notes to Chapter 8

1) Parzen (1962) and Jorgenson and Stephenson (1969).
2) The data from National Income Statistics in which all data are defined on the basis of "national production" had to be adjusted in such way that they are comparable with the data obtained from Input-Output Tables in which all data are defined on the basis of "domestic production." The concept of "net factor income from abroad" is devised to facilitate such an adjustment. However, since it is difficult to sub-divide the net factor income from abroad into the four industrial sectors, we treated all of the factor income from abroad as being contained in the transaction item of Sector 3.

Chapter 9
Quantity and Price Converters

One of the objectives of the new SNA system is to portray the structure of product-mix in a national economy by distinguishing clearly the concept of commodity from the concept of the industry producing it. We have emphasized repeatedly that the data of such a systematic social accounting scheme are indispensable for the purpose of constructing a multi-sectoral model.

The Japanese input-output tables for 1960 and 1965 have already clarified the concept of commodity, and their basic tables are constructed according to the activity base.[1] This differs from the concept of industry and basically follows the scheme of commodity classification adopted in the new SNA system. Based primarily on these input-output tables, the commodity data are reclassified into four major sectors our model. Note, therefore, that the classification of the four sectors in our analysis is based on the commodity concept. Even though these four sectors happened to be referred to as industries for the sake of convenience, the concept of "industry" in such a case does not correspond to an industry measured in terms of establishments.

We use an input-output table evaluated in terms of producers' price. Domestic transportation fee for goods and distributive trade margin are absorbed in our model collectively in the fourth sector (commercial and service sectors). On the other hand, models of the demand side have to be formulated using purchasers' prices which include transportation fee and trade margin. That is to say, in the case of price, the sum of shipment price from producers, transportation fee and transaction margin must be equivalent to its purchase price. In order to maintain consistency between producers' commodities and purchasers' demand items, both in terms of quantity and price, we need to have an input-output converter or quantity and price converters.[2]

Unlike the ideas behind the Brookings Model, we do not necessarily expect that each coefficient of the converter can be treated as a constant parameter throughout the period of observation.[3] It is conceivable that we would reach a stable relationship between the commodity classification on the production side and the demand item classification on the demand side if we could elaborate classification on both sides to such an extent that each component on one side finds its counterpart on the other. However, in our model which uses considerably aggregative concepts, it would be more appropriate for us to formulate, given the nature of the data, the converter in such a way that it can change from year to year.

In the ensuring discussion, we will explain the theoretical construct and the methodology of estimation of such a converter. A brief explanation of the data used in our estimation will also be included in the final section.

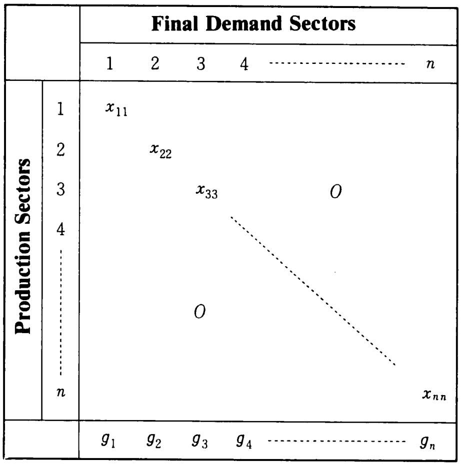
9.1 The Concept of Quantity and Price Converters

If we could disaggregate in our model goods and prices so elaborately that goods and prices on the production (supply) side and expenditure (demand) side correspond on a one-to-one basis, then the converter concept would be unnecessary. This is because points of equilibrated demand and supply of goods would appear only on the diagonal of the table of the final demand. Figure 9.1 depicts such an outcome.

The goods denoted as 1, ..., n in the production sector correspond exactly to the goods 1, ..., n in the demand sector, and accordingly so do their prices. Setting the problem of joint products aside for a moment, we can reduce the non-diagonal elements to zero by so reallocating goods in both demand and production sectors. If we were to define a converter in this case, it would be an unit matrix of $n \times n$ of which every element is defined as $(x_{ii}/g_j) = 1$. This implies that the converter can be taken as a constant parameter throughout the period of observation.

However, in cases of fairly aggregate multi-sectoral models which are opeational, non-diagonal elements are not usually reduced to zero. The values of such elements often vary from time to time depending on changes in demand and in the weights of joint products. In such cases it is difficult to treat the coefficient (i.e. x_{ij}/g_j) of the converter as a constant parameter throughout the period of observation. This is because the coefficients of the converter themselves reflect changes in product-mix on the producers' side and changes in taste on the purchasers' side. In other words, if we could elaborate sectoral classification of the model to such an extent that we could

Figure 9.1 THE CORRESPONDENCE BETWEEN THE CLASSIFICATION OF COMMODITIES IN THE PRODUCTION SIDE AND THE CLASSIFICATION OF DEMAND-ITEMS IN THE FINAL DEMAND SIDE



Note: (1) The notation x_{ii} represents transactions from the i-th production sector to the i-th final demand sector, and g_j represents the sum of the elements of final demand in j-th column.

(2) All off-diagonal elements of the matrix are zero. Zero indicates that there are no transactions.

regard the coefficient of the converter as a constant parameter then the converter itself would no longer be necessary.

The role of the converter in the Brookings model cited earlier is the role of weight coefficients which is designed to maintain a consistent relationship in quantity and price between the demand and production sectors and not as the parameters representing the structure of demand and supply. We too take the standpoint that the coefficient of the converter is a weight matrix which purports to cement production and demand sectors consistently for operational purposes in a fairly aggregate mult-sectoral model. If the nature of the converter is interpreted thus, its coefficient would not necessarily be stable because it reflects changes in the weight of commodity-mix and demand item-mix within each sector in our model, which employs a fairly aggregate sectoral classification scheme.

Table 9.1. presents the coefficients of converters computed from integrated input-output tables of four major sectors for 1955, 1960 and 1965 which are available for our use. The sectoral allocation ratio (expressed by the converter coefficient) has been changing obviously from time to time and therefore it could not be treated as a constant parameter.

Let us explain now the relationship between the quantity converter and the price converter. Suppose there are n sectors of production and m categories of final demand items. Let us suppose further that in the t-th period, the i-th production sector ($i = 1, \ldots, n$) supplies the quantity r_{ij}^t to the j-th final demand item ($j = 1, \ldots, m$) and if you sum all the demand items then the i-th sector supplies in total the quantity f_i^t. If this is the case, we have the following identity.

$$(9.1) \qquad f_i^t = \sum_{j=1}^{m} r_{ij}^t \qquad (i=1, \ldots, n).$$

Denoting the price and quantity of the j-th final demand item in the t-th period respectively by η_j^t and g_j^t, the total demand will be $\eta_j^t g_j^t$. Expressing the price of i-th product as p_i^t, we will have the following identity with respect to r_{ij}^t, $\eta_j^t g_j^t$, and p_i^t,

$$(9.2) \qquad \eta_j^t g_j^t = \sum_{i=1}^{n} r_{ij}^t p_i^t \qquad (j=1, \ldots, m).$$

That is to say, equations (9.1) and (9.2) represent respectively vertical and horizontal balances of the content of the final demand in money terms.

If $g_j^t \neq 0$ then from equation (9.1) we get

$$(9.3) \qquad f_i^t = \sum_{j=1}^{m} r_{ij}^t = \sum_{j=1}^{m} g_j^t r_{ij}^t / g_j^t.$$

When we define matrix B^t which contains elements r_{ij}^t / g_j^t arrayed along the i-th row and the j-th column, there will be a relation which may be expressed as:

$$(9.4) \qquad f^t = B^t g^t.$$

where

$$f^t = \begin{bmatrix} f_1^t \\ \vdots \\ f_n^t \end{bmatrix}, \qquad g^t = \begin{bmatrix} g_1^t \\ \vdots \\ g_m^t \end{bmatrix}, \qquad B^t = \begin{bmatrix} r_{11}^t / g_1^t & \cdots & r_{1m}^t / g_m^t \\ \vdots & & \vdots \\ r_{n1}^t / g_1^t & \cdots & r_{nm}^t / g_m^t \end{bmatrix}$$

This matrix B^t is a weight matrix by which vector f^t may be derived, when vector g^t is given. We define this matrix the quantity converter.

On the other hand, dividing both sides of equation (9.2) by g_j^t, we get

Table 9.1 THE COMPOSITION OF FINAL DEMAND BY

		(1) Business Consumption	(2) Private Consumptiion	(3) General Government Expenditure	(4) Fixed Capital Formation
1955	1	0.0692	0.0696	0.0023	−0.0015
	2	0.4785	0.4116	0.0420	−0.0008
	3	0.0525	0.0271	0.0514	0.9740
	4	0.3998	0.4917	0.9043	0.0282
1960	1	0.0332	0.0657	0.0006	0.0029
	2	0.4649	0.4229	0.0292	0.0117
	3	0.0622	0.0456	0.0413	0.9496
	4	0.4397	0.4658	0.9289	0.0359
1965	1	0.0446	0.0600	0.0001	0.0055
	2	0.3669	0.3648	0.0313	0.0217
	3	0.0400	0.0563	0.0471	0.9346
	4	0.5485	0.5188	0.9215	0.0382

Note: Sectors 1, 2, 3 and 4 denote respectively sectors of agriculture-forestry—fisheries, light manufacturing industries, heavy manufacturing industries, and commercial and service industries.
Source: Government of Japan, *Input-Output Tables*, 1955, 1960 and 1965.

(9.5)
$$\eta_j^t = \sum_{i=1}^{n} r_{ij}^{\,t} / g_j^{\,t} \cdot p_i^{\,t} .$$

According to the definition of matrix B^t, equation (9.5) may be written as

(9.6)
$$\eta^t = {}^T(B^t) P^t.$$

where ${}^T(B^t)$ is the transposed matrix of B^t and

$$\eta^t = \begin{bmatrix} \eta_1{}^t \\ \vdots \\ \eta_m{}^t \end{bmatrix}, \qquad P^t = \begin{bmatrix} p_1{}^t \\ \vdots \\ p_n{}^t \end{bmatrix}.$$

This matrix ${}^T(B^t)$ is a weight matrix by which the price vector η^t of final demand items may be derived when the price vector P^t for the production sector in the t-th period is given. We define this matrix as the price converter.

Equations (9.4) and (9.6) indicate that the quantity converter and the price converter are mutually transpositions one of the other. Therefore, all we have to do for the purpose of estimating quantity and price converters is to estimate matrix B^t from the observable data of f^t, g^t, $r_{ij}{}^t$, P^t, and η^t.

CLASSIFIED BY THE FOUR-SECTOR INPUT-OUTPUT TABLE

(5) Inventory Increase	(6) Export	(7) Special Procurement Export	(8) Import	(9) Total Output	(10) Custom Duties
0.5218	0.0234	0.0192	−0.4890	0.1275	
0.3367	0.4609	0.1819	−0.3807	0.2926	
0.1191	0.3129	0.2635	−0.1382	0.2404	
0.0224	0.2028	0.5355	0.0079	0.3394	
0.1596	0.0585		−0.3524	0.0847	−0.0446
0.4026	0.0660		−0.3957	0.2745	−0.6223
0.3553	0.5651		−0.2793	0.3493	−0.2740
0.0825	0.3103		0.0275	0.2915	−0.0591
0.1928	0.0215		−0.3070	0.0681	−0.1212
0.5344	0.3021		−0.4527	0.2542	−0.6421
0.1780	0.5184		−0.2207	0.3480	−0.2275
0.0948	0.1580		−0.0195	0.3298	−0.0092

It is evident from the form of converter B^t that by dividing both sides of equation (9.2) we will get the following relationship:

(9.7) $$\sum_i r_{ij}^t p_i^t / \eta_j^t = g_j^t .$$

Substituting equation (9.7) into each element of converter (9.4), we get

(9.8) $$\frac{\sum_i r_{ij}^t}{g_j^t} = \frac{\sum_i r_{ij}^t}{\frac{\sum r_{ij}^t p_i^t}{\eta_j^t}} = \frac{\eta_j^t \sum_i r_{ij}^t}{\sum_i r_{ij}^t p_i^t} .$$

If we can assume that the equation:

(9.9) $$\eta_j^t = \frac{\sum_i r_{ij}^t p_i^t}{\sum_i r_{ij}^t} .$$

is valid, then the value of equation (9.8) will become unity. Equation (9.8) is the column sum of j-th column of the converter matrix, while (9.9) implies that the price of j-th item η_j^t of the final demand sector is the weighted average of producers' prices p_i^t using r_{ij}^t as a weighting factor. Therefore, equation (9.8) indicates that so long as the itemized prices of the final demand sector are defined in such a way as to assure the validity of equation (9.9), then the column sum of the converter will be unity.

9.2 An Overview of the Methodology of Estimation

It will be useful to examine the estimation procedure employed by the Brookings Quarterly Model before attempting estimation of our converters. In the case of the Brookings model, we may distinguish two types of procedures: one is the type of estimation attempted by Fisher et. al. which is based on the assumption that the coefficients of converters are constant throughout the period of observation and the other is the somewhat improved method of estimation devised by Kresge.[4]

Define nominal value vectors for n sectors of production and m sectors of final demand as Y^t and Z^t respectively

$$Y^t = \begin{bmatrix} p_1{}^t f_1{}^t \\ p_2{}^t f_2{}^t \\ \vdots \\ p_n{}^t f_n{}^t \end{bmatrix}, \qquad Z^t = \begin{bmatrix} \eta_1{}^t g_1{}^t \\ \eta_2{}^t g_2{}^t \\ \vdots \\ \eta_m{}^t g_m{}^t \end{bmatrix}.$$

where p_i, f_i, η_i, g_i follow the notations in the previous section. Consider probability models and assume that the following relationship holds between the two sectors,

$$Y^t = BZ^t + u^t. \tag{9.10}$$

where B is the $n \times m$ coefficient matrix and u^t is a $n \times 1$ vector of random disturbances. Now we aim at estimating a constant matrix B from the time-series data of Y^t and Z^t for the period of observation.

In the Brookings model, demand vector Y^t is estimated from constant input coefficients A that are constant during the period of observation and sectoral production X^t using the relationship.

$$Y^t = (I-A)X^t. \tag{9.11}$$

Matrix B is estimated by applying the constrained least squares method to equation (9.10) with a constraint of suppressing the intercept using the estimates of Y^t and Z^t. Using the definitions of Y^t and Z^t, we have

$$\sum_{i=1}^{n} p_i{}^t f_i{}^t = \sum_{j=1}^{m} \eta_j{}^t g_j{}^t. \tag{9.12}$$

Thus, the row sum of estimated matrix $\hat{B}$ obtained in the neighborhood around the origin of equation (9.10) by the least squares method becomes unity and consequently satisfies the conditions of equations (9.8) and (9.9) explained in the previous section. One of the problems associated with this approach is that matrix B tends to be unstable due to multi-collinearity when equation (9.10) is fitted to the data using the least squares method.

The results of estimation according to this method of the Japanese data during the period of 1955 to 1965 is presented in Table 9.2. To make this estimation we followed the steps used for the Brookings model. That is, we estimated the data of final demand classified by production sectors using the constant input coefficients as of 1960, and made the estimated data the dependent variable of equation (9.10).

Production sectors are classified according to four groupings: (1) agriculture, forestry and fisheries, (2) light manufacturing, (3) heavy manufacturing, and (4) commercial and service industries. The sector of final demand is subdivided into five: (1) business consumption expenditures, (2) private consumption expenditures, (3) government purchases, (4) total domestic capital formation, and (5) exports of goods and services.

The data used were annual calender year data for 11 years from 1955 through 1965. It was difficult to increase the number of independent variables further because the degree of freedom is limited by the number of observations. The fit was generally good as seen from correlation coefficients in the last column of Table 9.2. However, estimated coefficients are not satisfactory both in terms of their poor statistical significance as indicated by the standard diviations in parentheses and of their signs which often contradict from the theoretical sign conditions of non-negativity. Compared with the result of Table 9.1 presented in the previous section, one may realize that the estimates thus obtained by the least squares method are quite unstable because of the influences of multi-collinearity.

In the case of the Brookings model, the suggested remedy for multi-collinearity was to eliminate those independent variables in equation (9.10) of which the value is presumed to be zero. However, as Table 9.1 suggests, there will be little room for us to take advantage of such *a priori* information when we integrate sectors of such a degree of aggregation as the four major sectors. The method of estimation of converter coefficients by a simplistic least squares method is not appropriate for our model or the following reasons: (1) the stability of the coefficient over time is not warranted and consequently theoretical assumptions and actual observations do not conform with each other, and (2) while having the virtue of operational simplicity the method is deficient in that it cannot alleviate the problem of multi-collinearity.

Table 9.2 ESTIMATED INPUT-OUTPUT CONVERTERS BY REGRESSION

Sectors	(1) Business Consumption Expenditure	(2) Private Consumption	(3) Government Consumption	(4) Capital Formation	(5) Export	r
1	0.0610 (0.0209)	1.0149 (0.2017)	−3.8520 (1.0128)	−0.3514 (0.0318)	−0.4815 (0.2741)	0.7285
2	0.0732 (0.0483)	1.4485 (0.4657)	−3.8352 (2.3383)	−0.3321 (0.0734)	−0.4562 (0.6327)	0.9638
3	−0.0246 (0.0720)	−0.3517 (0.6949)	1.4222 (3.4890)	1.1602 (1.0949)	0.2770 (0.9441)	0.9907
4	0.8904 (0.0633)	−1.1111 (0.6109)	7.2551 (3.0676)	0.5234 (0.0962)	1.6596 (0.8301)	0.9881
SCC	1.000	1.00063	0.9928	1.0001	0.9989 ≒1	

Notes: (1) Sectors 1, 2, 3 and 4 represent respectively agriculture-forestry-fisheries, light manufacturing industries, heavy manufacturing industries, and commercial and service industries.
SCC stands for the sum of converter coefficients.
(2) The estimates of converters are computed on the basis of the equation for each i-th sector

$$y_i = \alpha_{i1}x_1 + \alpha_{i2}x_2 + \alpha_{i3}x_3 + \alpha_{i4}x_4 + \alpha_{i5}x_5$$

where $\sum_i \alpha_{ij} = 1$ for each j-th item.
The notation r represents the correlation coefficient adjusted for the degree of freedom for regression of the above equation.

Kresge presented an improved method of estimation as an additional analysis accompanying the original version of the Brookings model.[5] He relaxed the restraint on constancy of the converter coefficient for the period of observation, allowed it to vary from year to year and adopted the method of checking the stability of the coefficient after the fact.

Denoting each element contained in the estimated matrix $\hat{B}$ of equation (9.10) by $\hat{b}_{ij}$ we can obtain the theoretical estimate $\hat{y}_i^t$ as

$$\hat{y}_i^t = \sum_j \hat{b}_{ij} z_j^t \,. \tag{9.13}$$

Compute the ratio α_i^t between the theoretical estimate $\hat{y}_i^t$ and the observed value y_i^t by the relation

$$\alpha_i^t = y_i^t / \hat{y}_i^t \,, \tag{9.14}$$

and modify each of $\hat{b}_{ij}$ in the j-th sector proportionately by the factor α_i^t.

$$\hat{\hat{b}}_{ij} = \alpha_i^t \hat{b}_{ij} \,. \tag{9.15}$$

This modification is repeated until α_i^t in equation (9.15) converges to unity for all the n sectors. In this method, since the modification factor α_i^t varies from year to year, so does the value of b_{ij} accordingly. The stability of the coefficient $\hat{b}_{ij}^t$ is examined by comparing from year to year the modified values of b_{ij}^t.

This method assures greater conformity with observed data in that the coefficient of converter changes over time. However, insofar as the initial value for iterative computation has to be obtained from the estimate of the least squares method, it is still subject to instability due to multi-collinearity. Moreover, since the improvement is made through an iterative method of computation the problem remains unsolved that the uniqueness of the solution of convergence may not be assured.

9.3 Construction of Converters

1. Formulation of the Converter and the Estimation Procedure

As far as the coefficient of converter is conceptualized using a probability model of basic equations (9.10), in addition to aforementioned methodological deficiencies the method is still subject to an important criticism. The problem is that equations (9.10) themselves formally represent demand and supply balances and consequently they overlap structural equations of demand and supply already prescribed in the model. In other words, the question at issue is whether or not the independence of the disturbance term of equation (9.10) between items and from time to time can be guaranteed.

As stated earlier, we take the position that the converters are not by themselves structural equations which indicate demand and supply balances but rather a cementing device used to maintain the conformity between production and demand sectors which tend to diverge in a fairly aggregate multi-sectoral model. Having explained this basic standpoint, let us proceed to explain the estimation procedure of our converter.

Suppose that the data f_i^t, p_i^t, η_j^t ($i = 1, \ldots n, j = 1, \ldots, m$) are available for each period necessary to estimate B^t ($t = 1, \ldots, k$), and that the detailed matrix of final demand having $p_i^t r_{ij}^t$ in i-th row and j-th column is available only for the first and k-th periods.

Define y_i, z_j, x_{ij} as follows:

$$
\begin{aligned}
y_i^t &= p_i^t f_i^t \;, \\
z_j^t &= \eta_j^t g_j^t \;, \qquad (9.16) \\
x_{ij}^t &= p_i^t r_{ij}^t \,.
\end{aligned}
$$

That is, equations (9.16) imply that, in the t-th period, the i-th production sector supplies the value x_{ij}^t of goods to the j-th final demand item, and supplies altogether the value y_i^t of goods, and conversely the j-th final demand item demands all in all the value z_i^t of goods. Thus we may write,

(9.17) $$y_i^t = \sum_{j=1}^{m} x_{ij}^t , \qquad (i = 1, \ldots, n)$$

(9.18) $$z_j^t = \sum_{i=1}^{n} x_{ij}^t . \qquad (j = 1, \ldots, m)$$

Now, an element b_{ij}^t of matrix $\boldsymbol{B}$ in the i-th row and the j-th column is expressed as

(9.19) $$\hat{b}_{ij}^t = r_{ij}^t / g_j^t = x_{ij}^t / p_i^t g_j^t .$$

Assuming that between the estimate $\hat{b}_{ij}^t$ of b_{ij}^t and the estimate $\hat{x}_{ij}^t$ of x_{ij}^t the following relationship is valid

(9.20) $$\hat{b}_{ij}^t = \hat{x}_{ij}^t / p_i^t g_j^t ,$$

and, based on this relationship, let us estimate $\boldsymbol{B}^t$ indirectly by estimating $\boldsymbol{X}^t$ instead of trying to estimate $\boldsymbol{B}^t$ directly.

In estimating x_{ij}^t let us assume the following.

For any t-th period between the first and k-th periods, the ratio c_{ij}^t of the i-th product to the total demand z_j^t of the j-th final demand sector and the ratio d_{ij}^t of the i-th item of final demand to the total supply y_i^t of the i-th production sector can be approximated as follows when the data of x_{ij}^1, x_{ij}^k, y_i^1, y_i^k, z_j^1, z_j^k are given:

(9.21) $$c_{ij}^t = \left(1 - \frac{t-1}{k-1}\right)(x_{ij}^1 / p_i^1)/(z_j^1 / \eta_j^1) + \left(\frac{t-1}{k-1}\right)(x_{ij}^k / p_i^k)/(z_j^k / \eta_j^k) ,$$

(9.22) $$d_{ij}^t = \left(1 - \frac{t-1}{k-1}\right) x_{ij}^1 / y_i^1 + \left(\frac{t-1}{k-1}\right) x_{ij}^k / y_i^k .$$

Equations (9.21) and (9.22) are respectively approximate of rates of change of structures of supply c_{ij}^t and demand d_{ij}^t approximated by the average rate of change between the first and k-th periods. This assumption should not be unreasonable if the first is not far removed from each k-th period.

The amount x_{ij}^t is estimated to take the value $\hat{x}_{ij}^t$ so that it satisfies the vertical sum y_i^t and horizontal sum z_j^t and at the same time minimizes the sum of squares of deviations from respective interpolated values of equations (9.21) and (9.22).

That is, under the constraints

(9.23) $$y_i^t = \sum_{j=1}^{m} x_{ij}^t \qquad (i = 1, \ldots, n) \ ,$$

(9.24) $$z_j^t = \sum_{i=1}^{n} x_{ij}^t \qquad (j = 1, \ldots, m\text{-}1) \ ,$$

the estimate $\hat{x}_{ij}^t$ will be obtained by finding a value which will minimize the following quantity,

(9.25) $$Q^t = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} [(\frac{\hat{x}_{ij}^t}{d_{ij}^t y_i^t} - 1)^2 + (\frac{\hat{x}_{ij}^t / p_i^t}{c_{ij}(z_j^t / \eta_j^t)} - 1)^2] \ .$$

The following points are worthy of attention here: (1) in the case of $c_{ij}^t = 0$ and $d_{ij}^t = 0$ we presume that $x_{ij}^t = 0$ and do not include it in the objective function. This is because it is evident *a priori* that products are not supplied from the j-th production sector to the i-th final demand sector, and (2) the reason why equation (9.24) applies to cases $j = 1, \ldots, m-1$ is because once x_{ij}^t is determined then, by the definition of the content of the final demand, all of the remaining factors will be determined uniquely since $i=1, \ldots, n$ and $j=1, \ldots, m-1$.

Denoting undetermined Lagrange multipliers by $\lambda_1, \ldots, \lambda_n; \mu_1. \ldots, \mu_m (\mu_m = 0)$, from the objective function

(9.26) $$V^t = Q^t - \sum_{i=1}^{n} \lambda_i (y_i^t - \sum_{j=1}^{m} \hat{x}_{ij}^t) - \sum_{j=1}^{m} \mu_j (z_j^t - \sum_{i=1}^{n} \hat{x}_{ij}^t) \ .$$

As the first order conditions which are necessary conditions for the minimum of $\hat{x}_{ij}^t$ we have:

(9.27)
$$\frac{\partial V^t}{\partial \hat{x}_{ij}^t} = \frac{1}{d_{ij}^t y_i^t} (\frac{\hat{x}_{ij}^t}{d_{ij}^t y_i^t} - 1) + \frac{(1/p_i^t)}{c_{ij}^t (z_j^t / \eta_j^t)} [\frac{\hat{x}_{ij}^t / p_i^t}{c_{ij}^t (z_j^t / \eta_j^t)} - 1] + \lambda_i + \mu_j = 0 \ ,$$
$$(i = 1, \ldots, n; \ j = 1, \ldots, m; \ \mu_m = 0) \ .$$

(9.28) $$\frac{\partial V^t}{\partial \lambda_i} = -y_i^t + \sum_{j=1}^{m} \hat{x}_{ij}^t = 0 \quad (i = 1, \ldots, n) \ ,$$

(9.29) $$\frac{\partial V^t}{\partial \mu_j} = -z_j^t + \sum_{i=1}^{n} \hat{x}_{ij}^t = 0 \quad (j = 1, \ldots, m\text{-}1) \ .$$

The first order conditions (9.27), (9.28) and (9.29) form the first order simultaneous equations with $n \times m + n + m - 1$ variables for each of $\hat{x}_{ij}^t$, λ_i, and μ_j. The rank of the coefficient matrix is generally $n \times m + n + m - 1$ and thus x_{ij}^t can be uniquely determined.

The objective function V^t is the second order function of $\hat{x}_{ij}^t$, λ_i, and μ_j, and thus the solutions of this system of first order simultaneous equations will assure that the objective function V^t will reach a unique extreme value. While holding the coefficients of λ_i and μ_j as zero, if you allow $\hat{x}_{11}{}^t$ and $\hat{x}_{22}{}^t$ approach infinity and $\hat{x}_{12}{}^t$ and $\hat{x}_{21}{}^t$ approach negative infinity, then the value of function V^t would approach infinity.

Therefore, x_{ij} obtained from this system of simultaneous equations is the solution which minimizes Q^t of equation (9.25) under the conditions of equations (9.23) and (9.24).

Next, by rewriting the first order conditions let us reduce a simultaneous system of $n+m-1$ simultaneous equations with respect to λ_i and μ_j.

(9.30)

$$[(\frac{1}{d_{ij}{}^t y_i{}^t})^2 + (\frac{1/p_i{}^t}{c_{ij}{}^t(z_j{}^t/\eta_j{}^t)})^2]\,\hat{x}_{ij}{}^t = [\frac{1}{d_{ij}{}^t y_i{}^t} + \frac{1/p_i{}^t}{c_{ij}{}^t(z_j{}^t/\eta_j{}^t)}] - \lambda_i - \mu_j$$

$$(i=1,\ldots,n;\ j=1,\ldots,m;\ \mu_m=0),$$

$$\sum_{j=1}^{m}\hat{x}_{ij}{}^t = y_i{}^t \qquad (i=1,\ldots,n),$$

$$\sum_{i=1}^{n}\hat{x}_{ij}{}^t = z_j{}^t \qquad (j=1,\ldots,m\text{-}1).$$

If we define such that

(9.31) $$S_{ij} = \frac{(d_{ij}{}^t y_i{}^t)^2\,(c_{ij}{}^t(z_j{}^t/\eta_j{}^t))^2}{(d_{ij}{}^t y_i{}^t)^2 + (c_{ij}{}^t(z_j{}^t/\eta_j{}^t))^2},\quad l_{ij} = \frac{1}{d_{ij}{}^t y_i{}^t} + \frac{1/p_i^t}{c_{ij}{}^t(z_j{}^t/\eta_j{}^t)},$$

then equation (9.30) will be rewritten as

(9.32) $$\hat{x}_{ij}{}^t = S_{ij}\cdot l_{ij} - S_{ij}\lambda_i - S_{ij}\mu_j \qquad (i=1,\ldots,n;\ j=1,\ldots,m;\ \mu_m=0).$$

Substituting (9.32) into the last two equations of (9.30), we have

(9.33) $$\sum_{j=1}^{m} S_{ij}l_{ij} - \lambda_i\sum_{j=1}^{m} S_{ij} - \sum_{j=1}^{m-1}\mu_j S_{ij} = y_i{}^t \qquad (i=1,\ldots,n),$$

(9.34) $$\sum_{i=1}^{n} S_{ij}l_{ij} - \sum_{i=1}^{n}\lambda_i S_{ij} - \mu_j\sum_{i=1}^{n} S_{ij} = z_j{}^t \quad (j=1,\ldots,m\text{-}1).$$

By substituting solutions λ_i and μ_j of simultaneous equations (9.33) and (9.34) into equation (9.32), we can get a solution of $\hat{x}_{ij}{}^t$. Then by substituting the estimate $\hat{x}_{ij}{}^t$ into (9.20), we can get matrix $\hat{\boldsymbol{B}}^t$ uniquely.

The concept of matrix $\boldsymbol{B}^t$ presented here is based on a breakdown of final demand which is interpreted as a transaction table between production sectors and final demand sectors. In order to estimate equation (9.26), the value of final demand by sectors y_i^t are necessary. In case of the Brookings model, the time-series data of y_i^t were estimated from the data of an input coefficient matrix in a bench-mark year and values of sectoral productions in order to supplement the paucity of the data of y_i^t. In this case, however, input coefficient matrices are subject to instability as are coefficients of converters.

Variations of input coefficients need to be analyzed in terms of two aspects: (1) changes in technological structure of the supply side caused by technological changes or, more specifically, enlargement of scales of production and changes in product-mix, and (2) changes in demand structure stemming from changes in relative prices and increases in income. At the present state of art, it is difficult to introduce explicitly factors which change input structures into analysis of multi-sectoral models which use a fairly aggretage input-output table.[6]

Leontief has suggested the possibility of analyzing changes in input coefficients due to technological changes in the closed system by introducing the concept of productivity coefficient and production coefficient. Such changes are conceptualized in his suggested approach as a compound effect of the factor A_j (productivity coefficient) which transmits a change in an inputs of a certain industry and the factor A_i(production coefficient) which changes proportionately input structure of all the industries by substitution of inputs.

It is the RAS method developed by Stone which is basically a method of interpolating the changes in input coefficients by replacing formally Leontief's productivity effect and production effect respectively by a fabrication effect and a substitution effect.[7].

For our model, it will be necessay to estimate input coefficients and converters for interim years between 1955, 1960 and 1965 for which input-output tables are unavailable.

While we need to incorporate the laws which regulate changes in input structures into our model explicitly, we nevertheless have to rely at the present stage on some form of interpolation in our four sectoral model. We also attempted an application of the RAS method, but we were unable to get results satisfactory for our purposes. This failure seems to be related to the fact that the RAS method depends on an iterative computational procedure.

We, therefore, extended the estimation procedure of converters described above to intermediate input sectors. In this case, for the data of y_i^t we used values of sectoral productions in place of the total final demand, and for vector Z^t the sum of intermediate inputs were added to elements of the vector. Input coefficients can be computed according to their definition after elements of intermediate transactions $\hat{x}_{ij}$ ($i = 1, \ldots, 4; j = 1, \ldots, 4$) have been estimated.

Our method of estimation of input coefficients, is also subject to the criticism that it is basically an interpolation formula like the RAS method. This problem deserves further research especially where the sectoral classification of the model is being elaborated.

2. Estimation

Data editing plays an integral role in estimating converters and input coefficients for the model based on equation (9.26).

There are three input-output tables available for our use; those for the years 1955, 1960, and 1965. Although the 1955 table is not exactly comparable with the other two tables, we exploited the data of all three tables to the extent that the precision of the data permitted, to make an integrated classification of four major sectors.

Another important issue relates to the question of how to construct the data for interim years for which input-output tables are not available. We have to rely on the National Income Statistics for those years. Since the National Income Statistics are compiled on the "national" concept, while input-output tables are dependent on the "domestic" concept, some adjustments between the two sources of data are necessary. But even after such conceptual adjustments are made there still remain in the currently available data some statistical discrepancies.

In compiling the data for our model, we decided to reclassify and modify the National Income Statistics in order to connect them to input-output tables. It is sufficient here to note some points concerning such adjustment which directly pertain to the estimation of converters.

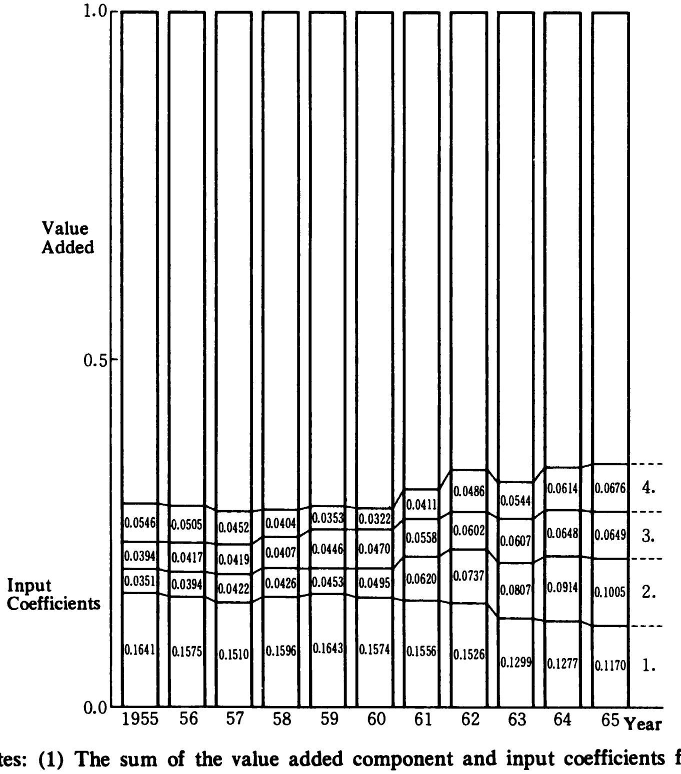
(1) Production sectors in input-output tables are classified into four major sectors (agriculture-forestry-fisheries, light manufacturing industries, heavy manufacturing industries, and commercial and service industries which are the same in the sectoral classification of our model.

(2) The table of value added of the input-output table lumps together under the title of operating surplus such items as income from unincorporated enterprises, income from property, income from private corporation, and government's income from property and entrepreneurship. Therefore, details of value added are supplemented by the National Income Statistics after necessary conceptual adjustments are made between "national" and "domestic" concepts.

(3) Even after conceptual adjustments were made, there still remained considerable statistical discrepancies between input-output tables and the national income statistics of the order of nearly 7 percent in terms of the

total value of production for the 1965 data and 7 percent in terms of the total intermediate inputs for the same data. Since our data are based primarily on input-output tables, we make them consistent by adjusting the national income statistics item by item. The ratios used for adjustment for 1965 are also applied to national income statistics of other years for which input-output tables are unavailable.

Figure 9.2 THE COMPOSITION OF INPUTS TO SECTOR 1—VALUE ADDED RATIO IN SECTOR 1 AND INPUT-COEFFICIENTS FROM OTHER SECTORS INTO SECTOR 1



Notes: (1) The sum of the value added component and input coefficients from other sectors is unity.

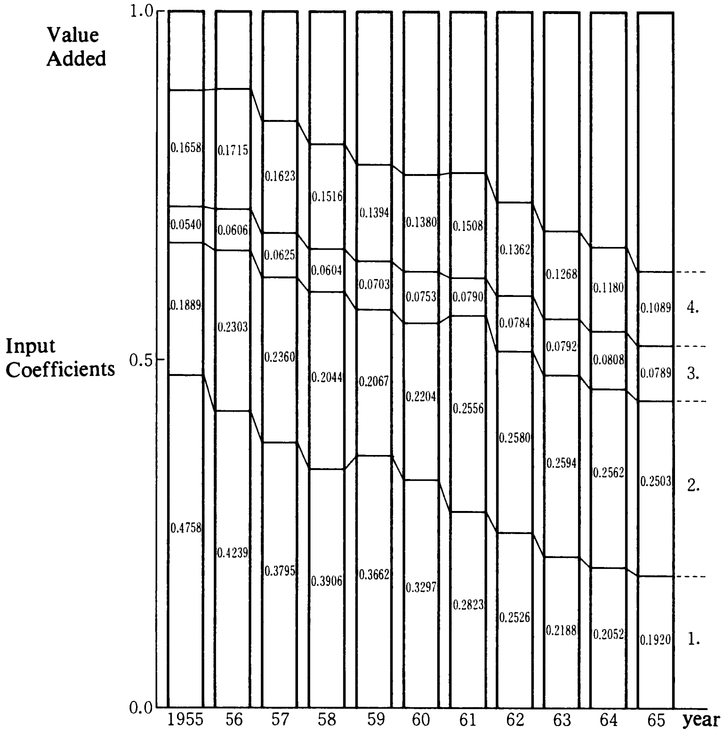
(2) The numbers 1, 2, 3 and 4 of input coefficients represent, respectively, the input from Sector 1, from Sector 2, from Sector 3 and from Sector 4 to the agricultural sector (Sector 1). The figures of input coefficients for each year represent the estimated input-coefficients from various sectors to the agricultural sector.

(3) For sources of the data used for this computation, see the explanation of estimation procedures in Section 3.2 of this chapter.

(4) For analysis of personal consumption expenditures we tabulated estimates of transactions between the four industrial sectors and five major consumption expenditure items (i.e. food, clothing, light and fuel, housing, and miscellaneous) for the three years 1955, 1960 and 1965 based on the basic table and the item classification table of the Family Expenditure Survey. This was because it was necessary to have converter coefficients for each of the expenditure items in order to analyze consumption demand.

(5) Similarly for total domestic capital formation, we estimated transaction tables for 1955, 1960 and 1965 for each of the components; private investment for productive facilities, personal investment for housing, and government's fixed capital formation.

Figure 9.3 THE COMPOSITION OF INPUTS TO SECTOR 2—VALUE ADDED IN SECTOR 2 AND INPUT-COEFFICIENTS FROM OTHER SECTORS INTO SECTOR 2



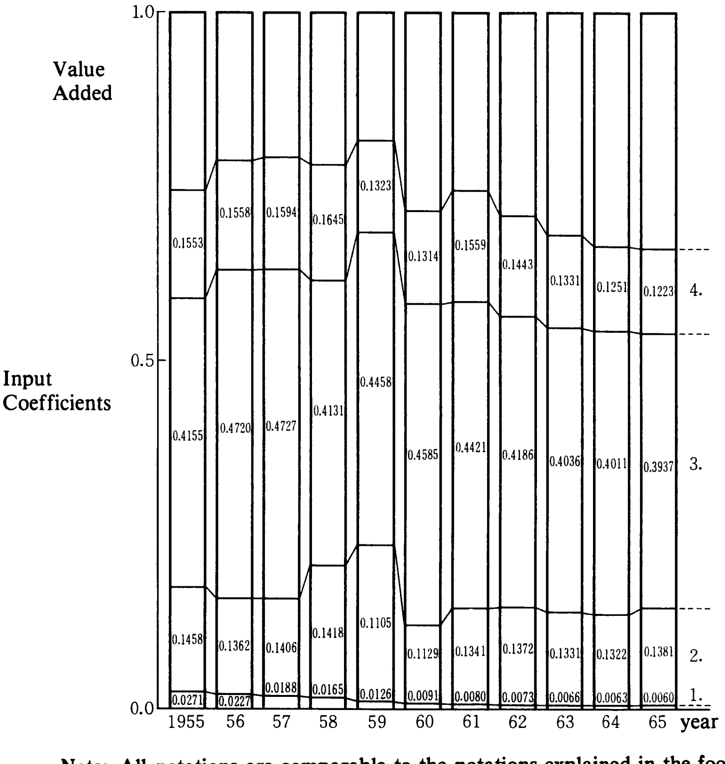
Note: All notations are comparable to the notations explained in the footnotes to Figure 9.2 except that the value added is the value added of Sector 2 and the input coefficients are the input coefficients to Sector 2 from various sectors.

(6) As for the prices of final demand items we used itemized implicit deflators from the National Income Statistics, and for production deflators we used the data integrated from 60 sectoral deflators. Since these two series of deflators do not necessarily conform to one another, the column sums of estimated converters ***B*** will not necessarily become unity.

Although many of the data problems described above were left unsolved, we attempted to estimate matrices ***B*** from the data of sectoral productions, final demand and intermediate inputs which are all adjusted to the basic classification scheme of input-output tables.

Since three input-output tables were available for use, we used the three

Figure 9.4 THE COMPOSITION OF INPUTS TO SECTOR 3—VALUE ADDED IN SECTOR 3 AND INPUT COEFFICIENTS FROM OTHER SECTORS INTO SECTOR 3



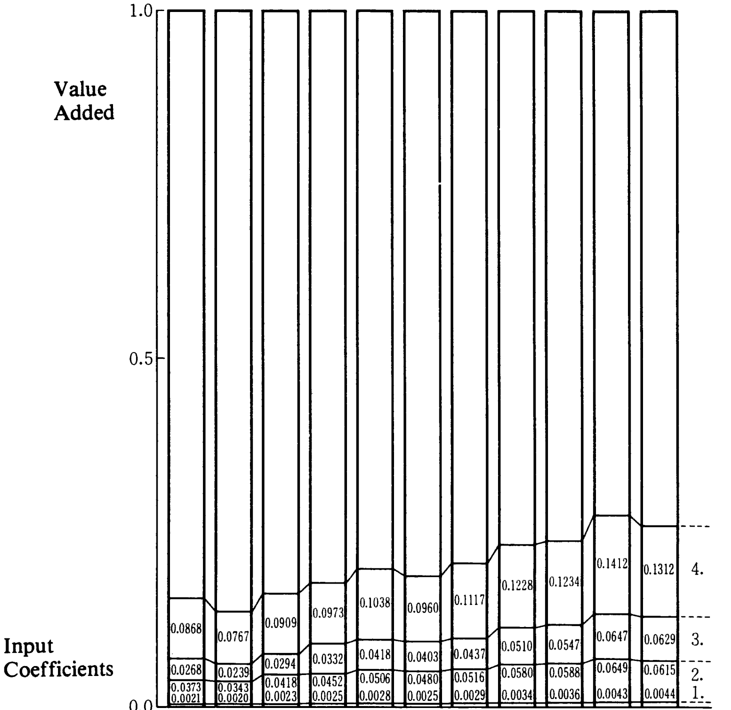
Note: All notations are comparable to the notations explained in the footnotes to Figure 9.2 except that the value added is the value added of Sector 3 and the input coefficients are the input coefficients to Sector 3 from various sectors.

years for which they are available as bench-mark years. We sub-divided the period of 11 years of observation into two sub-periods using 1960 as a dividing point. There are four production sectors and 17 demand sectors including sectors of intermediate inputs.

Using the estimated converters, we can obtain time-series of integrated input-output tables of four sectors from 1955 to 1965.

Figures 9.2 through 9.5 illustrate time-series changes of input coefficients. The changes in input coefficients are shown in the form of changes in input vectors of production sectors. As seen from the figures, each of the input coefficients is not necessarily stable. Changes in light manufacturing industries (sector 2) and service industries (sector 4) are particularly apparent.

Figure 9.5 THE COMPOSITION OF INPUTS TO SECTOR 4—VALUE ADDED IN SECTOR 4 AND INPUT COEFFICIENTS FROM OTHER SECTORS INTO SECTOR 4

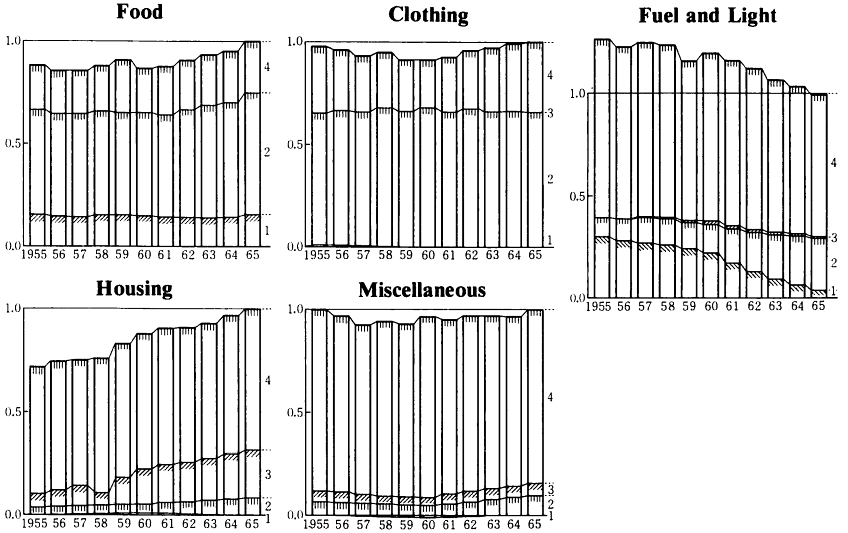


Note: All notations are comparable to the notations explained in the footnotes to Figure 9.2 except that the value added is the value added of Sector 4 and the input coefficients are the input coefficients to Sector 4 from various sectors.

While in the light manufacturing sector the ratio of value added has shown a remarkable increase, the service sector has, in contrast recorded a substantial decline. Common in all sectors is the tendency of intermediate inputs to increase noticeably from the second and third sectors. The input ratio from the fourth sector shows an increasing trend but it is rather modest. Movements in input coefficients deserve re-examination using more autonomous information based on more elaborately classified data.

Figure 9.6, panels *a* through *e*, shows converter coefficients b_{ij} for five consumption expenditure items. The value of $\sum_{i=1}^{n} \hat{b}_{ij}{}^{t}$ is not necessarily unity in this case. This fact is interpreted as reflective the aforementioned in-

Figure 9.6 THE COMPOSITION OF EACH MAJOR ITEM OF PRIVATE CONSUMPTION BROKEN DOWN BY COMPONENT COMMODITIES OF VARIOUS SECTORS



Notes: (1) The number 1, 2, 3 and 4 on the left side of each panel represent Sectors 1, 2, 3 and 4 whose commodities compose the major consumption items of each panel.

(2) The vertical length of the rectangle measures the coefficient of the converter which converts the quantity of each final demand item into commodities of various sectors. And these coefficients can also be used as weights used to compute the price of each consumption item from the prices of commodities of various sectors.

(3) The sum of converter coefficients, as illustrated by the total length of each rectangle, may not be unity because of the partial lack of consistency between the data of output deflators and of deflators of consumption items.

(4) For sources of the data used for this computation, see the explanation of estimation procedures in section 2 of this chapter.

consistencies associated with the price data. To examine changes in $\hat{b}_{ij}^t$ is itself to check the changes in demand structures after the fact. However, the validity of estimated coefficients can be assessed legitimately only when these coefficients are incorporated within the system of the entire model.

Let us present the converted prices, by demand item, of supply prices in the production sectors listed in Table 7.6 in Chapter 7.

The result of such a conversion is presented in Table 9.3. The validity of this conversion also has to be judged in connection with the total structure of the model, in which the quantity and price converters will play an integral role as shown by the attached flow chart of our model.

Notes to Chapter 9

1) The activity of any industrial sector is defined on the basis of technology by which a certain commodity is produced. We call such a classification of industrial activity as a "commodity-based classification."
2) Fisher and Others (1965) and Duesenberry and Others (1965).
3) Kresge (1969).
4) Kresge, *op.cit.*
5) Fisher and Others, *op.cit.*
6) Leontief (1941).
7) University of Cambridge (1963).

Table 9-3 ESTIMATED SUPPLY PRICES OF DEMAND ITEMS

Year	(1) Food		(2) Clothing		(3) Fuel and light		(4) Housing		(5) Miscellaneous		(6) Business Consumption	
	OB	ES	OB	ES	OB	ES	OB	ES	OB	ES	OB	ES
1955	0.7160	0.7575	0.8260	0.8818	0.8310	0.8837	0.4830	0.5253	0.8340	0.7312	0.6740	0.7430
1956	0.7020	0.7178	0.8290	0.8501	0.8410	0.8426	0.5350	0.5357	0.8720	0.6897	0.6890	0.6866
1957	0.7250	0.7368	0.8310	0.8465	0.9070	0.9221	0.5760	0.5861	0.9000	0.7108	0.6970	0.7251
1958	0.7150	0.7176	0.8160	0.8194	0.8910	0.8921	0.6120	0.6196	0.8750	0.7052	0.7020	0.7099
1959	0.7170	0.7014	0.8080	0.7869	0.8710	0.8699	0.6580	0.6563	0.8740	0.7247	0.7240	0.7192
1960	0.7400	0.7009	0.8220	0.7702	0.9130	0.8826	0.7040	0.6712	0.8950	0.7196	0.7480	0.7165
1961	0.7800	0.7712	0.8530	0.8429	0.9520	0.9236	0.7640	0.7336	0.9220	0.7510	0.7820	0.7486
1962	0.8300	0.8197	0.8950	0.8824	0.9760	0.9448	0.8000	0.7698	0.9440	0.8066	0.8410	0.7961
1963	0.8970	0.8966	0.9390	0.9385	0.9870	0.9849	0.8590	0.8599	0.9630	0.8901	0.8920	0.8928
1964	0.9730	0.9802	0.9730	0.9663	0.9910	0.9892	0.9300	0.9501	0.9800	0.9702	0.9290	0.9402
1965	1.0000	1.0212	1.0000	1.0287	1.0000	1.0307	1.0000	1.0377	1.0000	1.0331	0.9980	1.0383
Year	(7) General Government Consumption Expenditure		(8) Private fixed Capital Formation		(9) Housing Investment		(10) Fixed Capital Formation by Government		(11) Inventory Increases		(12) Exports	
1955	0.4860	0.5276	0.7970	0.8934	0.6180	0.6953	0.7350	0.8261	0.8770	0.9044	1.0020	1.0867
1956	0.5110	0.5105	0.8860	0.8943	0.6430	0.6488	0.7920	0.7989	0.9400	0.9533	1.0340	1.0494
1957	0.5540	0.5650	0.9570	0.9588	0.6930	0.6933	0.8460	0.8468	0.9430	0.9507	1.0260	1.0392
1958	0.5800	0.5821	0.9300	0.9921	0.6840	0.7335	0.7990	0.8556	0.8960	0.9149	0.9496	0.9720
1959	0.6030	0.6049	0.9350	0.9158	0.7240	0.7083	0.8140	0.7966	0.9240	0.9052	0.9790	0.9600
1960	0.6600	0.6368	0.9570	0.8806	0.7610	0.6978	0.8430	0.7735	0.9200	0.8640	0.9860	0.9243
1961	0.7230	0.6930	0.9860	0.9435	0.8770	0.8383	0.9020	0.8625	0.9450	0.9265	0.9670	0.9416
1962	0.7740	0.7406	0.9860	0.9584	0.9150	0.8897	0.9350	0.9091	0.9470	0.9344	0.9540	0.9339
1963	0.8400	0.8379	0.9830	0.9961	0.9440	0.9581	0.9420	0.9555	0.9780	0.9818	0.9750	0.9811
1964	0.9200	0.9305	0.9910	0.9890	0.9780	0.9844	0.9740	0.9764	0.9850	0.9913	0.9940	0.9982
1965	1.0020	1.0383	1.0000	1.0461	1.0010	1.0493	1.0000	1.0472	1.0010	1.0233	1.0000	1.0369

Note: OB and ES, respectively, stand for the observed values and estimates of supply prices.
The units of prices are standardized so as to make the observed prices for 1965 unity.

Chapter 10
Estimation of Consumption Demand

In our model, final demand is classified through the input-output scheme into household and business consumption, private investment, government expenditure and external demand (exports). These categories are further subdivided into demand for outputs of various industrial sectors.

This chapter deals among components of the final demand with the determination of a vector of consumption demand in the household sector. The data used in our analysis are concerned primarily with five major categories of private consumption expenditure details of which are obtainable from the National Income Statistics. These are food, clothing, light and fuel, housing and a "miscellaneous" category.

Viewed from the standpoint of general equilibrium, personal incomes and personal consumption are simultaneously determined. It is convenient however to analyze changes in consumption assuming income as given for our present purpose of constructing a consumption part of the model. The interdependent relationship will be incorporated eventually when the entire model is constructed.

A complete theory of household behavior could be developed by first analyzing the income-leisure preference of the given number of employable members of the household, or in short, by formulating a model of household labor supply. It would then be necessary to consider the division of income between consumption and saving and to proceed finally to analyze the distribution of consumption expenditure between different expenditure categories and commodities. As a theoretical construction, one has to deal with these three stages of analysis simultaneously. Here, however, for simplicity, the model is formulated in such a way that the rate of labor force participation within a household is given exogenously.

Neither is the division of income between consumption and saving treated in this model within the framework of consumer preference because neat and articulate model on this aspect of household behavior has not been developed yet fully that we can utilize for our purposes. We use, for the moment, instead an empirical macro relationship of which the degree of approximation has been ascertained.

$$E = a + by + E_{-1}.$$

Personal saving separated from personal income according to this equation now comprises, together with government's saving, the national saving which is presumably equal to national investment. On the other hand, personal consumption of a variety of expenditure items constitutes a vector of household consumption which is an important component of the final demand.

However, the classification of consumption items in the National Income Statistics and that of sectors in the Input-Output Tables are not identical. The itemized consumption demand, therefore, has to be converted to the sectoral consumption demand using a converter specially built for this purpose. Through this conversion, consumer demand for goods and services will now compose, together with demands for investment goods and intermediate goods, the total demand for all the industrial sectors, and this will interact with the total supply regulated by supply side conditions.

10.1 The System of General Equilibrium Type Consumer Demand Functions

The allocation of the total consumption budget into spending on various items is described by a system of consumption demand functions. As a component of the entire model designed to analyze the general interdependence among sectors of the economy, we also need to use a system of general equilibrium type consumer demand functions. So, we use a system of demand functions which is based on the notion that the demand for a good is a function not only of its price but also of the prices of all other goods and of the total volume of expenditures made by consumers.

In deriving the system of general equilibrium type demand functions, we need to specify the preference function of consumers. From the system of structural equations consisting of equalities between marginal rate of substitution and relative price ratios and a budget constraint we derive a set of demand functions as a reduced form which corresponds to a vector of consumer demand. The parameters of demand functions as reduced forms are determined by preference parameters of the structure.

The number of variables in a general equilibrium type demand equation varies with the number of items of consumption unlike a partial equilibrium type demand function. Therefore the statistical estimates of coefficients in the equation tend to be subject to the risk of biases due to multi-collinearity. When we have the preference function explicitly spelled out, however, since both the sign and the range of meaningful values of preference parameters are theoretically constrained by necessary and sufficient conditions of utility maximization, theoretical interpretation of parameters of reduced form equations can readily be made and we can avoid theoretically meaningless numerical functions.

10.2. The Linear Expenditure System

We have examined in the past various alternative specific forms of preference function in the light of usefulness for empirical analysis.[1] In this book, we chose to use the Bernoulli-Laplas type preference function, which has the virtue of formal simplicity as well as a sufficiently high degree of empirical approximation.

Expressing the utility indicator U, we can write the preference function for the five items as

$$U = \sum_{i=1}^{5} \alpha_i \log(a_i + q_i), \tag{10.1}$$

where q_i represents the quantity consumed of each item and a_i and α_i are parameters.

Differentiating equation (10.1) with respect to the consumed quantity of each item q_i, marginal utility becomes

$$U_i = \frac{\partial U}{\partial q_i} = \frac{\alpha_i}{a_i + q_i}. \tag{10.2}$$

From this, it has to be the case that $\alpha_i > 0$ in order to satisfy the condition of positive marginal utility.

Denoting the price of each good by p_i, equalities of marginal utilities are derived from equation (10.2) as

$$\frac{\alpha_1}{(a_1 + q_1)p_1} = \frac{\alpha_2}{(a_2 + q_2)p_2} =, \ldots, = \frac{\alpha_5}{(a_5 + q_5)p_5} = \lambda, \tag{10.3}$$

where λ is the common value of equilibrium. Transposing denominators and numerators of equations (10.3), we may write

$$\frac{a_1}{\alpha_1} p_1 + \frac{1}{\alpha_1} p_1 q_1 =, \ldots, = \frac{a_5}{\alpha_5} p_5 + \frac{1}{\alpha_5} p_5 q_5. \tag{10.3'}$$

Expressing the expenditure for each item as $p_i q_i = E_i$, then equilibrium conditions are

$$(10.3)'' \qquad a_1 p_1 + E_1 = \frac{\alpha_1}{\alpha_2}(a_2 p_2 + E_2) =, \ldots, = \frac{\alpha_1}{\alpha_5}(a_5 p_5 + E_5).$$

Solving this simultaneously with the budget constraint

$$(10.4) \qquad E = E_1 + E_2 + \ldots + E_5 ,$$

we get the system of demand functions for different items as follows:

$$(10.5) \qquad \begin{aligned} E_1 &= \frac{\alpha_1}{\sum_i \alpha_i} E - \frac{\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5}{\sum_i \alpha_i} a_1 p_1 + \frac{\alpha_1}{\sum_i \alpha_i}(a_2 p_2 + a_3 p_3 + a_4 p_4 + a_5 p_5), \\ &\vdots \\ E_5 &= \frac{\alpha_5}{\sum_i \alpha_i} E - \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{\sum_i \alpha_i} a_5 p_5 + \frac{\alpha_5}{\sum_i \alpha_i}(a_1 p_1 + a_2 p_2 + a_3 p_3 + a_4 p_4). \end{aligned}$$

Since equations (10.5) are such that the expenditure for each item E_i is a linear function of total consumption expenditure E and all the prices p_i (i=1,...,5). *R.* Stone referred to them as a linear expenditure system.[2] This system, as it always holds with the general equilibrium type demand function, rigorously satisfies the required condition of equality between income and expenditures. And as will be seen clearly from the following form which is obtained by dividing both sides of each equation (10.5) by its own price

$$(10.6) \qquad \frac{E_1}{p_1} = q_1 = \frac{\alpha_1}{\sum_i \alpha_i}\frac{E}{p_1} - a_1 \frac{(\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)}{\sum_i \alpha_i} + \frac{\alpha_1}{\sum_i \alpha_i}\left(\frac{a_2 p_2 + a_3 p_3 + a_4 p_4 + a_5 p_5}{p_1}\right),$$

the criterion of 0 degree homogeneity of demand with respect to income and prices is also satisfied rigorously.

10.3. The Linear Expenditure System and Shifts in Preference

In equation (10.5), it is noted that the coefficient of the total consumption expenditure E is determined independently from prices p_i.

The fact that the slope of the expenditure expansion line obtained from the usual cross-sectional Family Budget data changes from year to year has been known from the 1930s when the notable analyses of Allen and Bowley were made, or even earlier.[3] This may suggest that the coefficient of E in a demand equation should change due to changes in prices from year to year. The coefficient of total consumption expenditure E can be obtained as a function of prices when the system of demand functions are derived from the

Gossen type preference function.

However, since the coefficient of E is independent from p_i in the case of the outright linear expenditure system, it may not be interpreted thus and consequently would not match the cross sectional data. Any theory should be capable of offering a consistent explanation for alternative data sets on the same object of observation. Frisch and Haavelmo called this property the autonomy of theories.[4] In this sense, the linear expenditure system expressed as (10.5) is deficient in terms of autonomy in that it is incapable of explaining both time-series and cross-sectional data both consistently.

We have examined this point closely in our earlier studies.[5] In terms of actual data analysis, we have to treat a household as a consumer. The number of household members usually varies in either time-series data or cross-sectional data. Therefore, we need to formulate our theoretical model in such a way that changes in household membership can explicitly be taken into account.

In the case of the preference function (10.1) the simplest method of adjustment is to decompose the parameter a_i as

$$a_i = a_{i0} + b_i m \qquad (m \text{ is the family size}). \tag{10.7}$$

Then the coefficient of price p_i of equation (10.5) would explicitly contain the variable m.

In the cross-sectional data of the Family Expenditure Survey a fairly strong correlation is usually observed between income and family size across different income groups. Therefore, because the change in the number of household members m across income groups is reflected in the term which contains p_i in equation (10.5), it is not impossible to infer that the regression coefficient of E_i on E changes from year to year.

However, the coefficient of E changes from year to year even though we separate the effect of E from the effect of m by means of multiple regression in which total consumption expenditure E and household members m are both included explicitly as independent variables. Thus, the gap between the linear expenditure system and cross-sectional data will not be bridged by merely introducing the effect of family size alone.

We have ascertained through our previous empirical studies that in order to offer a consistent and compatible explanation for time-series changes in cross-sectional data we have to admit the possibility of cross-sectional as well as inter-temporal shifts in consumers' preference itself.[6] We have also found further that the shift in preference depends on the accumulated consumption by each household of each item in the past. We call this the "habit formation effect".

Taking the preference function (10.1) for instance by decomposing parameter a_i we can write

$$a_i^t = a_{i0} + b_i m^t + c_i H_i^t . \tag{10.8}$$

Since the subscript t represents a unit time period, say a year, m^t implies the family size for t-th year, and H_i^t is a habit potential for i-th good for t-th year which may be measured by $H_i^t = \sum_{t=0}^{t-1} q_i^t$.

Substituting a_i^t of equation (10.8) into equation (10.5), the coefficient of p_i in equation (10.5) will include $b_i m^t + c_i H_i^t$. If we apply this equation (10.5) to cross sectional data, since the level of past consumption is generally higher the higher the present income level, the habit potential correlates closely and positively with income level, and as such the habit potential appears to be a function of income or total consumption expenditure E.

Therefore, the regression coefficient of consumption expenditure of each item E_i on total consumption E can now change from year to year, putting aside the effect of family size, because of differences in habit potential between income groups. The linear expenditure system and cross-sectional data are now finally cemented in this way.

Since our present model is applied to time-series data, shifts in preference through habit formation will be confined to inter-temporal shifts. The time-series shifts of demand functions have been treated conventionally by means of introducing a time-trend term. In our case too it is be conceivable to use a time-trend term to represent habit formation as a very crude appproximation since our habit formation term H_i^t also changes monotonically with the passage of time. But it is important to note that in our case we introduce the term H_i^t not merely for the purpose of formally allowing for secular shifts.

We introduce the habit formation term for the purpose of maintaining theoretical autonomy, in the sence that our model can prove a consistency between theory and measurement in cases where it applies both to time-series and to cross-sectional data. It should be born in mind that the linear expenditure system can preserve theoretical autonomy only when it allows explicitly for shifts in preference.

10.4 Estimating the System of Demand Functions

As can be seen clearly by substituting equation (10.8) into demand equations (10.5) for various items, consumption expenditure E_i and consequently quantities consumed q_i will change with changes in the family size m and habit potential H_i even though the total consumption expenditure E_i and prices p_i are kept constant. While the number of household members m is determind extraneously largely by sociological factors, habit potential H_i itself certainly changes from year to year through the accumulated effect of past consumption itself. Consequently, the consumption vector (q_1, q_2, ..., q_5) would not be maintained intact even though income, prices, and family size remain unchanged. In other words, the consumption vector is bound to change due to changes in habit formation.

In applying this system of demand functions to statistical data, we compute consumption expenditure per household on the one hand by dividing the aggregate total personal consumption expenditure and itemized expenditures obtained from the annual reports of National Income Statistics by the total number of households, and on the other compute the average family size by dividing the population by the number of households. Then combine E^t, E_i^t, and m^t thus obtained respectively with itemized consumption deflator p_i^t. In case when we integrate E_i and q_i estimated from the demand functions back into the total system of the model, we simply inflate them again by the total number of households.

Table 10.1 ACTUALLY MEASURED PREFERENCE PARATMETERS

	a_{i0}	b_i	c_i	α_i	λ			
Food	849356.81	−223747.52	−0.0426126	0.1978669	1955	240735.0	1962	732637.9
Clothing	−135583.07	23020.71	0.0064312	0.0297410	1956	313895.0	1963	842311.9
Fuel & light	59945.22	−16083.89	−0.0841886	0.010000	1957	353312.0	1964	946209.4
					1958	370390.2	1965	1039102.2
Housing	577876.99	−124827.65	−0.0555299	0.1935583	1959	409736.2		
Miscel-laneous	487519.73	−116077.72	−0.0787848	0.1479333	1960	512277.7		
					1961	611667.8		

Note: Estimates of parameters a_{i0}, b_i, c_i and α_i have been derived from the estimation of the following preference function

$$U = \prod^{i=1,5} (a_i + q_i)^{\alpha_i} ,$$

where U represents the preference indicator and q_i the quantity consumed of i-th item. This preference function takes into account specially the family size m and habit potential H_i for i-th item in the following way,

$$a_i = a_{i_0} + b_i m + c_i H_i .$$

λ represents estimates of the marginal utility of income.

The number of terms $a_i p_i$ in (10.5) corresponds to the number of consumption items. Our present model has five of them. But if we decompose a_i into two terms of family size and habit potential, then we would have 15 of them plus the term E, making 16 altogether. Consequently, the number of parameters contained in the system of reduced form demand functions for E_1 through E_5 will be as many as 80. Even though we apply the single equation least squares method for each of the reduced form functions for E_1 through E_5, in so far as mutual correlations among a set of so many variables are unavoidable the obtained estimates of parameters may well be biased due to the effect of multi-collinearity. The number of parameters increases as the number of items increases, and so does the danger of multi-collinearity. For this reason, application of the use of the least squares method to a reduced form demand function for each item is not appropriate for the purpose of analyzing the system of general equilibrium-type multi-item consumption demand functions.

Table 10.2 ACTUALLY MEASURED GENERAL EQUILIBRIUM-TYPE MULTI-ITEM DEMAND FUNCTIONS

$$\begin{aligned}
E_1 = {} & 0.3417E \\
& - 559148.0p_1 - 46326.0p_2 + 20482.0p_3 + 197449.0p_4 + 166576.0p_5 \\
& + (147297.0p_1 + 7865.0p_2 - 5495.0p_3 - 42651.0p_4 - 39661.0p_5)m \\
& + (0.02805p_1H_1 + 0.00220p_2H_2 - 0.02877p_3H_3 - 0.01897p_4H_4 - 0.02692p_5H_5)
\end{aligned}$$

$$\begin{aligned}
E_2 = {} & 0.0514E \\
& + 43621.0p_1 + 128620.0p_2 + 3079.0p_3 + 29678.0p_4 + 25038.0p_5 \\
& + (-11491.0p_1 - 21838.0p_2 - 826.0p_3 - 6411.0p_4 - 5961.0p_5)m \\
& + (-0.00219p_1H_1 - 0.00610p_2H_2 - 0.00432p_3H_3 - 0.00285p_4H_4 - 0.00405p_5H_5)
\end{aligned}$$

$$\begin{aligned}
E_3 = {} & 0.0173E \\
& + 14667.0p_1 - 2341.0p_2 - 58910.0p_3 + 9979.0p_4 + 8419.0p_5 \\
& + (-3863.0p_1 + 397.0p_2 + 15806.0p_3 - 2156.0p_4 - 2004.0p_5)m \\
& + (0.00074p_1H_1 + 0.00011p_2H_2 + 0.08273p_3H_3 - 0.00096p_4H_4 - 0.00136p_5H_5)
\end{aligned}$$

$$\begin{aligned}
E_4 = {} & 0.3342E \\
& + 283889.0p_1 - 45317.0p_2 + 20036.0p_3 - 387472.0p_4 + 162949.0p_5 \\
& + (-74785.0p_1 + 7694.0p_2 - 5376.0p_3 + 83105.0p_4 - 38798.0p_5)m \\
& + (-0.01424p_1H_1 + 0.00215p_2H_2 - 0.02814p_3H_3 + 0.03697p_4H_4 - 0.02633p_5H_5)
\end{aligned}$$

$$\begin{aligned}
E_5 = {} & 0.2555E \\
& + 216972.0p_1 - 34635.0p_2 + 15313.0p_3 + 147621.0p_4 - 362981.0p_5 \\
& + (-57157.0p_1 + 5881.0p_2 - 4109.0p_3 - 31888.0p_4 + 86425.0p_5)m \\
& + (-0.01089p_1H_1 + 0.00164p_2H_2 - 0.02151p_3H_3 - 0.01419p_4H_4 + 0.05866p_5H_5)
\end{aligned}$$

Note: Notations are:

E_i: consumption expenditure for each item for i-th item,
E: total consumption expenditure,
p_i: price of i-th item,
m: family size,
H_i: habit potential for i-th item.

We therefore use a particular method of estimation in this book which we have developed through our previous studies. The methodology is such that we attempt complete determination of numerical values of parameters just as many times as the number of equations (10.3).

Since the sets of estimates of preference parameters obtained through the complete determination method presumably take the form of a multi-dimensional Cauchy distribution, we take the median of its marginal distribution as an estimate of each parameter. We can approximate fairly well by using the set of preference parameters chosen by this method in computing reduced form parameters of demand functions. However, in order to reinforce the numerical consistency of the set of preference parameters, we first choose among the Cauchy estimates the ones which are highly reliable and at the same time satisfy sign conditions required by theory and set them as point constraints. And then estimate a numerical set of

Table 10.3 FITNESS OF MULTI-ITEM CONSUMPTION FUNCTIONS

		Food	Clothing	Fuel and Light	Housing	Miscelleneous	COR. COEF.
1955	ES.	202600. 3	42518. 4	13726. 3	61789. 4	73392. 6	. 9997
	OB.	195019. 5	44523. 0	13992. 8	61761. 9	75324. 9	
1956	ES.	198205. 1	49224. 1	13282. 1	73583. 9	81108. 6	. 9974
	OB.	200160. 0	47324. 3	13752. 5	62065. 2	77582. 7	
1957	ES.	196753. 7	51576. 9	13310. 2	69255. 8	84074. 8	. 9991
	OB.	202351. 1	50824. 9	13492. 2	63414. 3	84601. 3	
1958	ES.	197833. 0	53487. 2	13714. 0	63380. 2	86435. 5	. 9996
	OB.	211216. 7	51543. 8	13484. 2	67757. 1	91306. 3	
1959	ES.	204109. 5	56797. 9	14347. 6	64142. 4	92046. 4	. 9978
	OB.	215576. 9	53766. 6	13805. 9	75677. 2	97587. 8	
1960	ES.	226854. 3	61858. 7	15600. 1	84656. 3	112135. 9	. 9999
	OB.	219746. 3	62800. 5	14980. 6	84182. 4	109868. 9	
1961	ES.	223431. 7	65672. 5	15914. 5	87274. 6	122270. 9	. 9996
	OB.	220981. 6	66610. 4	15769. 1	90875. 1	119636. 3	
1962	ES.	229130. 0	69842. 3	16795. 3	100564. 5	132856. 5	. 9994
	OB.	226421. 5	71988. 7	17173. 4	95664. 8	134546. 0	
1963	ES.	234417. 6	73344. 2	17960. 6	105374. 4	142621. 8	. 9989
	OB.	233370. 7	75007. 7	18705. 5	103912. 8	150773. 3	
1964	ES.	249540. 1	77465. 5	19549. 3	115780. 2	160160. 1	. 9985
	OB	245302. 2	78199. 8	19921. 9	114380. 7	168642. 7	
1965	ES.	242528. 6	79522. 9	20588. 2	108689. 6	162461. 6	. 9987
	OB.	247377. 4	80288. 9	21815. 7	114013. 7	176156. 3	
COR. COEF.		. 9326	. 9924	. 9883	. 9523	. 9931	. 9977

Note: (1) Every consumption is evaluated at constant price in 1970.
(2) COR. COEF. stands for correlation coefficient. OB and ES represent observed value and estimated value respectively.

preference parameters by means of constrained weighted regression of the system of equilibrium equations using an index of precision of itemized data which is reflected in the outlook of the above observed Cauchy marginal distribution.

The estimates of preference parameters are presented in Table 10.1. Numerical demand functions by items, which are based on the results in Table 10.1 are shown in Table 10.2.

The goodness of fit of estimates of itemized consumption expenditures, obtained by controlling total consumption expenditure E and family size m, to actually observed expenditures is shown by Table 10.3. The result seems to be fairly satisfactory. Note, however, that this is the result of an interpolation test within the consumption block in isolation. The ultimate judgement on the goodness of fit would be ascertained from the result of the final test, which is made after integrating the system of consumption demand functions into the entire system of the economic model.

10.5 The Precision of Partial Equilibrium Approximation

In the preceding analysis, we have estimated a system of general equilibrium-type consumer demand functions using observed data. Now let us examine how closely the partial equilibrium-type demand function can approximate the general equilibrium-type demand functions.

When we write the demand function for each item derived from the linear expenditure system as

$$(10.6)\quad q_1 = \frac{\alpha_1}{\sum_i \alpha_i}\frac{E^t}{p_1{}^t} + \frac{\alpha_1}{\sum_i \alpha_i}\left(\frac{a_2 p_2{}^t + a_3 p_3{}^t + a_4 p_4{}^t + a_5 p_5{}^t}{p_1{}^t}\right) - a_1 \frac{(\alpha_2+\alpha_3+\alpha_4+\alpha_5)}{\sum_i \alpha_i},$$

the first term of the right hand side is the total consumption expenditure E divided by the price of the good in question p_1, the second term is the linear combination of prices for the rest of goods divided by the price of the good in question, and the third term is a constant.

As an example of the oft-used partial equilibrium-type demand function which has a form analogous with this, we may present a demand function

$$(10.9)\qquad q_1 = A_1 \frac{E^t}{P^t} + B_1 \frac{P^t}{p_1{}^t} + C_1 .$$

As a measurement for P^t, a general deflator of consumption or general consumers' price index is usually used. So, the first term in the right hand side is often treated simply as the real total consumption expenditure (or real household income) and the second term as relative prices. The second term could be interpreted to mean an approximation of a general equilibrium-type function in the sense that since the assumption of *ceteris paribus* clause may not be valid in time-series data and, consequently, prices of other goods have to be taken into account simultaneously. Klein has employed a similar method of approximation of general equilibrium in his aggregation procedure which we refered to earlier.

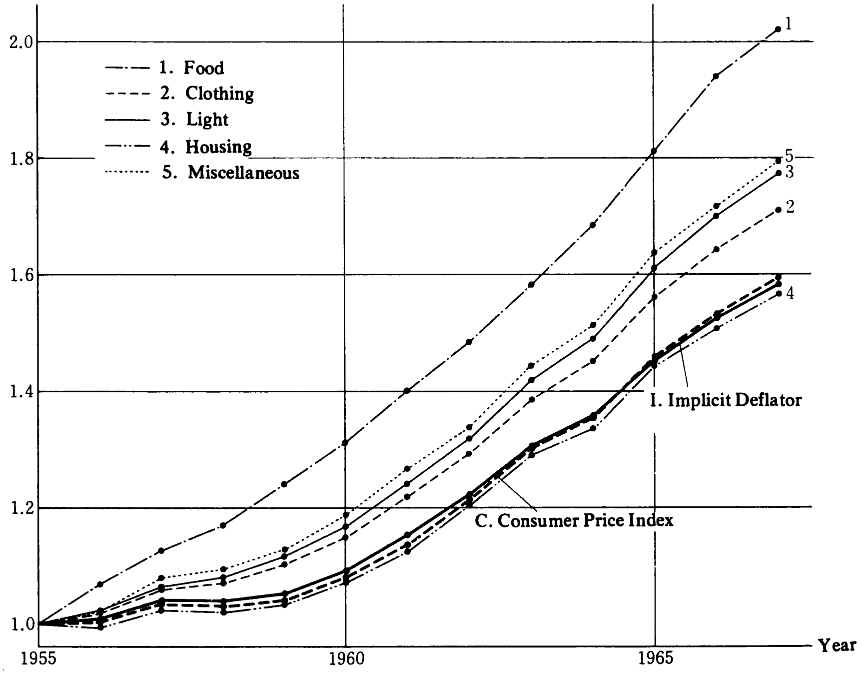
It can be seen from a comparison of (10.6) and (10.9) that the denominator of the first term of the right hand side is the price of the good in question in the case of general equilibrium-type demand function whereas it is the general price level in the partial equilibrium approach. This difference implies that the meaning of real purchasing power differs between the two approaches. This, however, may be adjusted easily by replacing P in (10.9) by p_1.

As for the second term, while the numerator in the case of the general equilibrium-type equation (10.6) is a linear combination of prices of other goods p_i weighted by respective preference parameters a_i it is represented in the case of the partial equilibrium-type equation (10.9) by a general price index as a proxy. Since the general price index is a kind of linear combination of all the prices weighted by the respective consumption expenditure or the quantities consumed either at a bench-mark time or a time of comparison, it resembles its counterpart in equation (10.6). They differ, however, according to whether or not the price of the good in question is itself included and what kind of weights are attached to prices. To put it differently, while the term of relative prices in the general equilibrium-type equation contains general price indices which vary with respect to specific items, the partial equilibrium-type equation merely uses the ordinary general price index.

Let us now examine quantitatively how closely the partial equilibrium-type demand function approximates the general equilibrium-type equation. To facilitate such a comparison, we compute the value of the numerator of the second term of the right hand side of the general equilibrium-type demand function for each year and for each item using the estimates of preference parameters a_i and then compare these series of estimates with the movements of the general implicit deflator of consumption and an official general consumers' price index. The results are shown diagramatically in Figure 10.1.

Figure 10.1 indicates that there are considerable differences from item to item in the value of the linear combination of other prices contained in the

Figure 10.1 A COMPARISON OF COMPOSITE PRICE INDICES DERIVED FROM MULTI-ITEM CONSUMPTION FUNCTIONS, CONSUMER PRICE INDEX, AND THE IMPLICIT DEFLATOR



Note: In analysis of consumption using partial equilibrium-type demand functions, a general price index such as CPI or the implicit deflator is ordinarily used. In the case of multi-item consumption functions on the other hand, we use deflators of individual consumption items explicitly. An examination of the degree of conformity between the composite price indices constructed on the basis of these deflators and all ordinary general price index will serve as a test of the legitimacy of the use of general price index in partial equilibrium-type consumption functions.

second term on the right hand side of equation (10.6). The differences between items increase over the period of observation and the pattern of change differs from year to year. In sum, the value of the linear combination deviates significantly from the movement of the ordinary general price index in all the items except expenditure on housing.

Our observation above has shown that the precision of the oft-used partial equilibrium approximation is not as high as it is usually thought. When we apply partial equilibrium-type equations such as (10.9) to the data for the purpose of demand projections, we often find that the estimated equations have a high degree of statistical significance, and to this extent, they are highly useful for projecting demand for specific commodities. But our earlier finding suggests that a set of these partial equilibrium-type equation could not serve as a substitute for general equilibrium analysis because they together would not approximate the performance of general equilibrium-type equations with an acceptable degree of precision.

The validity of Keynesian macro analysis has been questioned recently in the face of increasingly complex problems associated with the determination of prices. The level of prices in general is obviously not the only important problem. Changes in the relative price structure and its interplay with the structure of consumption are also important questions. It is desired, especially under these circumstances, that the general equilibrium theory itself is directly translated into empirical analysis.

10.6 Actually Measured Consumers' Preference

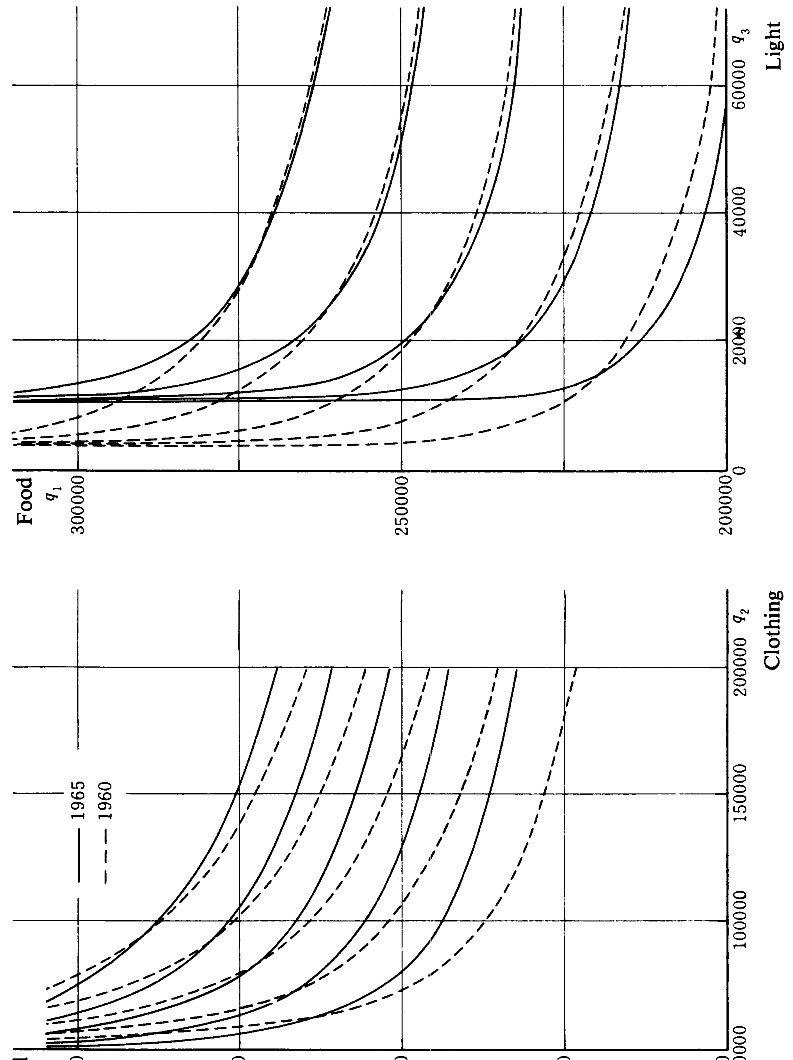
1. Measured Indifference Maps

Now let us try to visualize actual consumers' preferences graphically using the above estimated numerical parameters. Panels of Figure 10.2 present indifference curves drawn on the basis of the measured preference functions. Since it is excessively cumbersome to illustrate combinations of all five items, let us simply present combinations of foodstuff with each other item in turn.

When the curvature of the indifference curve is sharp, allocation of expenditures between the two goods will not change much with changes in relative prices, since the degree of complementarity is strong and the range in which the equilibrium point shifts is small. In contrast, when the curvature is small and the indifference curve takes the form of nearly straight line, there is a high degree of substitutability between the two goods. In other words, even a slight change in relative prices would shift the equilibrium point a long way along the indifference curve. In other words, allocation of expenditures in this case varies greatly with changes in relative prices. Examination of our graphs of measured indifference curves suggests that food is complementary with clothing, and light and fuel while it is substitutable with housing and miscellaneous goods and services. The result that the substitutability is strengthened, as typically observable in the relationship between food and clothing, as the level of indifference curve gets higher, seems to substantiate what is said in text-books.

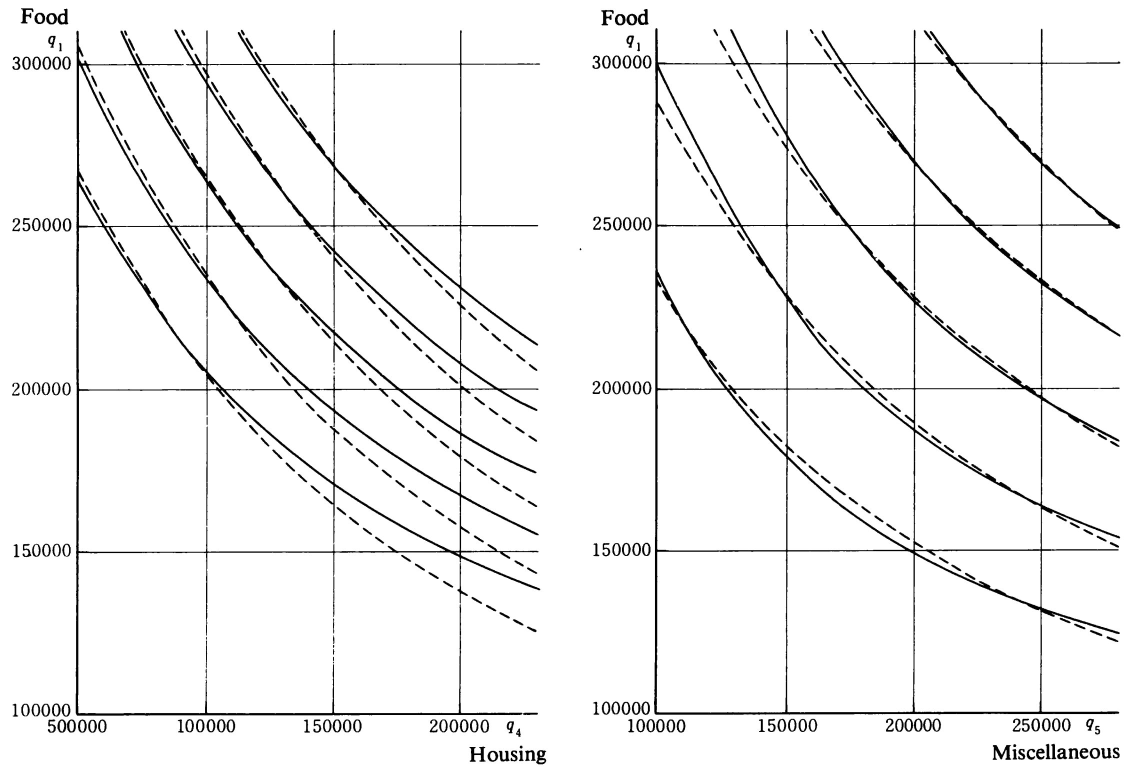
In Figure 10.2, solid curves represent indifference curves observed in 1965 and dotted curves represent those for 1960. In drawing them, the family size m is standardized as of 1965 for all the observations. Therefore, the visible

Figure 10.2 Actually Measured Indifference Maps



Note: Family size is fixed at the 1965 value, while habit potentials vary from 1960 to 1965.

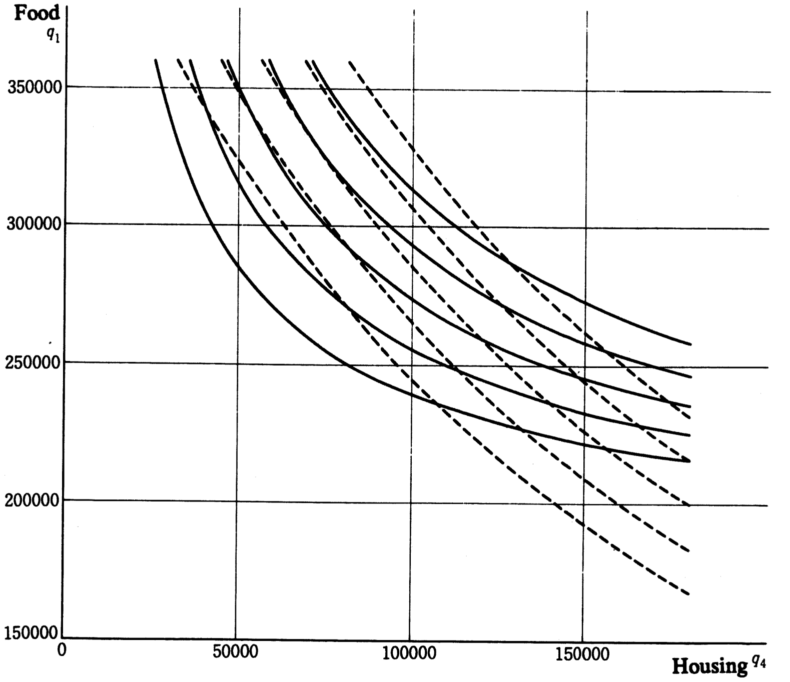
Figure 10.2 ACTUALLY MEASURED INDIFFERENCE MAPS (CONTINUED)



shifts between 1960 and 1965 represent only the shifts due to the habit formation effect. In the case of preference between food and clothing, from 1960 to 1965 curves seem to have inclined more toward the left side and also in the direction of stronger complementarity. In the case of choice between food and fuel and light, the indifference curves shifted to the right indicating increased consumption of light and fuel for the same level of food consumption. Between food and housing, it seems that the complementarity grew somewhat stronger from 1960 to 1965. The same was true with the preference between food and miscellaneous goods the curvature of the indifference curve of which has become somewhat greater.

Figure 10.3 demonstrates how the shape and position of indifference curves between food and housing would change if the average number of household members declines from 4.2 to 3.0 while holding the habit potential constant as of 1965.

Figure 10.3 Shifts in Indifference Curves Due to Changes in Family Size



Note: The indifference curves expressed by solid and dotted lines are based on family sizes of 4.2 and 3 persons, respectively.

The results show that the larger the family size m the larger the curvature, while the smaller the size the smaller the curvature. This result may be interpreted to mean that the larger the family size the more complementary do food and housing become and the smaller the size the more they become substitutable.

Yet, it must be noted that indifference curves of different family sizes have similar curvatures within the range below a certain level of household consumption, namely 80000. But that the curvature begins to differ when expenditure for housing exceeds that level.

The result exhibited in Figure 10.3 thus implies that the preference between food and housing varies significantly with changes in family size. That is, in the case of a large family consumption expenditure for housing will not increase rapidly even though the relative price for housing declines while in the case of a small family even a slight decline in relative price of housing will trigger a sharp increase in spending for housing.

2. Marginal Utility Curves

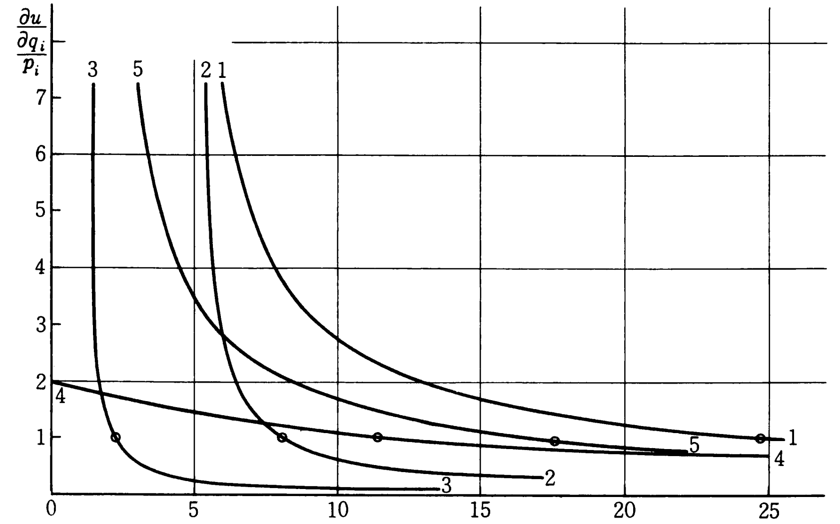
Indifference curves are instrumental for the purpose of expressing the pattern of consumers' preference visually. However, they are subject to a limit in that the preference space becomes multi-dimensional as the number of items increases and two dimensional illustration of a highly complex multi-dimensional space often becomes prohibitively tedious. Thus, the illustration of the same thing by marginal utility curves is also useful as a supplement.

Figure 10.4 presents actually measured marginal utility curves which are drawn holding the number of household members m and habit potential H constant as of 1965 which is the base year for itemized deflators.

The indicator along the horizontal axis measures the annual consumption expenditure per household. To the extent that the indicator is standardized as of the base year, it may also be interpreted as indicating the quantity consumed at constant prices.

The shapes and positions of these marginal utility curves illustrate the difference in the degree of indispensability among different consumption items. For example, if you draw a horizontal line at the level of 2.5 and see where marginal utility curves cut across that line, you will find the amount of each item consumed. At the level of 2.5 in terms of the marginal utility indicator, itemized consumption expenditures are arrayed in descending order such as food, miscellaneous, clothing, light and fuel, and housing reflecting exactly relative positions of marginal utility curves so arrayed from above. However, at the level of marginal utility indicator 3.0, the relative positions

Figure 10.4 ACTUALLY MEASURED MARGINAL UTILITY CURVES



Notes: (1) The vertical axis measures the marginal utility of each consumption item per unit of currency. The horizontal axis meausres the quantity of consumption at the 1965 constant prices. The units are 100 thousands of yen.

(2) The numbers 1, 2, 3, 4 and 5 attached to marginal utility curves indicate, respectively, the curves for food, clothing, fuel and light, housing and miscellaneous items.

of clothing and miscellaneous items are reversed. Above this level of marginal utility the relative positions of the curves remain unchanged, that is, in order of precedence food, clothing, miscellaneous, light and fuel, and housing.

The equal marginal utilities at high levels indicate a case of small total consumption expenditure or income. Therefore, the fact that the marginal utility curves are arrayed in the descending order of food, clothing, miscellaneous, light and fuel, and housing in the range corresponding to low incomes implies that the degree of indispensability is reflected also by this order. In the neighborhood of indicator 7.0 the slopes of the marginal utility curves are quite steep, especially those of clothing, light and fuel, which are almost vertical. This implies that the item is so necessary that the amount of consumption could not be reduced any further even if income were reduced to a lower level. In other words, this situation indicates that the household consumption of that item is approaching the minimum critical necessity level. An examination of Figure 10.4 reveals that food, clothing, miscellaneous, and light and fuel are more like necessities and consumption of them would not easily be reduced below a certain critical level. In contrast, consumption for housing changes considerably with changes in income. A feature of housing consumption is that it is cut substantially when income is low and increases considerably when income increases. In other words, consumption on housing is a sensitive indicator of the standard of living. The allocation of marginal utility curves indicates that a high standard of living can only be said to have been attained when a stage is reached in which housing consumption reaches a comparatively high level.

Even with items characterized generally as necessities, the right and lower portion of the marginal utility curves have flatter slopes and the amount of consumption tends to increase considerably once income increases beyond a certain level. In the case of food for example, the consideration of supplying nourishments tends to be dominant when the level of consumption is low, while an element of enjoyment will become important as consumption increases. Similarly in the case of clothing, the observed change in the slope is interpreted to mean that the function of protecting the human body from cold weather is relatively more important at low levels of consumption, while aesthetics and fashion become more important at high levels of consumption.

A word of caution must be added quickly regarding the interpretation of Figure 10.4. The fact that housing expenditure falls to zero below the 290,000 yen level of total annual consumption on the chart represents a hypothetical case in which the income of a household having the preference pattern of the average household as of 1965 is cut by half abruptly, and should not be taken to imply that households with incomes lower than half the average income as of 1965 did not actually spend on housing. The preference function for actual lower income households in that year have habit potentials which differ from those of the average household and consequently the marginal utility curves should be different from those shown in Figure 10.4.

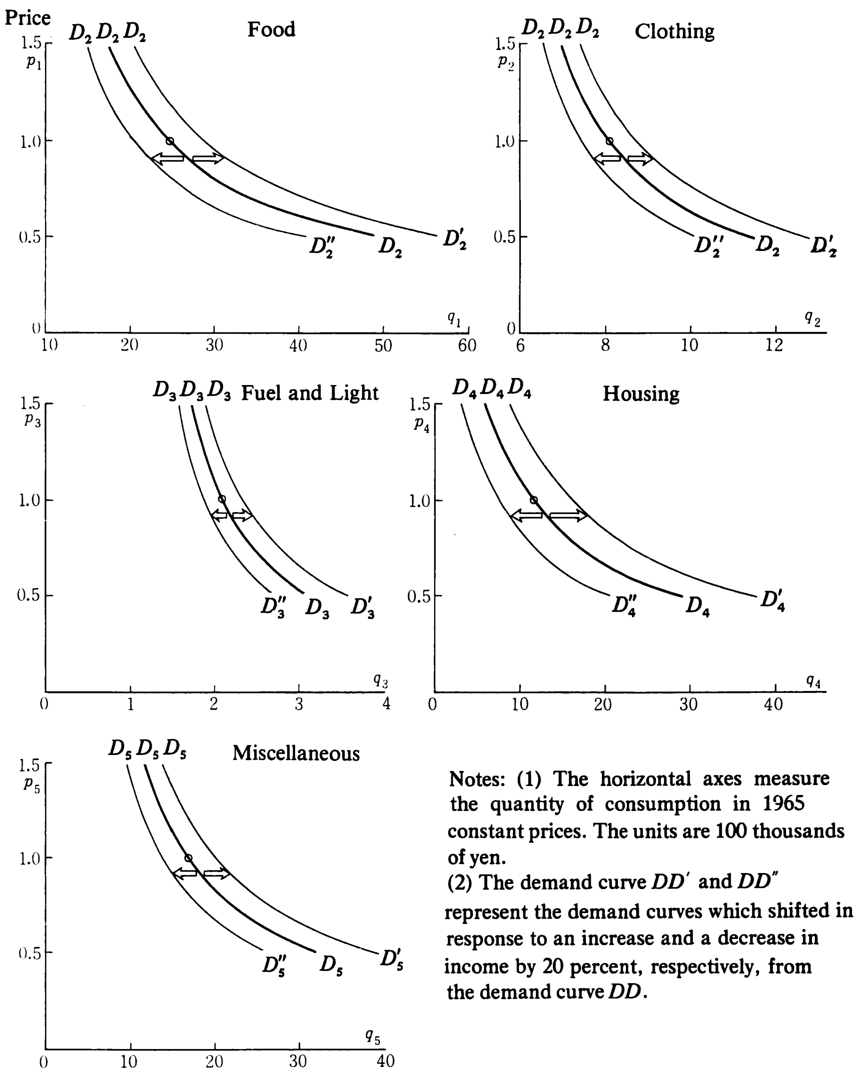
What Figure 10.4 depicts, in other words, is the effect of hypothetical changes in income on the structure of consumption with a given set of marginal utility curves, and not a cross sectional comparison of the structure of consumption in different income groups.

In the era when J.R. Hicks was drawing income-consumption curves on an indifference map, he did not have to distinguish the effects of hypothetical changes in income from cross-sectional comparisons among actual income groups, since shifts in preference were for theoretical purposes assumed to be absent. But at the present state of art, they should be distinguished rigorously.

3. Marshallian Demand Curves and the Elasticity

Figure 10.5 describe the concrete shapes of Marshallian demand curves computed on the basis of measured parameters of itemized general equilibrium-type demand functions.

Figure 10.5 MARSHALLIAN DEMAND CURVES REDUCED FROM THE MEASURED PREFERENCE FUNCTIONS



Notes: (1) The horizontal axes measure the quantity of consumption in 1965 constant prices. The units are 100 thousands of yen.
(2) The demand curve DD' and DD'' represent the demand curves which shifted in response to an increase and a decrease in income by 20 percent, respectively, from the demand curve DD.

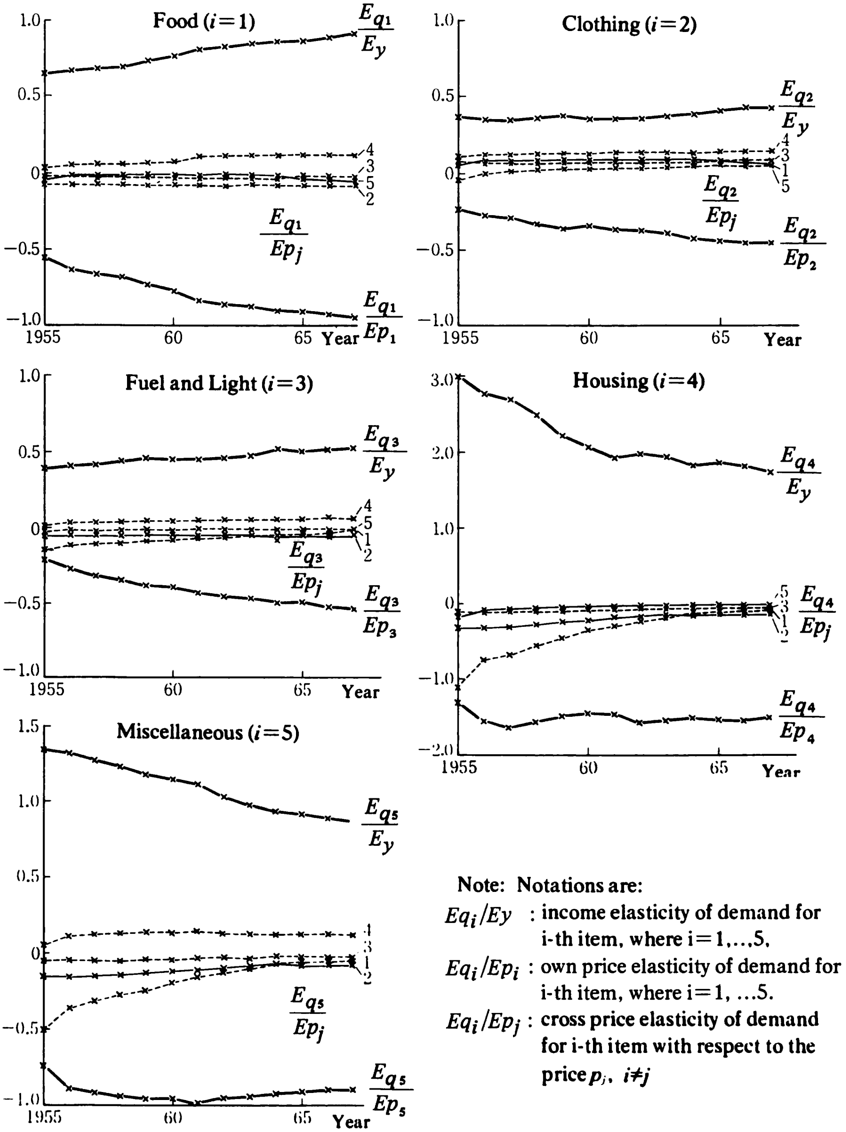
Needless to say, the Marshallian demand function is describable by tracing the locus of the volume of demand for a good corresponding to changes in its price holding other prices constant. In addition, the chart shows how much the demand curve would shift in response to a hypothetical 20 percent increase or decrease in income relative to the actually observed income level. The graph is drawn for 1965, the base year of the deflator, and the round point indicates the point of the observed volume (value) of demand. The horizontal scale axis represents the quantity demanded in terms of 10 thousands of yen per year measured at 1965 constant prices. By multiplying the volumes of these itemized demands by the number of household in the economy, and by converting the itemized quantities and prices into quantities and prices of commodities classified by industrial sectors through a converter, we can transplant them in the vector of final demand of an input-output table as sectoral consumption demand curves. They constitute demand curves for commodity groups together with demand curves for investment goods, exports and government spending, and in turn confront the supply curves for commodity groups derived earlier in Chapter 7.

Figure 10.6 presents the values, based on our itemized general equilibrium-type demand functions, of such Marshallian concepts as the price elasticity of demand, cross price elasticity, and income elasticity obtained in the neighborhood of actually observed annual values.

Let us examine at changes in the income elasticity of demand for each item in turn. In the process of increased household income during the period from 1955 to 1967, the income elasticity of demand for foods increased from 0.65 to 0.92, that for clothing has stayed relatively stable around the level of 0.4, and that for light and fuel increased from 0.4 to 0.53. In contrast, the income elasticity of demand for housing decreased from 3.0 to 1.8 and that for the miscellaneous category declined from 1.35 to 0.9. Generally speaking, income elasticities for those items of which the elasticity is less than unity have increased and decreased for those with elasticities greater than unity. It should be noted, however, that the income elasticity of demand for housing declined sharply until around 1960 and kept declining moderately after that and yet it stayed at a level much higher than unity. This is an alternative expression of the aforementioned fact that housing consumption is given the highest priority when people want to enrich their lives as incomes increase rapidly.

Now let us shift our focus to the price elasticity. Price elasticities have changed in the direction of increased flexibility for food from −0.56 to −0.94, clothing from −0.22 to −0.45, and light and fuel from −0.2 to −0.53. On the other hand, the price elasticity of demand for housing varied between −1.2

Figure 10.6 ICOME ELASTICITY, PRICE ELASTICITY AND CROSS PRICE ELASTICITY IN THE NEIGHBORHOOD OF OBSERVED EQUILIBRIUM POINTS FOR YEARS 1955 TO 1967



Note: Notations are:

Eq_i/Ey : income elasticity of demand for i-th item, where i=1,...,5,

Eq_i/Ep_i : own price elasticity of demand for i-th item, where i=1, ...5.

Eq_i/Ep_j : cross price elasticity of demand for i-th item with respect to the price p_j, $i \neq j$

and −1.3, and that for miscellaneous items fluctuated between −0.9 and −1.0. That the absolute value of price elasticity is less than unity implies that the item in question is a necessity, except for the case of housing. But the fact that the elasticity became more flexible year after year may be interpreted to mean that consumption patterns have been changing even for the same item toward a lower degree of indispensability.

The absolute value of cross-elasticity is generally smaller than the income elasticity or price elasticity. It is interesting to note that while cross-elasticities are negative among food, clothing, light and fuel and miscellaneous items, the cross-elasticity of each of these items with respect to the price of housing is positive. This implies that while for the four items of necessities changes in prices of other goods give an income effect, changes in the prices of housing give a substitution effect to demand for other goods. For instance, while an increase in the price of clothing will lead to a decrease in the demand for food through a reduction of a consumer's purchasing power an increase in the price of housing will have the effect of increasing demand for food by cutting down his spending on housing.

In contrast, the cross-elasticities of prices of other goods with respect to housing consumption are all negative, which implies an income effect is operative. This is interpreted to mean that an increase in the price of necessities will cut down the real purchasing power of a consumer and will lead to diminished consumption of housing.

The cross-elasticities of housing consumption and miscellaneous consumption with respect to food prices were quite large (in absolute values) around 1955 but grew smaller steadily after that. This seems to imply that when income was low the effect of an increase in food prices in cutting down the purchasing power was quite large but the effect has been diminished with increases in income and decreases in family size.

The fact that as many as 16 out of the total of 20 cross-elasticities associated with the five items take negative values implies that as an impact of price changes, the income effect generally outweighs the substitution effect. Recall that Hicks, in his book *Value and Capital*, defended the Marshallian position by saying that the income effect or the effect of a price change on marginal utility of money is negligible when the expenditure for a commodity is small.[7] However, the assumption of constant marginal utility of money will not hold in our case where the total consumption is sub-divided into five major categories. Even if the number of categories was increased to as many as 20 or 30, considerable theoretical errors would still be unavoidable. In other words, it is difficult from the beginning to introduce assumptions of partial equilibrium analysis into a multi-sectoral model such as ours. However, as we have demonstrated, it is advantageous for heuristic reasons to express the results of general equilibrium analysis in terms of Marshallian concepts.

4. Increases in Potential Demand and Reactions in Prices

In Japan, from 1955 to 1965, personal disposable income increased by 3.5 times from 6.36 trillion yen to 22.22 trillion yen. The standard of living undoubtedly rose during this period. But, at the same time, consumers' prices increased especially markedly after about 1960. With increased income, consumption of various items naturally increases. The coefficient of consumption for each item with respect to total consumption, as listed in Table 10.2, tells us for example that an increase of total consumption by 1000 yen will increase consumption of food by 342 yen, clothing by 51 yen, light and fuel by 17 yen, housing by 334 yen, and miscellaneous items by 256 yen. The fact that the marginal rate of consumption is highest for food apparently contradicts Engel's Law, as can be seen from the cross-sectional data of the Family Budget Survey. However, as Simon Kuznets pointed out, time-series movements do not always seem to be what Engel's Law suggests. Kuznets suggested that this is because not only the quality of food is enriched but also because substitution of industrially processed food for home-made food occurs.[8]

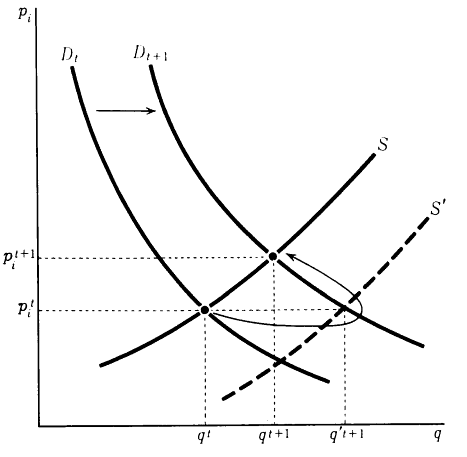
Needless to say, the actually observed change in consumption for each item from year to year is a result of the combined effects of changes in family size, habit potential and relative prices in addition to the effects of increased income and total consumption. Yet, it seems safe to say that consumption of food and housing is likely to increase considerably in the process of economic growth on the ground that the marginal rate of consumption for these items represented one third of the increment in total consumption during the period of observation.

Examining this in terms of demand curves in Figure 10.5, we can see increases in consumption for various items in terms of consumption units since the horizontal axis measures annual amount of consumption at 1965 constant prices. If we measure the increases in consumption of various items at the price level of 1.0 along the vertical axis, the increases are 40000 units for food, 4000 units for clothing, 2000 units for light and fuel, 40000 units for housing, and 30000 units for miscellaneous goods and services.

Figure 10.7 indicates what will happen in the market when demand increases due to increased income.

With an increase in income, the demand will potentially increase from q^t to q'^{t+1}. But since the price will increase simultaneously from p^t up to p^{t+1}

Figure 10.7 A SHIFT IN THE DEMAND CURVE AND POTENTIAL DEMAND



Note: The shift of the demand curve from D_t to D_{t+1}, as indicated by horizontal arrow, represents a shift due to an increase in income.

because of a reaction from the supply side, an equilibrium point will be reached at q^{t+1}. If on the other hand the supply curve shifts at the same time to the position S', then the potential increase in demand would be realized without triggering an increase in the price. If the supply curve shifted further to the rightward than S', then with the additional effect of increased demand due to the decreased price, the amount of consumption would increase to a level greater than q'^{t+1}.

An ideal type of economic growth would be perhaps the one in which the supply curve shifts at least as far to the right as does the demand curve so that potential increases in demand will be satisfied every year without increasing prices. An increase in the price due to a rightward shift in the demand curve while the supply curve remains intact may be said to be the case of demand-pull inflation. At any rate, it is quite abnormal, even from the viewpoint of the free competition principle of the Neo-Classical economists, for a situation where the supply curve lags behind shifts in the demand curve to prevail is the long run. When the price increases temporarily in some sector of the economy due to an increase in demand unmatched by a commensurate increase in supply, the rate of return in that sector will increase. Then new investments will naturally be concentrated in that sector and supply capacity will consequently be increased. This process will be continued until the supply capacity eventually meets the size of demand to stabilize the price level. In the case of a good which is importable, the excess demand will be met by increased imports if an excess supply exists outside the country.

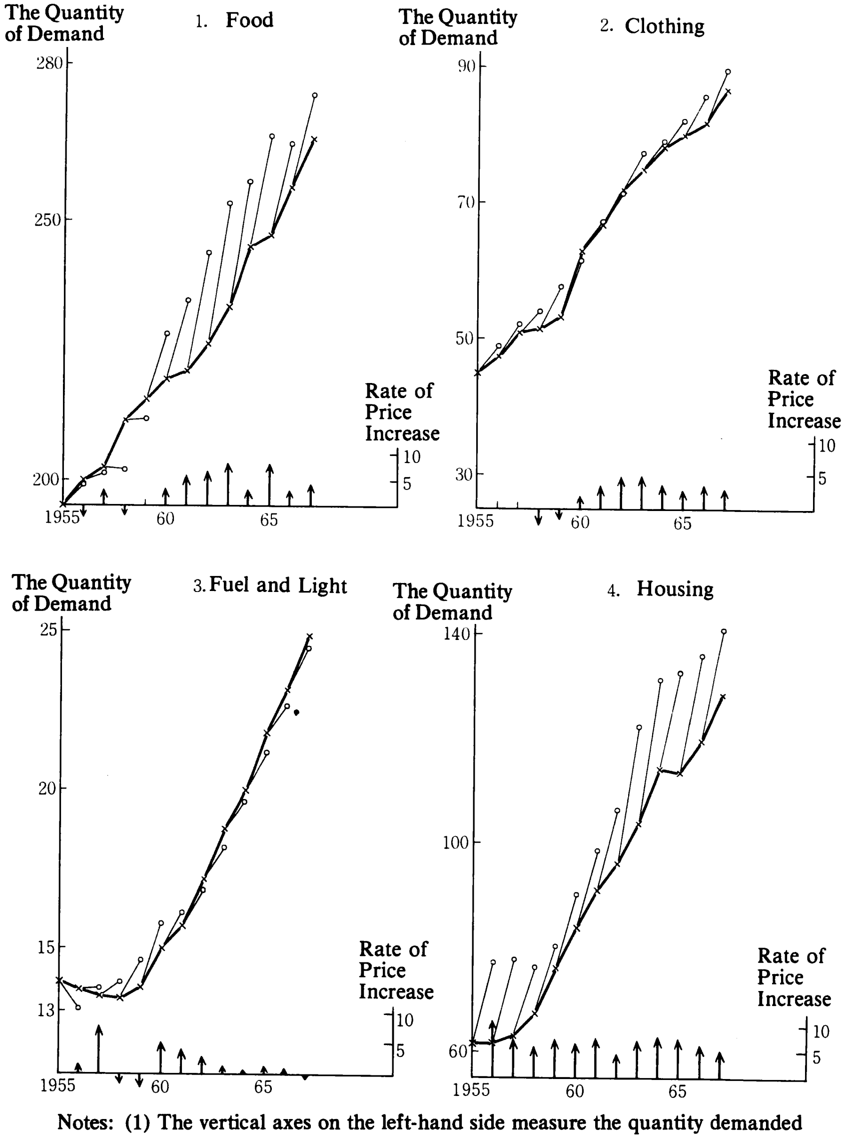
There are cases in which such an automatic market adjustment function will not operate: on the supply side, for example, there are cases in which there exist supply bottlenecks due to natural causes, inadequate mobility of labor or capital due to governmental intervention and import restrictions, restricted supplies of commodities due to private monopoly etc, and on the demand side, for example, an excessive increase in nominal income thanks to excessive money supply etc. Whatever the reasons may be, the desired pattern of economic growth should be attained with appropriate increases in supply to meet increases in demand. Policy interventions are required precisely for the purpose of removing obstacles which disturb efficient allocation of resources and which give rise to discrepancies between demand and supply.

Figure 10.8 is drawn, based on this viewpoint, to show how supply capacity increased in response to increases in demand generated by increases in income during the period from 1955 to 1967. The thick line indicates movements of the volume of demand (in 1965 constant prices) from year to year, which is estimated by giving the observed values in corresponding years of income, the number of household members, habit potential, and prices for the demand function for each item (shown in Table 10.2).

The thin line on the other hand indicates the estimated demand obtained by providing observed values for income, the number of household members, habit potential of corresponding years while giving prices for the previous years to the demand function for each item. In other words, the latter expresses the potential demand for each year before prices change as they did during the preceding year. Therefore, the year in which the thick and thin lines conform with each other is the year in which the supply curve shifted to match a shift in the demand curve while the year in which the thin line is above the thick line is the year in which the supply failed to increase sufficiently to meet the increased demand and consequently an increase in potential demand was compressed by increased prices.

In the lower part of each of the panels 1 to 5 of Figure 10.8, the rate of price change is shown for each item. Needless to say that the volume of consumption of each item is not determined solely by its price and that this picture alone can not clarify completely the relationship between shifts of demand and supply curves. In fact, the major purpose of our total model of the economy is precisely to analyze this point legitimately. However, a glance

Figure 10.8 TIME-SERIES MOVEMENTS OF POTENTIAL DEMAND



Notes: (1) The vertical axes on the left-hand side measure the quantity demanded for a year in 1965 prices.

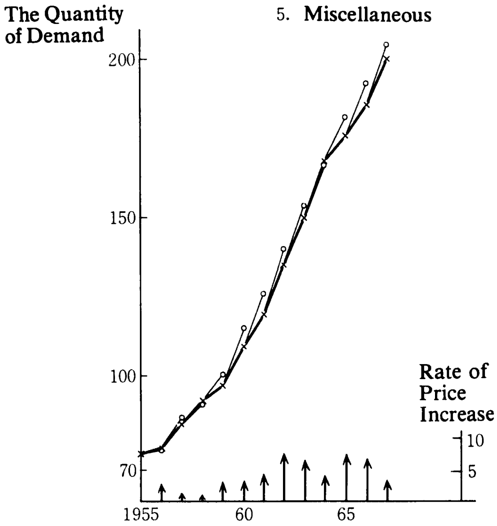
(2) The vertical axes on the right-hand side measure the rate of increase in the price of each respective consumption item expressed in terms of percentage points.

(3) The notation ——o represents a potential increase in demand from year to year which is calculated from the demand function assuming that the price of the consumption item is held unchanged.

(4) The notation x——x represents the actually observed change in demand.

(5) The notation → represents the actually observed rate of increase in the price of each item.

Figure 10.8 TIME-SERIES MOVEMENTS OF POTENTIAL DEMAND (CONTINUED)



Note: For notations, see Figure 10.8 in the previous page.

at Figure 10.8 shows clearly that the increases in prices have deterred increases in potential demand in the process of income growth.

For clothing, light and fuel and miscellaneous goods and services, the increased potential demand has been more or less fulfilled by an increased capacity of supply. In contrast, the performance of supply has been quite poor in cases of food and housing. The potential demand has been compressed by increased prices clearly from 1960 in the case of food and for all the period of observation for housing. People welcome increases in income simply because they expect increases in purchasing power exactly corresponding to the increases in income assuming unconsciously that the level of prices prior to the increases in income will remain unchanged. In other words, they expect that the increase in potential demand will be realized without modification. There is no point in obtaining an increase in income if it is offset by an increase in prices.

In 1965 for example, a consumer would have been able to enjoy food consumption to the tune of 266,000 yen at constant prices during that year thanks to an increase in income from the previous year. In fact the consumption was cut by 16,000 yen due to an 8 percent increase in food prices. For housing, 132,500 worth of annual consumption was anticipated during

1965, but it was in fact compressed to 110,000 yen. The proportion of the cut was 6 percent for food while it was as much as 24 percent for housing. In 1956, the cut in demand for housing reached as much as 25 percent. The performance in improvement in the standard of living during the process of economic growth since the mid-1950's has been poor in cases of consumption of food and housing. Housing consumption was especially depressed. Since, as suggested by our earlier analysis, housing consumption serves as the single most important indicator reflecting the enrichment of people's lives, it is doubtful even under the phenomenal economic growth whether the standard of living has been improved much in substance insofar as housing consumption has been disturbed this much by the poor supply conditions.

Notes to Chapter 10

1) The Analysis presented in this chapter is heavily dependent on a number of our previous studies. Since it is difficult to describe those studies in detail because of the limited space of this chapter, interested readers are referred to the following publications: Tsujimura (1961), Tsujimura and Sato (1964), Tsujimura and Kutsukake (1966), Tsujimura (1967), Tsujimura (1968) and Tsujimura (1964).
   For the sake of readability, we denote price by p_j in stead of the notation of price η_{cj} which is used in describing the total system of our model.
2) Stone (1953 and 1954).
3) Allen and Bowley (1935).
4) Haavelmo (1944)
5) Tsujimura (1961) and Tsujimura (1964).
6) A general survey of theories on shifts in preferences is given in Tsujimura (1967). Also for a useful survey on this subject, see Houthakker and Taylor (1966). We would like to express our profound thanks to Professor H.S. Houthakker for his many helpful comments which he gave during the process of our research. It is needless to say, however, that we are solely responsible for any mistakes remaining in our study.
7) Hicks (1939).
8) Kuznets (1961).

Chapter 11
The Estimation of the Structure of Investment Demand

We have ascertained in Chapter 7 that the supply schedules derived on the basis of the theoretical concepts of the SFS production function and the anticipated demand function were verified in the light of empirical data. The supply schedules thus verified would shift with changes in the capacity of production. Therefore, it would be necessary to explain endogenously the time-series changes in the capacity of production for us to understand fully the properties of the supply side of the economy in the process of development. It is this question that this chapter is dedicated. The principal task of this chapter, therefore, is to explain endogenously the expansions of productive capacities of different sectors from the viewpoint of producer's investment behavior.[1]

In manufacturing and service industries, the capacity of production of each sector at the beginning of each period corresponds to the level of capital stock of that sector, as described by the SFS production function. Therefore, changes in the capacity of production may be explained by specifying endogenously the mechanism by which the level of capital stock changes. In other words, this means to explain the annual demand for investment goods by means of a profit maximizing behavior of the firm.

In agricultural sector, on the other hand, we have formulated in the short-run under the given capital stock that its labor force is the residual which remains after subtracting the amount of employment from the total labor force in the economy. In this situation, the level of output in the shortrun would be determined by pre-determined or exogenous variables such as the level of capital stock, the amount of employment, and the area of cultivated land. This implies that the level of output is not necessarily always that level of output which results in profit maximization in the short-run. Therefore, the investment behavior in the agricultural sector may be interpreted as adjusting the level of capital stock in order to make the supply capacity optimal.

There exists a rich literature on the theory of investment. Nevertheless, this subject still remains to be the least developed area in terms of empirical analysis. This is due largely to the difficulty associated with designing the appropriate scheme of empirical analysis of the concept of investors' subjective prospect for the future such as the long-term expectations of a firm as suggested by Keynes.[2]

In describing the investment behavior in manufacturing and service sectors in this chapter, we will introduce an analytical tool of what we call the "long-term anticipated demand function" in order to analyze empirically the long-term expectations of a firm. The optimal level of capital stock will be derived on the basis of technological conditions of production and anticipated demand for the future. The investment will then be described as an adjustment between this optimal level and the actual level of capital stock. We assumed that the adjustment to the optimal level would be completed after a gestation period of one year for all sectors. Haavelmo has emphasized years ago the need to specify the process of adjustment.[3] Adjustment processes have been analyzed empirically since then by applying various distributed lag functions.[4] However, we did not incorporate complex lag structures into our analysis. We would be willing to incorporate appropriate lag structures into the formulation of our model in the future if stable and empirically valid lag structures are found. Our basic intention at this stage, however, is to see how much of the observed variation will be explained by the model with the simplest assumption on the process of adjustment.

In the agricultural sector, we described the investment behavior assuming that producers invest on the basis of future expected prices of their agricultural products unlike the long-term anticipated demand used in the case of manufacturing and service sectors. We treat the investment behavior in the agricultural sector in this way because the price of agricultrual produce, represented by rice, is specified as an exogenous variable.

11.1 Investment Demand in Manufacturing and Service Sectors

1. The Formulation of Investment Demand

Let us begin our discussion by stating once again the SFS production function explained in detail in Chapter 7.

$$Q_j = aK_j^b \quad , \tag{11.1}$$

$$L_j = cK_j^d \quad , \tag{11.2}$$

$$X_j = Q_j h_j^\alpha \quad , \tag{11.3}$$

where a_j, b_j, c_j, d_j and α_j are parameters representing technological conditions, Q_j is the capacity of production for a unit period for the j-th sector, L_j is the number of workers employed in the j-th sector, X_j is the amount of output for a unit period for the j-th sector and h_j is actual hours of operation for a unit period. For simplicity unless otherwise stated the subscript j attached to the parameters will henceforth not be written explicitly.

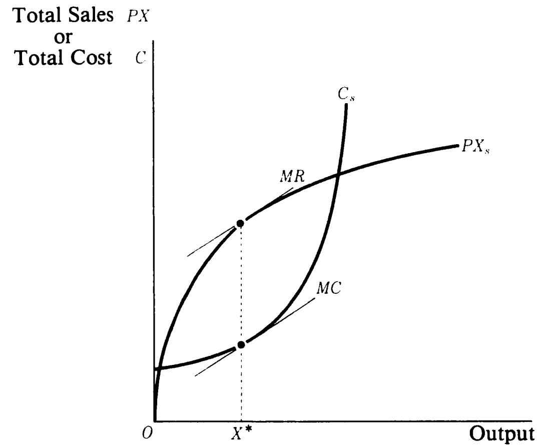
The investment in the current period can be sub-divided into supplementary investment to maintain the current capacity of production and expansionary investment to expand the capacity of production in the future. Let us assume that expansionary investment increases the productive capacity only after a one year gestation period. In other words, this is to assume that the capital stock existing at the beginning of each period always satisfies the optimal level of capital stock and the needed adjustment is always completed within a year. Although it is conceivable to introduce complex distributed lags, the first appropriate task for us to do is to examine how effective the hypothesis would be which simply assumes a uniform one-year lag. According to this hypothesis, the capacity of production which has been determined by capital stock K_j^t at the beginning of the t-th period will be increased by the beginning of the $(t+1)$-th period by the amount corresponding to the increment in the capital stock due to expansionary investment. The investment demand in this situation is generated, therefore, for expansionary purposes supported by the firm's anticipation of increases sales in the future.

Figure 11.1 illustrates the determination of the optimal level of output which maximizes profits in the short-run under the given capacity of production. The curve PX_s indicates the sales revenue and C_s the cost. The profit maximizing output X^* is the level at which marginal revenue MR equals marginal cost MC. Given the level X^*, we can calculate hours of operation h_j from equation (11.3).

The short-run cost curve C_s in the t-th period depends upon the capital stock at the beginning of that period K_j^t, which in turn has been accumulated by the optimal investments in the past. The purpose of our estimation of the investment demand is to find out the path of changes in the optimal capital stock. The long-run profit maximization by means of optimal capital accumulation may be explained diagramatically by Figure 11.2.

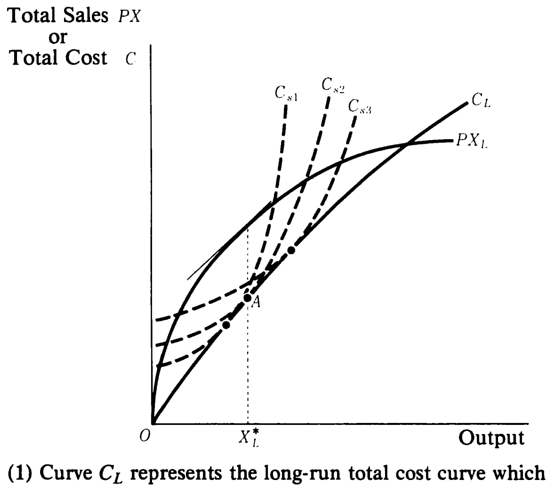
Curve C_L in Figure 11.2 is the long-run cost curve which is an envelope line of short-run cost curves C_s for different levels of capital stock K. The profit

Figure 11.1 AN ILLUSTRATION OF THE DETERMINATION OF THE PROFIT-MAXIMIZING AMOUNT OF SUPPLY IN THE SHORT RUN



Notes: Curves indicated by C_s and PX_s, respectively, represent the total cost curve and the total sales curve. The notations *MR* and *MC* stand for marginal revenue and marginal cost, respectively.

Figure 11.2 AN ILLUSTRATION OF THE DETERMINATION OF THE PROFIT MAXIMIZING AMOUNT OF SUPPLY IN THE LONG RUN



Notes: (1) Curve C_L represents the long-run total cost curve which is derived from the envelope of the short-run cost curves C_{s1}, C_{s2} and C_{s3}, and curve PX_L represents the total sales curve derived from the long-run anticipated demand function.
(2) The level of output X_L^* is determined at the point of profit maximization where the marginal revenue, expressed by the slope of th tangent to the total sales curve, equals the marginal cost, expressed by the slope of the tangent to the long-run cost curve.

maximizing output $X_L{}^*$, which is determined by the sales curve PX_L and the cost curve C_L according to the principle of $MR = MC$, will determine the relevant short-run cost curve, for example C_{s2} in the case of Figure 11.2, which is tangent to the long-run cost curve C_L at point A. Therefore, the optimal long-run output $X_L{}^*$ should correspond uniquely to a particular level of the capital stock.

When investment plans are made, the planners will determine the capacity of production taking into account the normal hours of operation designed for the equipment. In terms of the SFS production function, which was explained in Chapter 7, it is implied that the planner will detemine the optimal level of capital stock in the long-run on the basis of a certain normal hours of operation $h_j{}^*$.

The analytical framework of the optimal capital accumulation as illustrated by Figure 11.2 consists of two elements: one is the anticipated demand for the future as shown by the curve PX_L, and the other is the cost function as indicated by the curve C_L.

We shall again formulate the anticipated demand function to represent the firm's anticipation for the future demand for its output. This anticipated demand function, however, should be distinguished from the short-run anticipated demand function discussed earlier in Chapter 7. In the case of the short-run anticipated demand function, the focus was solely the firm's anticipation for the price elasticity of demand while the level of demand itself was presumed to be determined in the short-run by a demand-supply balance in the market. On the other hand, in the case of the long-run anticipated demand function, it is necessary to know in addition the firm's anticipation of the level of future demand itself which will be taken into account for the firm to determine its optimal capacity of production for the future. Although the long-run anticipated demand function should be distinguished in this way from its short-run counterpart, we shall nevertheless use a simple expression, the "anticipated demand function", to mean in this chapter unless otherwise stated the long-run anticipated demand function.

The long-run cost function which is the other tool for the explanation of investment behavior should also be distinguished from its short-run countepart. The long-run cost function depends on anticipated future costs of labor, capital and raw materials. Let us specify the long-run cost function as follows.

$$C_j = L_j h_j w_j{}^* + K_j \eta_{Kp} (i^* + d_{ej}) + \sum_{i=1}^{4} p_i{}^* a_{ij} X_j + t_{Ij} p_j{}^* X_j \,, \qquad (11.4)$$

where C_j is the total cost for the j-th sector, $L_j h_j w_j^*$ is labor cost, $K_j \eta_{Kp}(i^* + d_{ej})$ is capital cost, $\Sigma p_i^* a_{ij} X_j$ is the cost for raw materials, the $t_{Ij} p_j^* X_j$ is indirect tax. Each of the unit costs w_j^*, $(i^* + d_{ej})$, p_i^* $(i = 1, ..., 4)$ is the cost anticipated by the firm when investments are made. It is often observed that anticipated increases in costs of labor and raw materials encourage entrepreneurs to adopt more capital intensive production technology which may be realized by increased investments, or anticipated conditions of cost of raising fund affect investment behavior. However, we assume the following relation for w_j^* and p_j^*.

(11.5) $$w_j^* = w_j^t.$$

(11.6) $$p_i^* = p_i^t \quad (i \neq j), \qquad p_j^* = p_{Aj}.$$

This means that the firm anticipates that the current level of wage and prices of goods other than its own will continue to prevail for the next period. On the other hand, the firm anticipates a certain future price for its own product which relates to the anticipated demand. The formulation of the anticipated demand will be explained later. As for the cost of capital, d_{ej} is the rate of depreciation which is assumed to be constant over time for each sector. η_{kp} is the price of investment goods and $K \cdot \eta_{Kp}$ therefore is the nominal value of capital stock, and thus i^* implies the unit cost of raising fund. Since tax rates are determined exogenously by the tax system, it may reasonably be assumed that the rate of indirect tax t_{Ij} for the current period will be effective for the subsequent period.

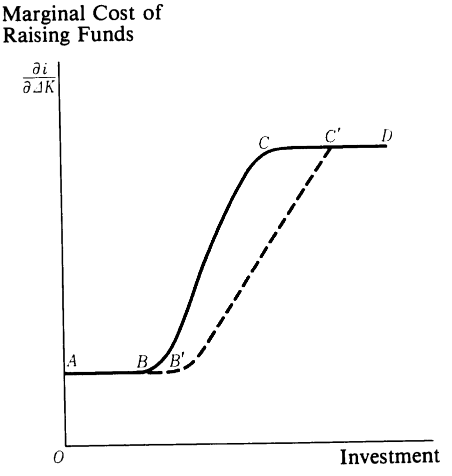
2. The Cost of Raising Funds and Investment

J. S. Duesenberry has hypothesized that the marginal cost of finance of a firm may be approximated by a curve as illustrated by Figure 11.3.[5]

The sources of funds of a firm is classified into three types: (1) finance from internal sources such as retained earnings and depreciation allowance, (2) borrowing from external sources, and (3) finance from issuing stocks and bonds. The range between A and B represents the finance from internal sources at a relatively low marginal cost, which is equivalent to the opportunity cost of the internal funds. The greater the size of investment beyond the level of the available internal funds, the more heavily the firm has to resort to external sources. Consequently the marginal cost of finance will increase as shown by the upward slope BC. When the marginal cost reaches the level of C, which is the level donimated by issuance of stocks and bonds, alternative methods of finance must be sought.

It is not easy to verify this hypothesis empirically. Many empirical studies of investment behavior in the United States have found on the contrary that

Figure 11.3 AN ILLUSTRATION OF A COST-SCHEDULE OF RAISING FUNDS FOR INVESTMENT



the cost of raising fund is independent of the asset composition, which is compatible with the hypothesis of Modigliani and Miller.[6] However, in the case of Japan, the Duesenberry's hypothesis has been found to be empirically valid by T. Ihara who explicitly took into account the high dependency of Japanese firms on borrowed funds and the relative inability of the inmature market.[7] The Duesenberry hypothesis also allows for the possibility of shifts of the marginal cost schedule, as shown for example by the shift from *ABCD* to *AB′C′D*, depending on the relative amount of internal funds or borrowed funds. Whether this hypothesis is valid when applied to the faily aggregate sectoral data of our model is yet to be seen.

Now, let us formulate an equation for the cost of raising fund following basically the Duesenberry's hypothesis.

$$(11.7) \qquad i^* = \delta\left(\frac{\eta_{Kp}\Delta K + D^t + \rho}{M^t}\right)^{\beta} + i,$$

where i^* is the unit cost of raising fund for the j-th sector. The subscript j is deleted for the sake of simplicity. η_{Kp} is the deflator for investment goods, D^t is the balance of debt at the beginning of the t-th period, M^t is the retained earnings for the t-th period, and i is interest rate loans discounts of all banks, δ, ϱ and β are parameters. Therefore, the total cost of finance will be expressed as

$$(11.8) \qquad K\eta_{Kp}(i^*+d_e) = K\eta_{Kp}\left\{\delta\left(\frac{\eta_{Kp}\Delta K+D+\rho}{M}\right)^{\beta}+ i+d_e\right\} .$$

The marginal cost of finance will be written as

(11.9)

$$\frac{\partial K^{t+1}\eta_{Kp}(i^*+d_e)}{\partial \eta_{Kp}\Delta K^t} = \frac{\partial K^{t+1}\eta_{Kp}(i^*+ d_e)}{\partial K^{t+1}} \cdot \frac{dK^{t+1}}{d\Delta K^t} \cdot \frac{1}{\eta_{Kp}}$$

$$= (i+d_e) + \delta\,(\eta_{Kp}\Delta K^t+D^t+\rho)^{\beta-1}M^{-\beta} \times \left\{\eta_{Kp}K^{t+1}(1+\beta)+D^t+\rho-\eta_{Kp}K^t\right\},$$

where $$K^{t+1} = K^t + \Delta K^t .$$

Equation (11.9) is an approximation of the curve ABCD of Figure 11.3 by a continuous function. The bottom level corresponding to the range AB is determined at the level of the opportunity cost $(i+d_e)$. The range CD is formulated to take a shape of sharply rising slope so that it reflects the situation in the financial market in the 1950s and 1960s in which funds were in short supply.

The theoretical constraints of the convexity of the marginal cost curve of equation (11.9) is given by

(11.10)

$$\delta\beta(\eta_{Kp}\Delta K^t+D^t+\rho)^{\beta-3}M^{-\beta}(\beta-1)\times\left\{\eta_{Kp}K^{t+1}(1+\beta)+3D^t+3\rho-3\eta_{Kp}K^t\right\}>0,$$

and of the convexity of the total cost curve of equation (11.8) is given by

(11.11)

$$\delta\beta(\eta_{Kp}\Delta K^t+D^t+\rho)^{\beta-2}M^{-\beta}\times\left\{\eta_{Kp}K^{t+1}(1+\beta)+2D^t+2\rho-2\eta_{Kp}K^t\right\}>0.$$

3. The long-run Anticipated Demand

We shall express the long-run anticipated demand function by a linear expenditure system, which is analogous in its form with the model of the short-run anticipated demand function. The long-run anticipated demand function may be written as

$$(11.12) \qquad p_{Aj}X_j = \alpha_{Lj}+\beta_{Lj}YY_j+\gamma_{Lj}p_{Aj} ,$$

where p_{Aj} is the anticipated price of the good of the j-th sector, and YY_j is the anticipated volume of demand for the j-th goods. YY_j is considered generally for the time being to be the function of pre-determined endogenous variables such as

$$YY_j^* = \alpha_{Lj}+\beta_{Lj}YY = g(GDP^{t-1}, Inventory^{t-1}, \ldots) .$$

By rearranging equation (11.12) we obtain

(11.13)
$$P_{Aj} = \frac{\alpha_{Lj}+\beta_{Lj}YY_j}{X_j-\gamma_{Lj}} = \frac{YY_j^*}{X_j-\gamma_{Lj}} \ .$$

Therefore, we get

(11.14)
$$\frac{\partial p_{Aj}}{\partial X_j} = -\frac{YY_j^*}{(X_j-\gamma_{Lj})^2} \ .$$

Since the sales revenue $P_{Aj}X_j$ is

(11.15)
$$p_{Aj}X_j = \frac{YY^* \cdot X_j}{X_j-\gamma_{Lj}} \ ,$$

we have

(11.16)
$$\lim_{X_j\to\infty} p_{Aj}X_j = \lim_{X_j\to\infty} \frac{YY^* \cdot X_j}{X_j-\gamma_{Lj}} = YY^* \ .$$

Thus, the anticipated sales revnue $P_{Aj}X_j$ will approach the level of YY^* asymptotically as X_j approaches infinity of $X_j \to \infty$. The parameters α_{Lj}, β_{Lj} and γ_{Lj} are generally not the same as those for the short-run anticipated demand function.

4. The Formulation of the Optimal Capital Stock

Thus far, we have formulated the production function which represents the technological constraints of production, the cost function for investment, and the anticipated demand function all of which are necessary to derive the optimal capital stock. We have assumed that the firm plans its capacity of production on the basis of normal hours of operation h_j^* when it determines the amount of investment. Let us assume here specifically that h_j^* is 200 hours per month (8 hours × 25 days).

The optimal stock of capital may be obtained from the following 9 equations which have been specified so far.

[The Production Function]

(11.17)
$$Q_j = aK_j^b \ .$$

(11.18)
$$L_j = cK_j^d \ .$$

(11.19)
$$X_j = Q_j h_j^\alpha = Q'_j h_j^* \left(\frac{h_j}{h_j^*}\right)^\alpha = Q_j h_j^{*\alpha} \ ,$$

where if

$$Q_j = Q'_j h_j^{*1-\alpha} \ , \quad h_j = h_j^* \quad \text{then,} \quad X_j = Q_j h_j^{*\alpha} \ .$$

[The Cost Equation]

(11.20) $$C_j = L_j h_j^* w_j^* + K_j \eta_{Kp}(i_j^* + d_{ej}) + p_{Aj} a_{jj} X_j + \sum_{\substack{i=1 \\ (i \neq j)}}^{4} p_i^* a_{ij} X_j + t_{Ij} p_{Aj} X_j \ .$$

(11.21) $$w_j^* = w_j^t \ .$$

(11.22) $$p_i^* = p_i^t \ (i \neq j) \ .$$

(11.23) $$i_j^* = \delta \left(\frac{\eta_{Kp} \Delta K_j^t + D_j^t + \rho}{M_j^t} \right)^{\beta} + i \ .$$

[The Anticipated Demand Function]

(11.24) $$p_{Aj} = \frac{YY_j^*}{(X_j - \gamma_{Lj})} \ .$$

[The Normal Hours of Operation]

(11.25) $$h_j^* = 200.0 \ .$$

Under the given capital stock K^t at the beginning of the current period t, let us derive the optimal capital stock K^{t+1} from the anticipated demand function for the next period and the cost equation.

Let us formulate the investment behavior of the firm in the form of maximizing the anticipated profits

(11.26) $$\Pi_j = p_{Aj} X_j - C_j \ ,$$

with respect to the capital stock K^{t+1} under the constraints of equations (11.17) to (11.25). We will delete subscript j and superscript t for the sake of simplicity unless they are specially necessary.

From the first order conditions of profit maximization, we obtain

(11.27)

$$\begin{aligned}
\frac{\partial \Pi}{\partial K} &= \frac{\partial p_A}{\partial X} \cdot \frac{\partial X}{\partial K} \cdot X + p_A \frac{\partial X}{\partial K} - \frac{\partial L}{\partial K} h^* w - (i^* + d_e)\eta_{Kp} - \frac{\partial i^*}{\partial K} K \eta_{Kp} - \frac{\partial X}{\partial K} \sum_{(i \neq j)} p_i a_{ij} \\
&\quad - \frac{\partial p_A}{\partial X} \cdot \frac{\partial X}{\partial K} X \cdot (a_{jj} + t_{Ij}) - p_A \frac{\partial X}{\partial K} (a_{jj} + t_{Ij}) \\
&= \frac{\partial p_A}{\partial X} \cdot \frac{\partial X}{\partial K} X (1 - a_{jj} - t_{Ij}) + \frac{\partial X}{\partial K} p_A (1 - a_{jj} - t_{Ij}) \\
&\quad - \frac{\partial L}{\partial K} h^* w - (i^* + d_e)\eta_{Kp} - \frac{\partial i^*}{\partial K} K \cdot \eta_{Kp} - \frac{\partial X}{\partial K} \sum_{(i \neq j)} p_i a_{ij} \ ,
\end{aligned}$$

where

$$\frac{\partial p_A}{\partial X} = - \frac{YY^*}{(X - \gamma_L)^2} \ ,$$

$$\frac{\partial X}{\partial K} = abK^{b-1}h^{*\alpha} \ ,$$

$$\frac{\partial L}{\partial K} = cdK^{d-1} \ ,$$

$$\frac{\partial i^*}{\partial K} = \delta \cdot \beta \cdot \eta_{Kp}(\eta_{Kp}\Delta K + D + \rho)^{\beta-1}M^{-\beta} \ .$$

Substituting this into equation (11.27) and rearranging, we can obtain the equation by which to determine the optimal capital stock K^{t+1} for the j-th sector (j = 2, 3, and 4) as follows.

(11.28)
$$\begin{aligned}
\frac{\partial \Pi}{\partial K} &= -\frac{YY^*}{(aK^b h^{*\alpha} - \gamma_L)^2}(1 - a_{jj} - t_{Ij})a^2 bK^{2b-1}h^{*2\alpha} \\
&\quad - cdK^{d-1}h^* w - \delta\eta_{Kp}(\eta_{Kp}\Delta K + D + \rho)^{\beta-1}M^{-\beta} \\
&\quad \times\left\{(1+\beta)\eta_{Kp}K^{t+1} + D + \rho - \eta_{Kp}K^t\right\} - (i + d_e)\eta_{Kp} \\
&\quad - abK^{b-1}h^{*\alpha}\sum_{(i\neq j)} p_i a_{ij} + \frac{(1 - a_{jj} - t_{Ij})YY^* abK^{b-1}h^{*\alpha}}{(aK^b h^{*\alpha} - \gamma_L)} \\
&= -\frac{(1 - a_{jj} - t_{Ij})YY^* abK^{b-1}h^{*\alpha}\gamma_L}{(aK^b h^{*\alpha} - \gamma_L)^2} - abK^{b-1}h^{*\alpha}\sum_{(i\neq j)} p_i a_{ij} \\
&\quad - cdK^{d-1}h^* w - \delta\eta_{Kp}(\eta_{Kp}\Delta K + D + \rho)^{\beta-1}M^{-\beta} \\
&\quad \times\left\{(1+\beta)\eta_{Kp}K^{t+1} + D + \rho - \eta_{Kp}K^t\right\} - (i + d_e)\eta_{Kp} = 0.
\end{aligned}$$

The demand for expansionary investment in the j-th sector will be the difference between the optimal capital stock K derived from equation (11.28) and the capital stock at the beginning of the current period K^t, or

(11.29)
$$\Delta K^t = K^{*t+1} - K^t .$$

Parameters a, b, c, d, and α which are associated with the production function have already been estimated. Therefore, our next task is to estimate the parameters of the anticipated demand function and the cost of raising fund function.

Since equation (11.27) is the necessary condition for profit maximization, the sufficient condition such as below also needs to be satisfied.

(11.30)

$$\frac{\partial^2 \Pi}{\partial K^2} = \frac{YY^*(1 - a_{jj} - t_{Ij})\gamma_L abK^{b-2}h^{*\alpha}\left\{b(aK^b h^{*\alpha} - \gamma_L) - (aK^b h^{*\alpha} - \gamma_L)\right\}}{(aK^b h^{*\alpha} - \gamma_L)^3}$$

$$-ab(b-1)K^{b-2}h^{*\alpha}\sum_{(i\neq j)}p_i a_{ij} - cd(d-1)K^{d-2}h^{*}w$$
$$-\eta_{Kp}{}^2\delta\beta(\eta_{Kp}\Delta K+D+\rho)^{\beta-1}M^{-\beta}\left\{(1+\beta)\eta_{Kp}+2D+2\rho-2\eta_{Kp}K^{t-1}\right\}<0.$$

11.2 Estimation

We can not use the ordinary least squares method in estimating the parameters of equation (11.27) since it is non-linear in its form. As an alternative to the least squares method, we will use a method of iterative computation. In this method, however, we need to give an initial value for each parameter.

1. The Cost of Raising Fund Function

The cost of raising fund function is given by

$$(11.31) \qquad i^* = \delta\left(\frac{\eta_{Kp}\Delta K+D+\rho}{M}\right)^{\beta}+i \ ,$$

and thus the total cost of raising fund may be expressed as

$$(11.32) \qquad K\eta_{Kp}(i^*+d_e) = K\eta_{Kp}\left\{\delta\left(\frac{\eta_{Kp}\Delta K+D+\rho}{M}\right)^{\beta}+i+d_e\right\},$$

where K is the optimal capital stock to be determined, η_{kp} is the deflator of investment goods, M is the retained earnings of private corporation, for the current period, D is the balance of debt at the beginning of the period, i is interest rate loans discounts of all banks and d_e is the rate of capital depreciation allowance. There are three parameters δ, β and ϱ. We use the data of capital stock at constant prices for each sector based on the assumption that the actual capital accumulation in each sector during the period of 1955 to 1965 has followed the optimal path of capital accumulation as indicated by equation (11.28).

It is quite difficult to give *a priori* information on parameters δ, β and ϱ in a suitable form to the fairly aggregate sectors of our model. To circumvent this difficulty, we obtained the initial values for these parameters by means of deriving theoretical constraints as follows.

If we assume $t_{Ij} = 0$ in the process of deriving equation (11.28), we can obtain

$$(11.33) \quad \frac{\partial \Pi}{\partial K} = \frac{\partial p_A}{\partial X}\left\{ab(1-a_{jj})K^{b-1}h^{*\alpha}X\right\}+ab(1-a_{jj})p_A K^{b-1}h^{*\alpha}$$

$$-cdK^{d-1}h^*w-(i+d_e)\eta_{Kp}-\frac{\partial i^{**}}{\partial K}-adK^{b-1}h^{*\alpha}\sum_{(i\neq j)}p_i a_{ij}=0\ ,$$

where

$$(11.34)\quad \frac{\partial i^{**}}{\partial K}=\delta\eta_{Kp}(\eta_{Kp}\Delta K+D+\rho)^{\beta-1}M^{-\beta}\left\{(1+\beta)\eta_{Kp}K^{t+1}+D+\rho-\eta_{Kp}K^{t}\right\}.$$

Equation (11.33) should hold regardless of the form of equation (11.34). Rearranging equation (11.33) with respect to $\partial p_A/\partial X$ we get

$$(11.35)\quad \frac{\partial p_A}{\partial X}=\frac{1}{a^2b(1-a_{jj})K^{2b-1}h^{*2\alpha}}\Big\{cdh^*K^{d-1}w+(i+d_e)\eta_{Kp}$$

$$+\frac{\partial i^{**}}{\partial K}+abK^{b-1}h^*\sum_{(i\neq j)}p_i a_{ij}-ab(1-a_{jj})p_AK^{t-1}h^{*\alpha}\Big\}.$$

Simplifying the notations as

$$F_1=a^2b(1-a_{jj})K^{2b-1}h^{*2\alpha}\ ,\quad F_2=cdh^*K^{d-1}w\ ,$$

$$F_3=(i+d_e)\eta_{Kp}\ ,\quad F_4=\frac{\partial i^{**}}{\partial K}\ ,$$

$$F_5=dbK^{b-1}h^*\sum_{(i\neq j)}p_i a_{ij}\ ,\quad F_6=ab(1-a_{jj})p_AK^{t-1}h^{*\alpha}\ ,$$

then we can write simply as

$$(11.36)\quad \frac{\partial p_A}{\partial X}=\frac{1}{F_1}\left\{F_2+F_3+F_4+F_5-F_6\right\}.$$

Although dp_A/dX should be derived from the anticipated demand equation of equation (11.24), its theoretical condition is

$$(11.37)\quad \frac{\partial p_A}{\partial X}<0\ ,$$

and the marginal revenue MR should be

$$(11.38)\quad MR=\frac{\partial p_A}{\partial X}X+p_A>0.$$

Therefore from equations (11.37) and (11.38), we obtain

$$(11.39)\quad 0>\frac{\partial p_A}{\partial X}>-\frac{p_A}{X}.$$

Substituting equation (11.36) into equation (11.39), we get

$$(11.40)\quad 0>\frac{F_2+F_3+F_4+F_5-F_6}{F_1}>-\frac{p_A}{X}.$$

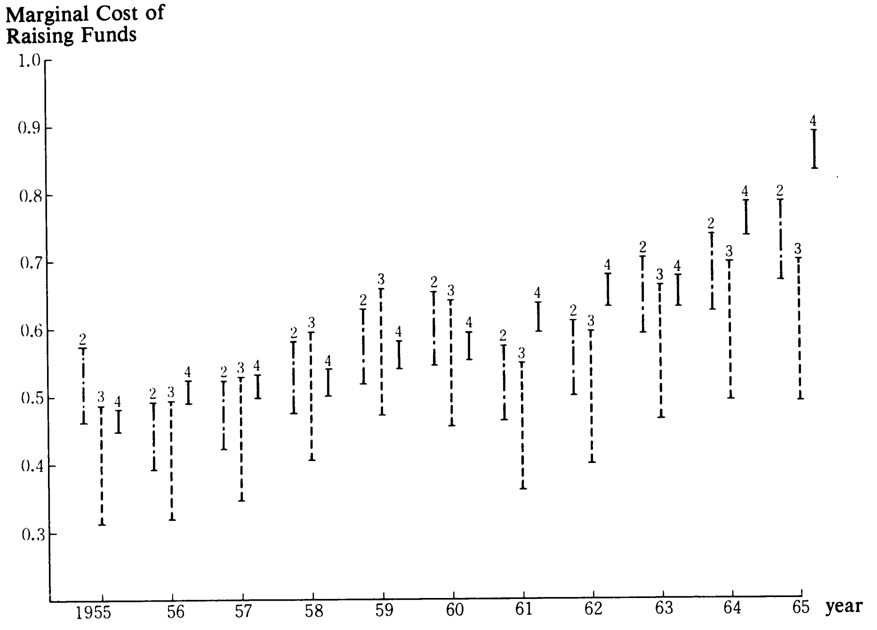
Consequently, we can derive the theoretical constraints on $\partial i^{**}/\partial K$ such as

(11.41) $$-F_2 - F_3 - F_5 + F_6 > F_4 = \frac{\partial i^{**}}{\partial K} > -\frac{p_A}{X} F_1 - (F_2 + F_3 + F_5 - F_6)\,.$$

The price p_A included in equation (11.41) is the anticipated price of the good of its own sector. However, we assume $p_A = p^t$ as the first approximation to determine quantitatively using equation (11.41) the legitimate range within which the value of $\partial i^{**}/\partial K$ is theoretically legitimate.

Figure 11.4 shows the theoretically legitimate ranges of $\partial i^{**}/\partial K$ computed on the basis of the data of parameters a, b, c, d, α of the production function, and $h^* = 200$, and the time series data of K^t, p_j^t, w^t, η_{kp} for the years from 1955 to 1965. Table 11.1 presents the upper and lower limits as well as the median of the computed ranges.

Figure 11.4 THEORETICALLY LEGITIMATE RANGES FOR MARGINAL COST OF RAISING FUNDS



Note: The ranges shown by vertical lines with numbers 2, 3 and 4 represent the theoretically legitimate ranges for marginal cost of raising funds, respectively, for Sectors 2, 3 and 4, which are derived from the theoretical constraint expressed by equation (11.41) in this chapter.

Table 11.1 INTERVAL ESTIMATES OF MARGINAL COST OF RAISING FUNDS

	Sector 2			Sector 3			Sector 4		
	U.	L.	M.	U.	L.	M.	U.	L.	M.
1955	0.56986	0.46460	0.5172	0.48783	0.31202	0.3999	0.46380	0.43316	0.4485
1956	0.49111	0.39022	0.4407	0.49677	0.31968	0.4082	0.52552	0.49227	0.5089
1957	0.52346	0.42011	0.4718	0.53063	0.34339	0.4370	0.53473	0.49945	0.5171
1958	0.58091	0.47658	0.5287	0.59642	0.40529	0.5009	0.54023	0.50478	0.5225
1959	0.62972	0.52281	0.5763	0.66060	0.47882	0.5697	0.58129	0.54369	0.5625
1960	0.65597	0.54700	0.6015	0.64145	0.46091	0.5512	0.59519	0.55683	0.5760
1961	0.57802	0.46975	0.5239	0.55430	0.36508	0.4597	0.63788	0.59689	0.6173
1962	0.61650	0.50658	0.5615	0.60059	0.40532	0.5030	0.68112	0.63723	0.6592
1963	0.71191	0.59768	0.6548	0.66928	0.46758	0.5684	0.76469	0.71699	0.7408
1964	0.74677	0.63034	0.6886	0.70600	0.50120	0.6036	0.79251	0.74232	0.7674
1965	0.79527	0.67551	0.7354	0.70972	0.49890	0.6043	0.89812	0.84382	0.8709

Note: Notations *U.*, *L.* and *M.* stand for, respectively, the upper boundary, lower boundary and the median of the theoretically legitimate range for marginal cost of raising funds.

The ranges thus derived are satisfactory regardless of the specific form of the equation which determines i^*. Now, we need to determine the values of parameters δ, β and ϱ so that $\partial i^{**}/\partial K$ satisfies each of the computed ranges.

In view of the results presented in Figure 11.4 and Table 11.1, the range for Sector 3 is relatively wide, or the theoretical constraints are less strict. This fact suggests that the fitting of equation (11.34) by the method will be relatively easier for Sector 3 compared with Sectors 2 and 4. So, let us first estimate the parameters for Sector 3 and then proceed to other sectors using the estimated values of the parameters as the initial values.

Assuming $D+\varrho=0$, and also $\beta=0$ in the third term of the right hand side of equation (11.34), we can approximate equation (11.34) simply as

$$(11.42) \qquad \frac{\partial i^{**}}{\partial K} \doteqdot \delta\eta_{Kp}(\eta_{Kp}\Delta K)^{\beta} M^{-\beta} = \delta\eta_{Kp}\left(\frac{\eta_{Kp}\Delta K}{M}\right)^{\beta} .$$

Then we estimate the regression equation of equation (11.42) whose dependent variable is the median of the range, denoted as MEI, for Sector 3 reported in Table 11.1.

$$\log(\text{MEI}) = -.45988032 + 0.22629015 \log\left\{\frac{\eta_{Kp}\Delta K}{M}\right\} .$$
$$(0.08842) \qquad (0.08169)$$

$$r^2 = 0.6784$$

Consequently we have the following estimates as the initial values for δ and β.

$$\hat{\delta} = 0.6313592\ .$$

$$\hat{\beta} = 0.22629015\ .$$

Needless to say, since these initial values are only crude approximations, these may differ considerably from the parameters of equation (11.34).

Therefore, we try the non-linear estimation of equation (11.34) using $\hat{\delta}$ and $\hat{\beta}$ as the initial values. The objective function in this case is given by

$$\phi = \sum_{t=1}^{11} [\text{MEI} - \delta\eta_{Kp}(\eta_{Kp}\Delta K+D+\rho)^{\beta-1}M^{-\beta} \times \left\{(1+\beta)\eta_{Kp}K^{t+1}+D+\rho-\eta_{Kp}K^{t}\right\}]^2\ , \tag{11.43}$$

where D is the balance of debt at the beginning of the period. However, since the suitable data of D for our model are hardly available, we made our estimation assuming

$$D^t = D^0 + \sum_{t=1}^{t-1} \eta_{Kp}\Delta K - \sum_{t=1}^{t-1} M, \quad \text{where } D^0 = 0.0\ .$$

It is conceivable in this context that parameter ϱ is to correct the result of approximation based on the assumption $D^0 = 0.0$ for $t = 1$.

We have fitted equation (11.43) by the iterative method using $\delta = 0.6313592$, $\beta = 0.22629015$, and $\varrho = 0.0$ as the initial values in such a way that the theoretical value of di^{**}/dK satisfies the ranges computed for all the years from 1955 to 1965. The result of the estimation is presented in Table 11.2. We have obtained the following estimates of parameters for Sector 3:

$$\delta = 0.30349119, \quad \beta = 0.15862120, \quad \varrho = 793.875.$$

For Sectors 2 and 4, we have carried out computations using equation (11.43) as the objective function. In this computation, we used the median MEI of the computed ranges for each sector reported in Table 11.1 and for the initial values the converged values of δ and β obtained for Sector 3. The initial value for ϱ was assumed to be 0.0. Table 11.3 and 11.4 present the results.

Since the value of parameter ϱ has converged to a negative value when equation (11.31) was used, we have reformulated i^* especially for Sector 4 as follows.

$$i^* = \delta\left(\frac{\eta_{Kp}\Delta K+D}{M+\rho}\right)^{\beta} + i\ . \tag{11.44}$$

Table 11.2 ESTIMATES OF MARGINAL COST OF RAISING FUNDS IN SECTOR 3

	U.	L.	ES
1955	0.48783	0.31202	0.4581
1956	0.49677	0.31968	0.4942
1957	0.53063	0.34339	0.4939
1958	0.59642	0.40529	0.5141
1959	0.66060	0.47882	0.5175
1960	0.64145	0.46091	0.4825
1961	0.55430	0.36508	0.5036
1962	0.60059	0.40532	0.5317
1963	0.66928	0.46758	0.5445
1964	0.70600	0.50120	0.5207
1965	0.70972	0.49870	0.5886

Note: Notations *U.* and *L.* represent, respectively, the upper and lower boundaries of the theoretically legitimate range for marginal cost of raising funds. *ES* stands for estimates of marginal cost of raising funds.

Table 11.3 ESTIMATES OF MARGINAL COST OF RAISING FUNDS IN SECTOR 2

	U.	L.	ES
1955	0.56986	0.46460	0.5015
1956	0.49111	0.39022	0.4908
1957	0.52346	0.42011	0.5203
1958	0.58091	0.47658	0.5625
1959	0.62972	0.52281	0.5892
1960	0.65597	0.54700	0.5470
1961	0.57802	0.46975	0.5481
1962	0.61650	0.50658	0.5928
1963	0.71191	0.59768	0.6044
1964	0.74677	0.63034	0.6332
1965	0.79527	0.67551	0.6784

Note: For notations, see the footnotes attached to Table 11.2.

Table 11.4 ESTIMATES OF MARGINAL COST OF RAISING FUNDS IN SECTOR 4

	U.	L.	ES
1955	0.46380	0.43316	0.4486
1956	0.52552	0.49227	0.5077
1957	0.53473	0.49945	0.5346
1958	0.54023	0.50478	0.5340
1959	0.58129	0.54369	0.5595
1960	0.59519	0.55683	0.5605
1961	0.63788	0.59689	0.6262
1962	0.68112	0.63723	0.6704
1963	0.76469	0.71699	0.7356
1964	0.79251	0.74232	0.7639
1965	0.89812	0.84382	0.8444

Note: For notations, see the footnotes attached to Table 11.2.

Therefore, in place of equation (11.34) we have

$$(11.45)\qquad \frac{\partial i^{**}}{\partial K} = \eta_{Kp}\delta(\eta_{Kp}\Delta K+D)^{\beta-1}\ (M+\rho)^{-\beta}\left\{(1+\beta)\eta_{Kp}K^{t+1}+D-\eta_{Kp}K^{t}\right\}.$$

Because of this revision, equations (11.28) and (11.30) must be revised for Sector 4 as follows:

$$(11.46)\qquad \frac{\partial \Pi}{\partial K} = -\frac{(1-a_{44})YY^{*}abK^{b-1}h^{*\alpha}\gamma_L}{(aK^{b}h^{*\alpha}-\gamma_L)^2} - abK^{b-1}h^{*\alpha}\sum_{(i\neq j)}p_i a_{ij}$$

$$-cdK^{d-1}h^{*}w-(i+d_e)\eta_{Kp}-\delta\eta_{Kp}(\eta_{Kp}\Delta K+D)^{\beta-1}$$

$$\times(M+\rho)^{-\beta}\left\{(1+\beta)\eta_{Kp}K^{t+1}+D-\eta_{Kp}K^{t}\right\}=0\ .$$

(11.47)

$$\frac{\partial^2 \Pi}{\partial K^2} = \frac{YY(1-a_{44})\gamma_L\cdot a\cdot b\cdot K^{b-2}h^{*}\left\{b(aK^{b}h^{\alpha}-\gamma_L)\cdot(aK^{b}h^{\alpha}-\gamma_L)\right\}}{(aK^{b}h^{*\alpha}-\gamma_L)^3}$$

$$-ab(b-1)K^{b-2}h^{*\alpha}\sum_{(i\neq j)}p_i a_{ij}-cd(d-1)K^{d-2}h^{*}w$$

$$-\eta_{Kp}{}^{2}\delta\cdot\beta(\eta_{Kp}\Delta K+D)^{\beta-1}(M+\rho)^{-\beta}\left\{(1+\beta)\eta_{Kp}K+2D-2\eta_{Kp}K^{t}\right\}<0\ .$$

Since the parameters thus estimated are constrained only by the theoretical constraints imposed by equation (11.31), these are still only the first order approximations of parameters associated with the optimal path of capital accumation as represented by equation (11.28).

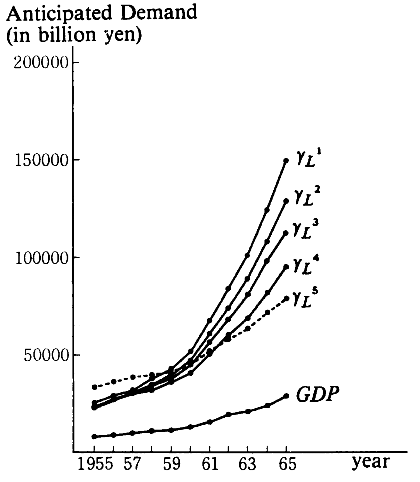
2. The Anticipated Demand Function

The parameters of the anticipated demand function will be derived by substituting the estimates of parameters $\hat{\delta}$, $\hat{\beta}$ and $\hat{\rho}$ of the cost of raising fund function into equation (11.28). Since the parameters of the production function are also known, equation (11.28) will become an equation which simply indicates the relationship between YY^* and γ_L when the data for variables K, h^*, p_j ($j = 1, \ldots, 4$), w, η_{kp}, D, M, i, d_e are given. In other words, by giving a specific value to parameter γ_L, we can impute the time series of the level of the anticipated demand, say for the period 1955 to 1965.

Theoretically speaking, $\gamma_L < 0$. Figure 11.5 shows alternative series of YY^* corresponding to alternative values of γ_L for Sector 2.

Figure 11.6 exhibits the levels of expected profits computed on the basis of profit equation (11.26) by giving values of anticipated demand YY^* which correspond to the values of γ_L for Sector 2. The profit equation is given by

Figure 11.5 TIME-SERIES MOVEMENTS OF THE LEVELS OF ANTICIPATED DEMAND CORRESPONDING TO ALTERNATIVE VALUES OF γ_L



Note: The lines indicated by $\gamma_L{}^i$ (i = 1, ..., s) represent the time-series movements of the levels of the anticipated demand for alternative values of $\gamma_L{}^i$:

$\gamma_L{}^1 = -4000$, $\gamma_L{}^2 = -5000$, $\gamma_L{}^3 = -6000$, $\gamma_L{}^4 = -9000$, and $\gamma_L{}^5 = -22000$.

The line indicated by GDP represents the time-series movement of the nominal gross domestic product with a lag of one year.

$$(11.48) \qquad \Pi = p_A X - C$$

$$= \frac{YY^*}{X-\gamma_L} X - \left\{ Lh^*w + K\eta_{Kp}(i^*+d_e) - p_A\, a_{jj}X - \sum_{(i\neq j)} p_i a_{ij} X \right\}$$

$$= \frac{(1-a_{jj})YY^* aK^b h^{*\alpha}}{(aK^b h^{*\alpha} - \gamma_L)} - \left[cK^d h^* w + K\eta_{Kp} \left\{ \delta\left(\frac{\eta_{Kp}\Delta K + D + \rho}{M}\right)^{\beta} \right. \right.$$

$$\left. \left. + i + d_e \right\} + \sum_{(i\neq j)} p_i a_{ij} aK^b h^{*\alpha} \right] .$$

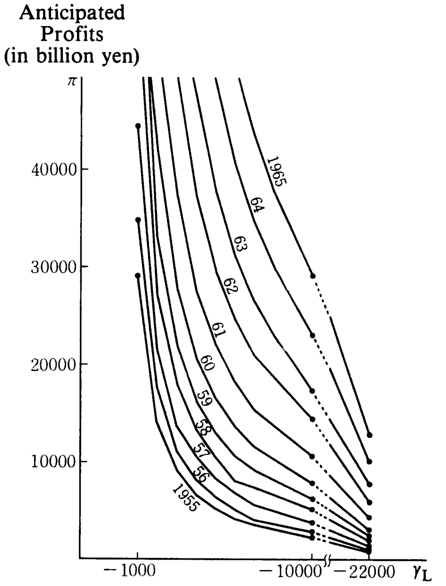
As long as the anticipated demand YY^* is the firm's anticipation of the future demand for its own product, it is very difficult to derive the appropriate function theoretically. The most appropriate method at this stage seems to be to select the variables which happen to show the greatest explanatory power out of various preceding indicators which is the index for firm's future perspectives. We decided to use in our case GDP^{t-1} for each sector as proxy variable of the most appropriate indicator for the amount of future demand. Out of a set of alternative series of anticipated demand as shown in Figure 11.5, we chose the series of anticipated demand which correlates most with the time series movement of GDP^{t-1}. The gamma value γ_L associated with that series was used as an initial value in an iterative convergence computation.

The series of anticipated profits exhibited in Figure 11.6 should correspond to the levels of profits attained during the current period which have been computed from equation (11.48) assuming $p_A = p_j^t$. This is the second condition by which to determine the initial value of γ_L.

On the basis of these two conditions, we adopted the following initial values for the three sectors under our consideration.
The coefficient of correlation between the anticipated demand YY^* and GDP^{-1} turned out to be 0.9969 for Sector 2, 0.9969 for Sector 3 and 0.9950 for Sector 4.

In an attempt to find out reliable variables which explain the movement of the anticipated demand YY^*, we have also tried regression analysis using the recursive method of regressing YY^* on alternative sets of independent variables selected among the preceding indicators such as accession rates, Dow-Jones industrial average in the Tokyo stock market, overtime hours, excess imports, GDP^{t-1}, official rate, and increasing in stock. The estimates of γ_L and YY^* thus derived are meaningful only as the initial values. Like the parameters δ, β and ϱ of the cost of raising fund function discussed earlier, estimation of these parameters also needs to be improved eventually by the non-linear estimation of equation (11.28).

Figure 11.6 ESTIMATES OF ANTICIPATED PROFITS CORRESPONDING TO ALTERNATIVE VALUES OF PARAMETER γ_L FOR DIFFERENT YEARS



Notes: (1) The anticipated profits corresponding to alternative values of γ_L are estimated using equation (11.48) on the assumption that the anticipated price is the same as the actually observed price, i.e., $p_A = p_j^t$.

3. The Demand Function for the Investment Goods

We have obtained so far approximate estimates of parameters of the cost of raising fund function and the anticipated demand function which are used to explain the demand for investment goods.

By giving the estimates of these parameters and of the parameters of the production function to equation (11.28), we can obtain the theoretically predicted values of optimal capital stock K^* for the period of 1955 to 1965. We can also obtain an estimate of net investment K^t from equation (11.29). When appropriate estimates are assigned to all the parameters of equation (11.28), its approximate solution can be obtained by the method of non-linear estimation such as the Newton method. Under our assumption of a one-year gestation period, the time-series of actual capital stock may be regarded as that of the optimal capital stock. Therefore, should the theoretically predicted values K^* derived from equation (11.28) differ from the actually observed values of capital stock, then the difference must be regarded as having been caused by the fact that the estimates of the parameters obtained earlier are only the first order approximations.

For the purpose of increasing the precision of the parameter estimates, we will attempt non-linear estimation by way of minimizing the objective function

$$\phi = \sum_{t=1}^{11} (K^{t+1} - K^{*t+1})^2 + \sum_{t=1}^{11} (\Delta K^t - \Delta K^{*t})^2 \quad ,$$

where

$$\Delta K^t = K^{t+1} - K^t, \quad \Delta K^{*t} = K^{*t+1} - K^{*t} .$$

Obviously, the objective variable ϕ is the function of parameters δ, β and ϱ of the cost of raising fund function and of γ_L and other parameters of the anticipated demand function. For the initial values to be used for the non-linear exploration, we shall use the estimates we have already obtained so far.

Because of limited degrees of freedom, we cannot try out alternative values for all the parameters simultaneously in the process of non-linear exploration. Therefore, we first explore the minimum value for ϕ by changing the values of four parameters δ, β, ϱ and γ_L while keeping YY^* constant. After having obtained the converged value, we shall again estimate the other parameters of the anticipated demand function by means of regressing the newly obtained theoretical values of YY^* recursively on the aforementioned variables which are regarded as the independent variables affecting YY^*. After having obtained the estimate of YY^* in this way, we again make a convergence computation of ϕ with respect to the four parameters δ, β, ϱ and γ_L. The minimum value for ϕ has been explored by repeating this computational procedure. The result obtained finally from this kind of repetitive computation is presented in Table 11.5 together with the already obtained estimates of parameters of the production function.

4. The Actually Measured Demand Function for Investment Goods in the Manufacturing and Service Sectors

As evidently seen from our estimation procedure explained above, we have been trying to minimize the gap between the theoretical specification and the specification for the empirical estimation. We tried to avoid the oft-used erroneous approach in which estimation is made by relatively simplistic equations which are devised to circumvent the difficulties associated with estimation and not necessarily derived strictly from the theoretical model. In contrast, in our approach, we have had to bear the burden of complex non-linear estimation. However, we have fulfilled our objective of identifying our theoretical model regorously.

Table 11.5 ESTIMATES OF STRUCTURAL PARAMETERS IN THE DEMAND FUNCTION FOR INVESTMENT GOODS

		Sector 2	Sector 3	Sector 4
$Q=aK^b$	$\log a$	−4.7953674	−1.0820038	−4.9290430
	b	1.15550111	0.99563189	1.1905800
$L=cK^d$	$\log c$	7.155600	5.05610	8.9533
	d	0.1789	0.3940	0.1885
$X=Qh^\alpha$	α	0.8173843	0.43188414	0.654125
	δ	0.0931333	0.3083738	0.09330591
	β	0.5325985	0.1519804	1.697421
	ϱ	2631.551	5575.125	5621.7490
	d_e	0.1217	0.1217	0.1588
	α_L	25419.738	2866.575	1543.1532
	β_L	2.2144873	4.5797085	4.2895909
	γ_L	−21769.9	−14060.67	−74968.75
	η_L	−956.61477	−2307.9288	−2592.9616
	ε_L			−89.71262
	λ_L	1501.5531	3121.8331	3581.5803
the value of objective function		−159106.0	−459822.3	−728369.9

Note: Parameters log a and b are the parameters of equation (7.6), log c and d are of equation (7.7), σ, β, ϱ and d_e are of equation (11.32). Parameters α_L, β_L, γ_L, η_L, ε_L and λ_L are the parameters of the long-run anticipated demand function

$$YY_j^* = \alpha_L + \beta_L x_1 + \gamma_L x_{2j} + \eta_L x_3 + \varepsilon_L x_{4j} + \lambda_L x_5 + u_j,$$

which has been found by the step-wise regression method to have most significant combination of relevant variables. In this equation YY_j^* stands for the anticipated demand, x_1 gross domestic product with one year lag, x_2 price of the output of the j-th sector relative to the general price index, x_3 the prime rate, x_4 inventory stock in the j-th sector with a one year lag, and x_5 a dummy variable allowing for changes in monetary policy.

We can assess, using the estimates of the parameters, the impact of changes in values of explanatory variables on investment demand. Out of the explanatory variables, retained earnings of private corporation and *GDP* presumably have a positive impact on investment demand. An increase in retained earnings will increase investment demand through a reduction in the cost of raising fund and an increase in *GDP* will increase investment demand through an increase of anticipated demand.

On the other hand, such variables as nominal wages, investment good prices, interest rate loans discounts of all banks, official rate, raw material prices and indirect tax rate are supposed to have a negative impact. Increases in nominal wages and prices of raw materials appear to increase investment demand by encouraging more capital intensive investment. However, one should note the fact that wages and prices usually increase with increases in the aggregate demand as represented for example by *GDP*. The net effect of an increase in wages or in raw material prices on the optimal level of capital stock, controlling the effects of other factors, is presumably negative.

An increase in the rate of interest will have negative effect on investment demand in two ways: one is by giving a depressing effect on the psychology of entrepreneurs and the other is by increasing the cost of raising fund. In our model, the former type of impact is represented by the prime rate included in the anticipated demand function and the latter type of impact is expressed by interest rate loans discounts of all banks included in the cost of raising fund function. Increases in prices of investment goods and in the rate of indirect taxes also have depressive impacts on investment demand through increasing the long-run cost of production.

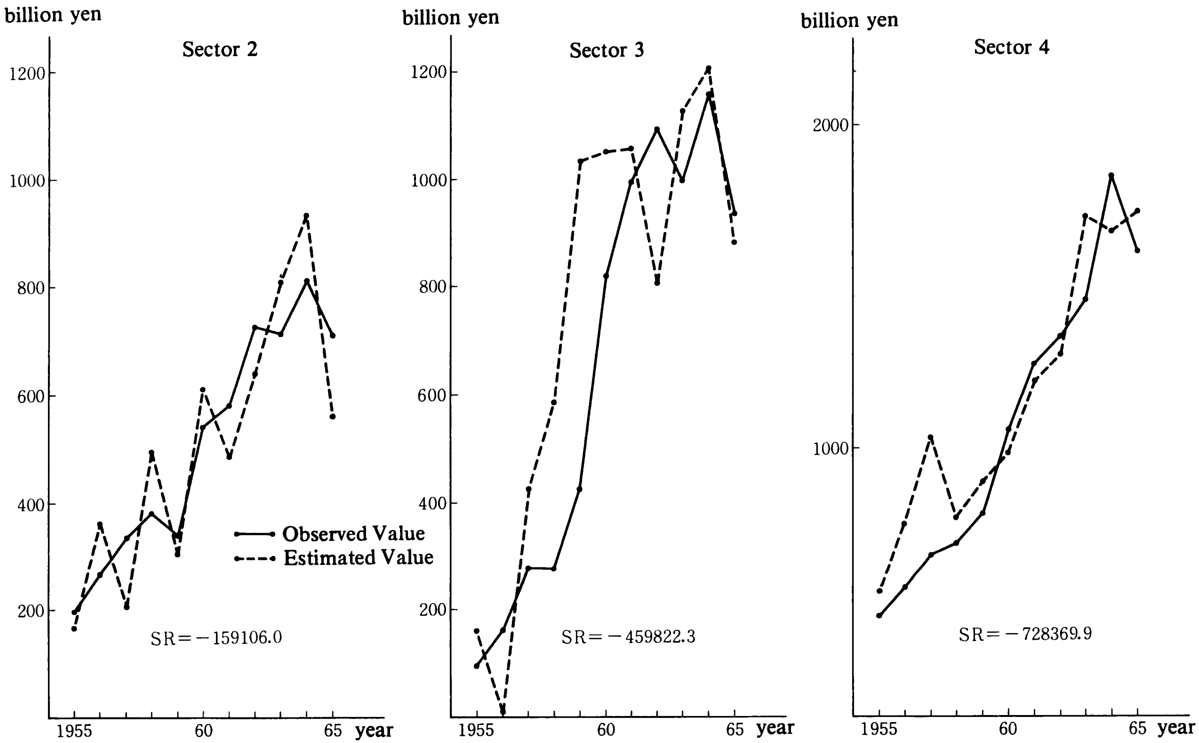
Table 11.6 presents the effect of a one percent increase in each these variables on the level of optimal capital stock.

The impact of an increase in retained earnings on investment demand is largest in Sector 2; one percent increase in retained earnings will increase the optimal capital stock by 0.1 to 0.2 percent. The impact of an increase in *GDP* on the volume of anticipated demand and hence its impact on capital stock turned out to be largest in Sector 3.

Other variables turned out to have negative effect on investment demand. It was found that changes in prices of investment goods had a large negative impact on investment demand. It was also found that the negative effect of an increase in the rate of interest through psychologically depressing the anticipated demand was greater than the negative effect through increasing the cost of raising fund. In the actual economy, these two effects take place simultaneously reinforcing with each other. It should be born in mind that the reported result in Table 11.6 is simply the result of a partial analysis of investment demand and not the result after taking into account the interactive effects of other segments of our entire model.

Figure 11.7 exhibits for three sectors separately the time-series movements of theoretically predicted values and actually observed values of investment demand. The figure shows that the theoretically-predicted values fluctuate more than the actually-observed values for all three sectors. This finding seems to suggest that somewhat finer adjustments are being made than what has been specified in our model in the process of determining the actual investment demand. This point deserves further investigation in the future.

Figure 11.7 ESTIMATES AND OBSERVED VALUES OF DEMAND FOR INVESTMENT



Note: Each of the dotted lines represents the time-series of estimates obtained on the basis of a single demand function for investment goods for each sector. The solid lines represent the time-series of actually observed data. SR stands for the sum of squared residuals.

Table 11.6 THE DEMAND ELASTICITIES FOR INVESTMENT GOODS WITH

Year	Sector 2	Sector 3	Sector 4	Year	Sector 2	Sector 3	Sector 4
Nominal Wages				Retained Earnings			
1955	−0. 0670	−0. 1253	−0. 5492	1955	0. 2678	0. 0782	—
1956	−0. 0476	−0. 1529	−0. 3224	1956	0. 2221	0. 0695	—
1957	−0. 0575	−0. 1369	−0. 3051	1957	0. 2105	0. 0547	—
1958	−0. 0377	−0. 1015	−0. 3009	1958	0. 2169	0. 0564	—
1959	−0. 0471	−0. 1235	−0. 1863	1959	0. 2040	0. 0434	0. 0233
1960	−0. 0434	−0. 0997	−0. 1924	1960	0. 1737	0. 0544	0. 0148
1961	−0. 0323	−0. 0798	−0. 1484	1961	0. 1614	0. 0443	0. 0330
1962	−0. 0356	−0. 0856	−0. 1278	1962	0. 1542	0. 0342	0. 0320
1963	−0. 0358	−0. 0725	−0. 1006	1963	0. 1487	0. 0396	0. 0387
1964	−0. 0308	−0. 0723	−0. 0899	1964	0. 1437	0. 0329	0. 0539
1965	−0. 0334	−0. 0771	−0. 0773	1965	0. 1421	0. 0337	0. 0492
Deflator of Investment Goods				GDP			
1955	−0. 8435	−0. 6263	−1. 0374	1955	0. 8436	2. 1296	2. 0747
1956	−0. 7296	−0. 6674	−0. 8955	1956	0. 8249	2. 0439	1. 4685
1957	−0. 7082	−0. 5886	−0. 7846	1957	0. 8612	1. 7522	1. 3077
1958	−0. 6967	−0. 5301	−0. 8168	1958	0. 8111	1. 6919	1. 3327
1959	−0. 6827	−0. 5679	−0. 7451	1959	0. 7455	1. 7656	1. 1177
1960	−0. 5861	−0. 4896	−0. 7405	1960	0. 7598	1. 3599	1. 0214
1961	−0. 5272	−0. 3991	−0. 6930	1961	0. 7531	1. 0819	0. 8741
1962	−0. 5338	−0. 3937	−0. 6711	1962	0. 6998	1. 0099	0. 7670
1963	−0. 4969	−0. 3692	−0. 6502	1963	0. 6711	0. 9099	0. 6579
1964	−0. 4772	−0. 3485	−0. 6383	1964	0. 6466	0. 8616	0. 5934
1965	−0. 4847	−0. 3471	−0. 6116	1965	0. 6268	0. 8003	0. 5202
Interest Rate of Loan Discounts of All Banks				Prime Rate			
1955	−0. 0670	−0. 0626	−0. 3051	1955	−0. 2678	−0. 7359	−0. 9153
1956	−0. 0635	−0. 0695	−0. 1791	1956	−0. 2697	−0. 8204	−0. 6806
1957	−0. 0574	−0. 0684	−0. 1734	1957	−0. 2488	−0. 6297	−0. 5667
1958	−0. 0566	−0. 0564	−0. 1720	1958	−0. 2452	−0. 5978	−0. 5804
1959	−0. 0471	−0. 0617	−0. 0931	1959	−0. 1883	−0. 5062	−0. 3493
1960	−0. 0543	−0. 0544	−0. 1036	1960	−0. 1628	−0. 3445	−0. 3108
1961	−0. 0430	−0. 0354	−0. 0825	1961	−0. 1183	−0. 2040	−0. 2144
1962	−0. 0474	−0. 0428	−0. 0639	1962	−0. 1186	−0. 1883	−0. 1758
1963	−0. 0359	−0. 0330	−0. 0387	1963	−0. 0819	−0. 1253	−0. 1087
1964	−0. 0308	−0. 0329	−0. 0357	1964	−0. 0616	−0. 0987	−0. 0809
1965	−0. 0334	−0. 0337	−0. 0281	1965	−0. 0585	−0. 0820	−0. 0633

RESPECT TO VARIOUS VARIABLES

Year	Sector 2	Sector 3	Sector 4	Year	Sector 2	Sector 3	Sector 4
Deflator of Sector 1				Deflator of Sector 4			
1955	−0.6695	−0.0470	—	1955	−0.2410	−0.2975	
1956	−0.5710	−0.0417	—	1956	−0.2538	−0.3198	
1957	−0.5168	−0.0274	—	1957	−0.2488	−0.3012	
1958	−0.4432	−0.0226	—	1958	−0.1981	−0.2707	
1959	−0.3923	−0.0247	—	1959	−0.1883	−0.2593	
1960	−0.3690	−0.0181	−0.0148	1960	−0.1737	−0.2176	
1961	−0.3335	−0.0089	—	1961	−0.1829	−0.2128	
1962	−0.2965	−0.0086	−0.0159	1962	−0.1661	−0.1968	
1963	−0.2612	−0.0066	—	1963	−0.1537	−0.1780	
1964	−0.2257	−0.0066	—	1964	−0.1334	−0.1710	
1965	−0.2173	−0.0096	—	1965	−0.1254	−0.1639	
Deflator of Sector 2				Indirect Tax Rates			
1955		−0.4071	−0.2441	1955	−0.2410	−0.0157	−0.0610
1956		−0.3893	−0.1433	1956	−0.1903	−0.0278	−0.3582
1957		−0.3559	−0.1308	1957	−0.1723	−0.0274	−0.0872
1958		−0.2933	−0.1290	1958	−0.1509	−0.0226	−0.0645
1959		−0.2592	−0.0931	1959	−0.1413	−0.0250	−0.0233
1960		−0.2357	−0.1036	1960	−0.1194	−0.0181	−0.0444
1961		−0.2217	−0.0825	1961	−0.1183	−0.0088	−0.0330
1962		−0.2139	−0.0799	1962	−0.1068	−0.0171	−0.0320
1963		−0.1978	−0.0542	1963	−0.0871	−0.0132	−0.0155
1964		−0.1842	−0.0539	1964	−0.0718	−0.0132	−0.0179
1965		−0.1832	−0.0422	1965	−0.0835	−0.0145	−0.0211
Deflator of Sector 3							
1955	−0.1071		−0.1831				
1956	−0.1269		−0.0716				
1957	−0.1340		−0.1308				
1958	−0.0943		−0.1075				
1959	−0.1177		−0.0931				
1960	−0.1194		−0.0740				
1961	−0.1183		−0.0659				
1962	−0.1068		−0.0639				
1963	−0.1025		−0.0464				
1964	−0.0923		−0.0539				
1965	−0.0919		−0.0422				

11.3 The Demand for Investment Goods in the Agricultural Sector

1. The Model

The structure of agricultural production is represented, as reported earlier in Chapter 7, appropriately by the Cobb-Douglas type production function, i.e.

$$X_1 = a_1 A_1^{b_1} L_1^{1-b_1} (K_1 + K_{g1})^{c_1} , \tag{11.49}$$

where A_1 is the cultivated acreages, L_1 is the number of workers employed, K_1 is capital stock and K_{g1} is the public capital stock allocated to the agricultural sector.

The size of the agricultural labor force is determined, in our model, by subtracting non-agricultural employment from the total labor force in the economy for a given level of capital stock at the beginning of the each period. The level of production X_1 is therefore determined by such exogenous variables as L_1 (which is pre-determined by the factors exogenous to the agricultural sector), A_1, K_1, and K_{g1}. Consequently, the output X_1 is not necessarily the optimal amount for profit maximization in the short-run. In view of this, one may interpret investment in agriculture as the adjustment of capital stock for the purpose of attaining the optimal supply capacity for the future under the fixed size of agricultural labor force L_1 which is given in the short-run. While the price of agricultural produce is taken as given exogenously in the short-run, the price which is taken into account in investment decisions is the future price anticipated on the basis of the time-series of past prices which have been given exogenously.

Let us formulate the equation of anticipated price

$$p^t_{A1} = \alpha + \beta p_1{}^t + \gamma \Delta p_1{}^t , \tag{11.50}$$

where p_{A1} is the anticipated price taken into account when investment in the t-th period is made, $p_1{}^t$ is the exogenously determined price of agricultural products in the t-th period, and $\Delta p_1{}^t$ is the rate of increase in the price during the t-th period.

The cost of raising fund function should be formulated taking into account the fact that the dependence on borrowed capital and the weight of juridical corporations are relatively small in the agricultural sector relative to other sectors. The cost of raising fund function is therefore expressed as

$$i^* = \delta (\eta_{Kp} \Delta K)^{\epsilon} + i , \tag{11.51}$$

where η_p is the price of investment good, i is interest rate of loan discounts of all banks, and δ and ε are parameters. Based on these assumptions, we can formulate the mechanism of determination of agricultural investment demand as follows.

Profit Π_1 is defined as

$$\Pi_1 = p_{A1} X_1 - C_1 \ . \tag{11.52}$$

The total cost C_1 is defined as

$$C_1 = L_1 h_1 w_1 + K_1 \eta_{Kp} (i^* + d_{e1}) + \sum_{i=1}^{4} p_i a_{i1} X_1 \ . \tag{11.53}$$

When the size of the labor force L_1 and the wage level per worker $h_1 w_1$ are given at the level at the beginning of the t-th period, and the price of investment good $\eta^t{}_{Kp}$ is given exogenously, then we can determine the profit maximizing capital stock $K_1{}^*$ subject to the constraint of the production function. The necessary condition for the profit maximization is

$$\frac{\partial \Pi_1}{\partial K_1} = p_{A1} \cdot \frac{\partial X_1}{\partial K_1} - \eta_{Kp} (i^* + d_e) - K \eta_{Kp} \left(\frac{\partial i^*}{\partial K_1}\right) - \frac{\partial X_1}{\partial K_1} \Sigma p_i a_{i1} = 0 \ . \tag{11.54}$$

From equations (11.49) and (11.51), we obtain

$$\frac{\partial X_1}{\partial K_1} = a_1 \cdot c_1 A_1{}^{b_1} (K_1 + K_{g1})^{c_1 - 1} \ . \tag{11.55}$$

$$\frac{\partial i^*}{\partial K_1} = \delta \epsilon (\eta_{Kp})^{\epsilon} (K^{t+1} - K^t)^{\epsilon - 1} = \frac{\epsilon (i^* - i)}{\Delta K} \ . \tag{11.56}$$

Substituting equations (11.55) and (11.56) into (11.54) and rearranging, we obtain

$$\begin{aligned} \frac{\partial \Pi_1}{\partial K_1} &= (p_{A1} - \Sigma p_i a_{i1}) \left(\frac{\partial X}{\partial K}\right) - \eta_{Kp} (i^* + d_e) - K \eta_{Kp} \left(\frac{\partial i^*}{\partial K}\right) \\ &= (p_{A1} - \Sigma p_i a_{i1}) a_1 \cdot c_1 A_1{}^{b_1} (K + K_g)^{c_1 - 1} \\ &\quad - K \eta_{Kp}{}^2 \left\{ \delta \epsilon (\eta_{Kp} \Delta K)^{\epsilon - 1} \right\} - \eta_{Kp} \left\{ \delta (\eta_{Kp} \Delta K)^{\epsilon} + i + d_e \right\} = 0. \end{aligned} \tag{11.57}$$

Solving equation (11.57) for K_1, we can get the optimal capital stock $K_1{}^*$. The demand for investment goods will be obtained consequently as

$$\Delta K_1{}^t = K_1{}^{*t} - K_1{}^{t-1} \ . \tag{11.58}$$

The gestation period of investment is assumed, like other sectors, to be one year and the necessary adjustment is assumed to be completed also within the period of one year.

2. Estimation

Since the parameters a_1, b_1, and c_1 of the production function are already known, all we have to do now is to estimate the parameters of the anticipated price function (11.50) and the cost of finance function (11.51). Substituting equation (11.51) into (11.57), we may rewrite equation (11.51) as

(11.59)

$$(p_{A1}-\Sigma p_i a_{i1})a_1\cdot c_1\cdot A_1{}^{b_1}L^{1-b_1}(K+K_g)^{c_1-1}-K\eta_{Kp}\frac{\epsilon(i^*-i)}{\Delta K}-\eta_{Kp}(i^*+d_e)=0.$$

The parameters in equation (11.56) have to be estimated, as in the case of non-agricultural sectors, by the method of non-linear estimation. We first have to specify the initial values for the non-linear exploration. Since observed values for p_{A1} are hardly available, we will use the actual price p_1 in place of the anticipated price p_{A1}. Replacing p_{A1} by p_1 and giving a concrete value for ε in equation (11.60), we can impute the time-series of i^* by

$$\text{(11.60)}\qquad \hat{i}^{*t}=\frac{(p_1-\Sigma p_i a_{i1})a_1\cdot c_1\cdot A_1{}^{b_1}(K+K_g)^{c_1-1}+\dfrac{\epsilon K\eta_{Kp}\cdot i}{\Delta K}-\eta_{Kp}d_e}{\left\{\dfrac{(1+\epsilon)K^{t+1}-K^t}{\Delta K^{t+1}}\right\}\cdot\eta_{Kp}}$$

Using the thus imputed values of $\hat{i}^*$ for 1955 to 1965, we can obtain estimates of $\hat{\delta}^t$ for each year during the same period which are compatible with equation (11.51) using the relationship

$$\text{(11.61)}\qquad \hat{\delta}^t=\frac{(\hat{i}^*-i)}{(\eta_{Kp}\Delta K)^{\epsilon}}\qquad t=1,\ldots,11\ .$$

Since $\hat{\delta}$ is a parameter, it should in principle be constant from year to year. Therefore, the estimate $\hat{\delta}^t$ derived from equation (11.61) is better the smaller the variance associated with it. Since $\hat{\delta}$ in equation (11.61) can be uniquely determined for an arbitrary value of ε, we choose the average of $\hat{\delta}^t$ as the first approximation of $\hat{\epsilon}$ which makes the variance of $\hat{\epsilon}$ smallest. The first approximation obtained in this way are

$$\hat{\epsilon}=3.2413,\quad \hat{\delta}=0.2476\times 10^{-6}\ .$$

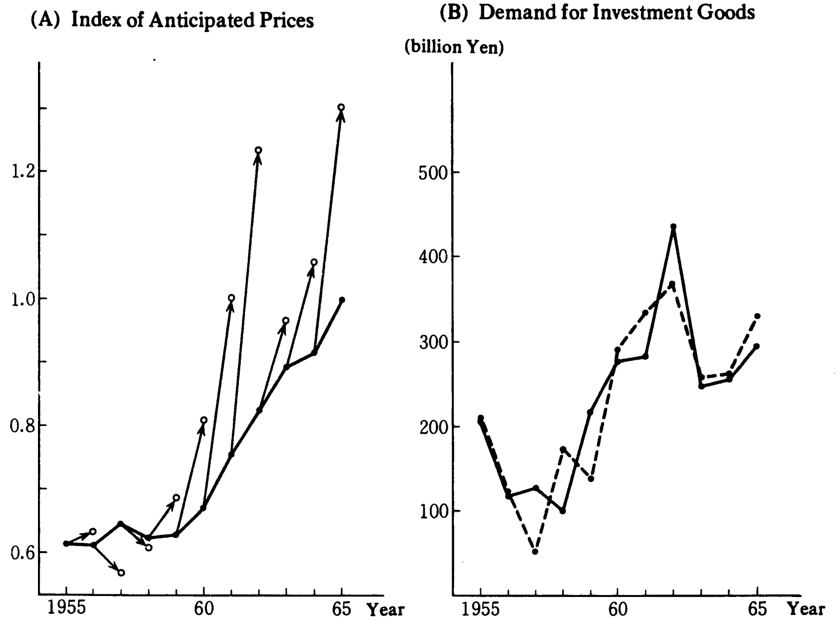
By substituting the first approximations of ε, δ into equation (11.59), we can now calculate the theoretically predicted values of the anticipated prices p_A by the following equation,

$$\text{(11.62)}\qquad \hat{p}_A=\left\{\frac{\eta_{Kp}(i^*+d_e)-K\eta_{Kp}\epsilon(i^*-i)/\Delta K}{a_1\cdot c_1\cdot A_1{}^{b_1}L^{1-b_1}(K+K_g)^{c_1-1}}-\sum_{(i\neq j)}p_i a_{i1}\right\}/(1-a_{11})$$

The time-series of the estimates of the anticipated price p_A are compatible with the first approximations of ε and δ. Using the series of p_A as the data for the dependent variable, we can estimate the parameters α, β and γ of the anticipated price function by applying the least squares method to equation (11.50).

The estimates of the parameters ε, δ of the cost of raising fund function and of parameters α, β and γ of the anticipated price function thus obtained are, needless to say, merely crude first approximations. We need to improve the precision of these approximations eventually by applying the method of non-linear estimation to the equation (11.59) using these estimates of the parameters as the initial values.

Figure 11.8 DEMAND FOR INVESTMENT GOODS IN THE AGRICULTURAL SECTOR



Notes: (1) In Panel (A), the notation •—• represnts the index of actually observed prices and →o represents the index of anticipated prices which is obtained on the basis of the estimated parameters of the anticipated price function. Both indices are standardized by setting the 1965 prices equal to 1.0.

(2) In Panel (B), the solid line represents the actually observed investment demand, and the dotted line the estimates of it derived from the estimated demand function.

In making the non-linear estimation, we set the objective function as

(11.63)

$$\phi = \sum_{t=1}^{11} [\{(1-a_{11})(\alpha+\beta p^t+\gamma\Delta p^t)-\Sigma p_i a_{i1}\} a_1 \cdot c_1 \cdot A_1^{b_1} L^{1-b_1} (K+K_g)^{c_1-1}$$
$$-\eta_{Kp}\{\delta(\eta_{Kp}\Delta K)^{\epsilon}+i+d_e\}-K\eta_{Kp}\delta\cdot\epsilon(\eta_{Kp})^{\epsilon}(\Delta K)^{\epsilon-1}]^2 .$$

and minimized the objective function ϕ with respect to parameters α, β, γ, δ and ε. Like the cases of other sectors, we used the pattern method of convergence computation in estimating the parameters. The process of convergence was quite quick and we obtained finally the following results:

$\alpha = -0.108824$, $\beta = 1.230032$, $\gamma = 3.38589$,
$\delta = 0.9822297 \times 10^{-8}$, $\epsilon = 2.2125234$.

Figure 11.8(A) shows the anticipated prices obtained on the basis of the final estimates of the parameters in contrast to the actually observed prices. Figure 11.8(B) presents the theoretically predicted values (dotted line) of investment demand in contrast to the actually observed values (solid line).

Notes to Chapter 11

1) Many useful surveys on investment functions are available. See for example Meyer and Kuh(1957), Eisner and Strotz(1963) and Jorgenson(1971).
2) Keynes(1936).
3) Haavelmo(1960) has emphasized the need to specify, on the basis of Neo-Classical theory of optimal capital accumulation, the adjustment process especially for the purpose of deriving the demand for investment goods. In contrast to Haavelmo's comparative static theory, Jorgenson(1963), (1965) and (1967) formulated the theory of comparative dynamics for investment demand also within the framework of Neo-Classical theory of optimal capital accumulation.
4) A formal specification of adjustment processes has been attempted early by Clark(1917) in terms of his simple flexible accelerator model. Numerous studies have been tried since then by means of applying alternative distributed lag models such as geometric distributed lag function, rational distributed lag function, etc. See for example Chenery(1952), Koyck(1954), Eisner(1960) and Jorgenson(1966).
5) Duesenberry(1958) and Meyer and Kuh(1957).
6) Modigliani and Miller(1958).
7) Ihara(1958).

Chapter 12
The Determination of Other Final Demands and Demand and Supply of Money

Of the major items of final demand, we have discussed so far the theory and measurement of private consumption demand and investment demand. In addition to these items the final demand includes such items as business consumption expenditure, general government consumption expenditure, gross fixed capital formation by government, private housing investment, inventory increases, demand for exports, and demand for imports etc. One of our objectives in this chapter is to explain how these items are dealt with in our model. The other objective is to explain the treatment of demand and supply of money in our model.

12.1 The Determination of Other Final Demands

The final demand items such as business consumption expenditure, general government consumption expenditure, the fixed-capital formation by government and private housing investment are treated as exogenous variables.

We have decided to treat business consumption expenditure as an exogenous variable for the following reasons. The data for this variable are available for the period of our investigation only for the years, 1955, 1960 and 1965, for which the Input-Output Tables are available. We obtained the data for years between these three years by interpolation. These tentative data obtained by interpolaton are naturally not sufficiently reliable to be analyzed by an independent model of firm's behavior. Moreover, it is quite difficult at this stage to introduce an effective hypothesis to explain the movements of a variety of business consumption expenditure such as expenditures for social relations, reception, etc. It is for these reasons that we treat business consumption expenditure as an exogenous variable.

General government consumption expenditure, the fixed capital formation and inventory increases by government are treated as policy variables. When these variables are given exogenously we can obtain the saving of general government according to the following formula of revenues and expenditures of general government.

Total revenue of general government
= Personal direct taxes and charges T_P + Corporation income taxes and charges T_c + Social insurance contributions TR_{sI} + Transfers from households and private non-profits institutions to government TR_{pG} + Transfers from the rest of the world to government TR_{RG} + General government income from property and entrepreneurship $\sum_{i=1}^{4} Y_{Gi}$. Indirect taxes T_I − Interest on public debt $\sum_{i=1}^{4} D_{CGi}$.

Total expenditure of general government
= General government consumption expenditure $\sum_{i=1}^{4} G_i$ + Transfers from government to households TR_{Gp} + Transfers from government to the rest of the world TR_{GR} + Current subsides $\sum_{i=1}^{4} S_{ci}$ + Inventory increase in general government $\sum_{i=1}^{4} I_{NVGi}$ + Gross fixed capital formation by government $\sum_{i=1}^{4} I_{Gi}$.

Saving of general government
= Total revenue − Total expenditure.

Private housing investment is also treated as an exogenous variable. Private housing investment appears to be affected significantly by such extraneous factors as the price of land and private assets holdings. Therefore, it may be possible to treat private housing investment endogenously in our model when sufficient information on these extraneous variables is made available.

Let us explain next our treatment of private inventory increases in some detail. Inventory increases should be explained, in principle, by demand-supply balances in the market, inventory finance and the level of optimal inventory stock. However, it is very difficult to construct elaborate sectoral models incorporating these elements because of the paucity of relevant data.

The data of net inventory increases obtainable from the Input-Output Table are classified by commodities and not by industries which hold them. In order for us to explain the incentives for holding inventories, we need on the one hand the disaggregated data for each of commodity inventories listed in the Input-Output Table, and on the other hand the data which indicate the holders of these inventories. The Input-Output Table of 1965 reports separately the detailed data on private net inventory increases classified by such categories as increase of finished goods in producer's stocks, increase of half-finished goods and work-in-process, increase of goods in dealer's stocks and increase of raw materials in stocks. However, it is quite difficult at this stage to reclassify these data by the sectors holding the inventories.

Because of this difficulty, we have taken an alternative approach. That is, we obtain from the National Income Statistics time series data of inventories classified by sectors holding them, for manufacturing and service sectors. We then assume that we can subdivide the inventory between the portion necessary for technological and institutional reasons and the portion which emerges due to changes in demand-supply balance in the market. We have used capital stock K_j at the beginning of each period and the rate of increase of output during the previous period GW_j^{t-1} as proxies representing the former and the latter portions, respectively. On the basis of these assumptions, we have formulated the following equation which determines the level of inventory stocks $SINV_j$,

$$SINV_j = \epsilon_{0j} + \epsilon_{1j}K_j + \epsilon_{2j}GW_j^{t-1} \quad (j = 2, 3, 4) \ , \tag{12.1}$$

where subscript j represents the number attached to each sector. Net inventory increases are obtained through the following relationship as,

$$INV_j = SINV_j^{t} - SINV_j^{t-1} \qquad (j = 2, 3, 4) \ . \tag{12.2}$$

The obtained result of estimation of equation (12.1) for the three sectors are respectively,

$$SINV_2 = -751.12357 + 0.57685501K_2 + 1577.4271GW_2^{t-1} \ , \tag{12.3}$$
$$(60.4058) \quad (0.01159) \quad (358.4357)$$
$$\bar{r} = 0.9986$$

$$SINV_3 = -211.52590 + 0.26928396K_3 + 914.56080GW_3^{t-1} \ , \tag{12.4}$$
$$(11.93521) \quad (0.02801) \quad (421.9863)$$
$$\bar{r} = 0.9780$$

$$SINV_4 = -315.70109 + 0.045739371K_4 + 497.86770GW_4^{t-1} \ , \tag{12.5}$$
$$(287.8357) \quad (0.002222) \quad (236.1142)$$
$$\bar{r} = 0.9927$$

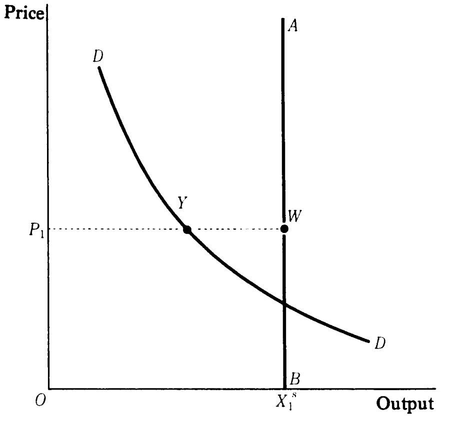
where the figures in parentheses are standard deviations of the respective parameters and r is the coefficient of correlation adjusted for the degree of freedom. The results for each sector are satistically significant. Net inventory increases are obtained from equation (12.2) using the results of equations (12.3) to (12.5). The net inventory increases classified by commodities will be obtained by converting these data into the commodity dimension through the quantity converters.

For the agricultural sector, as we have explained in detail in Chapter 7, the amount of supply X_1^s will be determined through the production function once the capital stock K_1 at the beginning of the period is given. The supply schedule in agriculture in this situation may be illustrated by a vertical line AB as shown in Figure 12.1.

The price p_1, on the other hand, is given exogenously. If the demand items except inventories are determined at point Y on the demand curve DD, then the excess supply YW will be determined after the fact. YW in this case corresponds to an inventory increase in the agricultural sector. This will constitute, together with inventory increases in manufacturing and service sectors, the vector of private inventory increases classified by commodities.[1]

In dealing with the balance of payments, we focus only on the current trade balance, outgoings and incomings of factor income from the rest of the world and transfer transactions are given exogenously and other capital transactions are not explicitly treated in our model.

Figure 12.1 AN EXPLANATION OF DEMAND-SUPPLY BALANCE IN THE AGRICULTURAL SECTOR



We have estimated export functions for the following six items in order to express more exactly the properties associated with export commodities. For these equations, η_{EXi} represents the implicit deflator for each good, EX_i is the real amount of export of each good, P_i is the world trade price of the i-th export good, and W_i is the quantity index of world trade of the i-th good. The figures in parentheses are standard deviations and $\bar{r}$ is the coefficient of correlation adjusted for the degree of freedom.

Foods and processed foods

$$(12.6) \quad \underset{}{\log(\eta_{EX1} \cdot EX_1/P_1)} = \underset{(5.8356)}{-18.283972} + \underset{(0.5768)}{2.2764686} \log(W_1/P_1)$$

$$\underset{(1.03411)}{-0.752038} \log(\eta_{EX1}/P_1) + \underset{(0.9110)}{3.0283893} \log(1/P_1) .$$

$$\bar{r} = 0.9742$$

Textiles

$$(12.7) \quad \eta_{EX2} \cdot EX_2/P_2 = \underset{(61.8192)}{-163.02734} + \underset{(0.005632)}{0.07900781} (W_2/P_2) + \underset{(76.6019)}{149.35880} (1/P_2) .$$

$$\bar{r} = 0.9879$$

Chemical Products

$$(12.8) \quad \log(\eta_{EX3} \cdot EX_3/P_3) = \underset{(2.5213)}{-12.16417} + \underset{(0.2793)}{1.873850} \log(W_3/P_3)$$

$$\underset{(0.8433)}{-1.7770} \log(\eta_{EX3}/P_3) - \underset{(2.2197)}{1.615010} \log(1/P_3) .$$

$$\bar{r} = 0.9898$$

Metal Products

$$(12.9) \quad \log(\eta_{EX4} \cdot EX_4/P_4) = \underset{(1.3664)}{-11.278056} + \underset{(0.1471)}{1.8533378} \log(W_4/P_4)$$

$$\underset{(0.21991)}{-2.175038} \log(\eta_{EX4}/P_4) . \qquad \bar{r} = 0.9940$$

Machinery

$$(12.10) \quad \log(\eta_{EX5} \cdot EX_5/P_5) = \underset{(3.1829)}{-11.54084} + \underset{(0.3059)}{1.7596699} \log(W_5/P_5)^{t-1}$$

$$\underset{(0.4181)}{-0.551537} \log(\eta_{EX5}/P_5) . \qquad \bar{r} = 0.9826$$

Miscellaneous Items

$$(12.11)\quad \eta_{EX_6} \cdot EX_6/P_6 = \underset{(40.1822)}{289.71353} + \underset{(0.002821)}{0.04809715}(W_6/P_6) - \underset{(129.3359)}{132.7732}(\eta_{EX_6}/P_6)$$

$$-\underset{(344.9308)}{308.548}(1/P_6) \qquad \bar{r} = 0.9975$$

P_i and W_i are treated as exogenous variables. These results are satisfactory in terms of statistical significance. Using these results we can compute the price elasticity of export for each good except textiles. Table 12.1 presents such elasticities. The elasticities for chemical and metal products are higher than those for foods, machinery and miscellaneous items.[2]

In estimating the export functions, we have used implicit deflators for 6 items. For the purpose of combining these 6 deflators with the four deflators of the four sectors of our model, we constructed price converters using the following equation,

$$(12.12)\qquad \eta_{EX_i} = a_{0i} + a_{1i}p_1 + a_{1i}p_2 + a_{2i}p_3 + a_{3i}p_4 \ .$$

This regression equation provides converter coefficients by means of expressing the price of export goods as the weighted average of prices of domestic sectoral products. Table 12.2 presents the estimates of converter coefficents for each export good.

On the other hand, coefficients of imports are obtained from time-series data on the basis of the relations,

$$(12.13)\qquad M_j = m_j X_j\ , \quad (j = 1, \ldots, 4)\ ,$$

Table 12.1 PRICE ELASTICITIES OF EXPORT GOODS

	Price elasticity
food	−0.7520
textiles	—
chemicals	−1.7700
metals	−2.1750
machinery	−0.5515
miscellaneous	−0.3183

Note: The price elasticities of export goods are derived from the estimated export functions expressed by equations (12.6) to (12.11).

Table 12.2 PRICE CONVERTERS OF EXPORT GOODS

	a_0	a_1	a_2	a_3	a_4
Food	0.728450	0.609670	−0.137830	−0.109500	−0.072400
Textile	−0.719850	−0.227640	2.051990	0.007228	−0.099670
Chemicals	2.849970	2.059050	−1.799260	1.901919	−4.133190
Metals	−0.906302	−0.019944	0.428554	2.896497	−1.467740
Machinery	2.427358	−2.277590	−1.682520	0.254914	2.332721
Miscellaneous	0.677619	0.973714	−1.097138	0.562961	−0.204150

Note: The converter coefficients are derived from the estimates of parameters of equation (12.12).

where M_j is import imported by the j-th sector, X_j is the product of the j-th sector, and m_j is the import coefficient for the j-th sector. Table 12.3 presents the estimates of import coefficients together with all the necessary data to compute them. As can be seen in the bottom row of Table 12.3, the estimated import coefficients have been quite stable for each sector.

12.2 Demand and Supply of Money

In order to determine the absolute levels of prices, we have to incorporate money demand and supply explicitly into our model.

In the well known Walrasian system of general equilibrium, the money price p_m of any one of n goods is used as a neumeraire to determine the relative money prices p_i/p_m ($i = 1, \ldots$, n) of the remaining $n - 1$ goods. The price p_m of the neumeraire in the physical sector will be determined by introducing either Fisher's equation of exchange, $MV = pT$ (M: the quantity of money; V: velocity; p: the level of prices; T: the amount of transaction) or Marshallian quantity equation, $M = kY$ ($k = 1/V$, $Y = pT$). Consequently, price p_i of each of the remaining $n - 1$ goods will become meaningful as a money price.[3]

While the monetary block will have to be treated more autonomously in the future, we considerably simplify the treatment of the monetary sector at this stage since our immediate objectives is to describe autonomously the physical aspect of the economy. For the sake of simplicity, we have assumed that demand for money consists only of demand for currency, which is classified by demand of individual persons, firms and the government. The last component of the demand, namely the government currency demand M_G

Table 12.3 IMPORTS AND IMPORT COEFFICIENTS

	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
Nominal Imports											
Sector 1	391. 0	458. 2	463. 8	397. 8	470. 8	581. 0	665. 9	628. 3	781. 2	844. 0	918. 7
Sector 2	391. 8	488. 8	651. 2	483. 9	540. 1	716. 5	871. 7	920. 4	1194. 5	1406. 6	1571. 5
Sector 3	133. 2	280. 8	472. 2	259. 7	375. 6	487. 3	677. 3	592. 2	654. 2	806. 3	721. 2
Sector 4	−25. 4	−30. 3	−41. 8	1. 3	7. 2	−63. 3	0. 2	6. 1	−15. 2	−28. 6	−44. 9
Output											
Sector 1	4084. 4	4070. 1	4259. 3	4313. 8	4614. 3	4663. 1	4816. 8	4911. 1	4878. 9	5036. 8	5101. 6
Sector 2	6952. 6	7697. 8	8391. 2	8409. 9	9371. 9	10729. 2	11912. 5	12793. 9	14608. 0	15881. 3	16965. 7
Sector 3	3650. 0	4743. 7	5721. 9	5747. 3	7528. 4	10196. 6	12941. 1	14281. 1	15910. 9	19335. 3	19932. 3
Sector 4	8678. 6	9885. 2	10974. 9	11812. 0	13146. 6	15239. 4	17862. 6	19859. 5	21881. 1	24920. 6	26583. 9
Deflators											
Sector 1	0. 6153	0. 6107	0. 6460	0. 6230	0. 6310	0. 6730	0. 7580	0. 8240	0. 8970	0. 9170	1. 0
Sector 2	0. 9453	0. 9429	0. 9590	0. 9160	0. 9300	0. 9480	0. 9700	0. 9700	0. 9940	0. 9910	1. 0
Sector 3	0. 8619	0. 9612	1. 0180	0. 9340	0. 9410	0. 9570	0. 9740	0. 9650	0. 9720	0. 9880	1. 0
Sector 4	0. 6395	0. 6783	0. 7210	0. 7200	0. 7580	0. 9520	0. 8010	0. 8510	0. 9070	0. 9520	1. 0
Real Imports											
Sector 1	635. 4624	750. 2866	717. 9567	638. 5233	746. 1173	863. 2987	878. 4960	762. 5000	870. 9030	920. 3926	918. 7000
Sector 2	414. 4716	518. 4007	679. 0407	528. 2751	580. 7526	755. 8017	898. 6598	948. 8659	1201. 7102	1419. 9343	1571. 5000
Sector 3	154. 5423	292. 1348	463. 8507	278. 0514	399. 1498	509. 1954	695. 3798	613. 7824	673. 0453	816. 0931	721. 2000
Sector 4	−39. 7185	−44. 6704	−57. 9750	1. 8055	9. 4986	−66. 4915	0. 2497	7. 1680	−16. 7585	30. 0420	−44. 9000
Import Coefficient											
Sector 1	0. 15558	0. 18434	0. 16856	0. 14801	0. 16169	0. 18513	0. 18238	0. 15526	0. 178500	0. 18273	0. 18008
Sector 2	0. 05961	0. 06734	0. 08092	0. 06282	0. 06197	0. 07044	0. 07544	0. 07417	0. 082264	0. 08937	0. 09263
Sector 3	0. 04234	0. 06158	0. 08107	0. 04838	0. 05302	0. 04994	0. 05373	0. 04298	0. 042301	0. 04221	0. 03618
Sector 4	−0. 00458	−0. 00452	−0. 00[illegible]28	0. 00015	0. 00073	−0. 00436	0. 00001	0. 00036	−0. 000766	0. 00121	−0. 00169

is treated as an exogenous variable. The demand equations of individual persons and of firms have been estimated, respectively, as follows:

$$(12.14) \qquad M_p = -41.07 + 0.09508\, Y_d \,, \qquad \underset{(25.10)\ (0.002022)}{} \qquad \bar{r} = 0.9977$$

$$(12.15) \qquad \log M_f = -9.5476 + 1.1115 \log\left(\sum_{t=1}^{4} p_i X_i\right) - 1.5087 \log i \,, \qquad \underset{(2.4170)\ (0.1045)\qquad (1.3242)}{} \qquad \bar{r} = 0.9916$$

where M_P is personal currency demand, M_f is currency demand of firms, Y_d is personal disposable income, $\sum_{i=1}^{4} p_i X_i$ is the nominal aggregate output and i is interest rates of loan discounts of all banks, which is given exogenously together with the prime rate. When the relevant variables in the physical sector are determined, the demands for currency of individual persons and firms are determined by these equations. The total money demand will be determined then by adding the exogenous government demand to these demand components in the private sector, or

$$(12.16) \qquad M^D = M_p + M_f + M_G \,.$$

Consequently, the amount of money supply M^S will be determined endogenously to match the demand M^D, that is

$$(12.17) \qquad M^S = M^D$$

After the quantity of money has been determined in this way, the price p_i in the physical aspect of our model will now become meaningful as money price.

Our formulation of the money market as described above reflects the situation of chronic over-borrowing phenomenon which prevailed in the 1950s and 1960s in the Japanese money market. During this period, the rate of interest was kept at a low level, which suggests that there has constantly existed a potential excess-demand for money. Had the interest rate been kept rigidly at a low level for some institutional reasons, the existing potential excess-demand may well have exerted some pressure upon the actions of the monetary authority. If this was indeed the case, then we may interpret the money supply as being determined endogenously. Although this interpretation justifies our simplistic specification of the monetary block, a more autonomous theorization of the monetary block needs to be developed in the future for our model.

Notes to Chapter 12

1) Metzler(1941), Darling(1959) and Lovell(1964).
2) Shishido(1966).
3) Hicks(1939), Scitovsky(1940), Pigou(1917), Patinkin(1956) and Hansen(1970).

Chapter 13
The Complete System of the General Equilibrium Model

The purpose of this chapter is to integrate the structural equations, which we have developed so far, into a single consolidated system. In section 13.1, we will summarize briefly the system of structural equations taking the obtained estimates of the parameters into account. In Section 13.2, we will discuss methods of solving a system containing non-linear equations, and also examine the stability of such a system. This problem arises from the fact that we cannot necessarily expect *a priori* that stable equilibrium solutions exist for our model which contains many peculiar properties of empirically found elements such as economies of scale associated with the technological conditions of production, habitual shifts in consumers' preference, possibility of imperfect competition, etc. In Section 13.3, we will examine the empirical validity of our model on the basis of the results of interpolation tests such as total and final tests which will have been carried out using the methods of finding the solutions as discussed in the preceding Section.

13.1 The System of Structural Equations and Solutions of the System

The system of equations presented at the end of this chapter shows the result of estimation of 166 structural equations of our model. The number referred in the text of this chapter implies the number attached to each of the equations listed at the end of this chapter. The reader is also referred to the flow-chart of our model appended at the end of this volume. The structure of our model expressed by the flow-chart may be divided into 4 blocks for expository convenience.

In the first block, the short-run behavior of firms is analyzed, taking the capital stock at the beginning of each period as given, within the framework of inter-sectoral dependency linked by the labor and intermediate input markets. The first block of the appended flow-chart corresponds to the segment of equations (1) to (21). The technological conditions of production in manufacturing and service sectors have been represented adequately by the SFS production function as discussed in detail in Chapter 7. The estimates of the parameters have revealed that economies of scale were clearly operative in manufacturing and service sectors. Also, it has been found that in the heavy manufacturing sector, whose capital intensity is higher than that of other sectors, the value of parameter α of the SFS production function (6) was quite small. This suggests that operation beyond the level of normal hours h^* may well lead to a sharp increase in the cost of production. The technological conditions in the manufacturing and service sectors are expressed by equations (2) through (10).

The supply schedules in manufacturing and service sectors have been derived on the basis of these technological conditions and the anticipated demand functions according to the principle of profit maximization. Equations (15), (16) and (17) represent the estimated supply schedules. The price elasticity of demand in the anticipated demand function turned out to be quite large for each sector, which gives the impression that firms anticipate highly competitive markets. This results, however, may be influenced importantly by the aggregative nature of the data due to the broad sectoral classification of our model.

The technological condition of production in the agricultural sector (sector 1) has been represented well by the Cobb-Douglas type production functon, as shown by equation (11). We have found on the other hand that the estimate of value added marginal productivity derived from equation (12) was comparable to the wage rate of unskilled workers in the manufacturing and service sectors. The supply equations (15), (16) and (17) in the manufacturing and service sectors may be interpreted in this situation also as to be the marginal productivity equations. Based on this finding, therefore, we have constructed a system of simultaneous equations of the supply equations in the non-agricultural sectors and the value added marginal productivity equation in the agricultural sector to determine wages and supply prices simultaneusly on the assumption of interdependence between the sectors linked by the labor market. Since we have been using only aggregate data for the labor market, we allowed for inter-sectoral wage differentials separately by introducing the empirical relationship of wage differentials as represented by equations (13) and (14) into our model. The empirical validity of this system of simultaneous determination of wages and supply prices has been examined in Chapter 7, Section 7.5.

In the second block, the distributions of income among different institutional sectors are described. The second block corresponds to equations (22) to (70) in terms of the list of equations at the end of this chapter. Personal income and personal disposable income are determined by equation (55) and (57), which in turn are dependent upon conpensation of employees: equations (31), (32), (33) and (34), income from unincorporated enterprises: equations (43), (44), (45) and (46), income from property: equations (47), (48), (49) and (50). Total personal consumption expenditures and personal savings are determined by equatins (58) and (59). On the other hand, income from private corporation are determined by equations (51), (52), (53) and (54). Retained earnings are determined by equations (67), (68), (69) and (70) after taking corporation income taxes and charges, income transfers and dividend payments into account. As for the revenue of the general government, indirect taxes: equations (22) to (25), personal direct taxes and charges: equation (56), and corporation income taxes charges: equations (62) through (66) are determined endogenously.

It is difficult this stage to incorporate explicitly the way of the institutional distribution of incomes into the equations as mentioned above, since it is difficult to reclassify the data organized by major commodity categories into the classification of institutional sectors. Because of this difficulty, many of the estimated equations mentioned above are merely equations representing empirical relationships without reflecting the changes in the institutional distribution of incomes.

In the third block, the determination of the final demand for different items is described. This block consists of equations (90) through (145).

Personal consumption demand functions are represented by equations (90) through (99). These equations show the results of estimation of five item consumption functions. The underlying utility indicator function is of the Bernoulli-Laplace type which also takes into account explicitly the habit formation effect. Details on the theory and estimation of the consumption functions have been discussed in Chapter 10.

The estimated investment demand functions are given by equations (100) through (117). The optimal capital stock has been determined for manufacturing and service sectors according to the long-run profit maximization of firms under the technological constraint represented by the SFS production function. The elasticities of substitution between capital and labor derived from the estimated SFS production functions for Sectors 2, 3 and 4 have turned out to have reasonable values.

We have also specified the long-run anticipated demand function to represent the likely market reactions in the long-run which firms anticipate when they determine the optimal amount of capital stock. The price elasticities of demand in the long-run derived from the estimated long-run anticipated demand functions turned out to be considerably smaller than those derived from the short-run anticipated demand functions. The determination of the optimal amount of capital stock is also constrained by costs of finance which are represented by the cost of raising fund functions.

The optimal capital stock in the agricultural sector has been obtained by introducing the price anticipation equation under the technological constraint expressed by the Cobb-Douglas type production function. This specification reflects the fact that the price of agricultural products is assumed to be given exogenously as a policy variable. These investment demands in different sectors have been discussed in detail in Chapter 11.

Export demand is specified in the form of export demand functions for 6 categories of exported goods which have been estimated by equations (118) to (130). As for imports, on the other hand, import coefficients have been estimated by equations (131) to (134). Inventory increases have been estimated by equations (136) to (145).

The fourth block deals with the money market and the balance between the aggregate investment and saving. This block consists of equations (150) to (166). The demand-supply balance of money is described in three segments: individual persons, firms and the government. The government demand for currency is given exogenously. The currency demand of personal demand for currency is assumed to depend on personal disposable income and is determined by equation (150). The currency demand of firms is assumed to be determined by the nominal amount of aggregate transaction and interest rates of loan discounts of all banks. Thus the total currency demand of government, individual persons and firms may be determined for a given level of interest rate. It is assumed here that the amount of supplied money is just sufficient the amount of demand. The amount of money in circulation then determines the absolute levels of prices. It is intended that this specification of the money market gives the simplest description of the Japanese money market during the 1950s and 1960s which has been dominated by chronic "over-borrowing."

Equations (153) and (154) are definitional equations representing the aggregate investment and aggregate saving, respectively. According to the Walrasian Law which governs the entire system of our general equilibrium model, the aggregate saving, represented by equation (154) which includes the government saving computed from the government revenue and expenditure, equations (159) and (160), should equal the aggregate investment, defined by equation (153). Equations (161), (162) and (164) to (166) are definitional equations which represent *GDP* and the sectoral growth rates.

In additon to these equations mentioned so far, price and quantity converters which convert between the commodity classification and the classification of final demand items are represented by equations (71) to (83) and (146) to (149).

This completes a brief review of the structural equations in our model corresponding to 166 endogenous variables. Needless to say, since all these structural equations are interdependent, the solutions to the model are to be obtained by solving these equations simultaneously.

The empirical validity of the model should be evaluated not only by whether each structural equation fits the observed data but also by whether the simultaneous solutions of the entire system represent the data generating mechanism of the variables.

If the system of the model were linear, then the simultaneous solutions could be obtained uniquely usually by solving the system of reduced form equations derived from the structural equations. However, it is difficult in our model to apply this usual method of solution using reduced forms, since our model includes many non-linear equations. Moreover, even if simultaneous solutions happend to be obtained by some method, it would still be difficult to give analytical proofs of the stability and uniqueness of the solutions.

13.2 The Stability of the Model and the Uniqueness of the Solution

In obtaining solutions to the system, we will first follow the logical flow of the system to find the solutions and then examine their stability and uniqueness. The solutions should be found at the points where demand-supply equilibria are attained simultaneously in all the sectors. The equilibrating process can start, in principle, from any endogenous variable which exists within the system unless a certain degree of simplicity and quickness are required in the process. However, in order to avoid the possible accumulation of errors and in order to limit computer costs, it is best to choose the simplest method of computation of the equilibria.

We have decided to start with the block of short-run supply equation (the first block) which is constrained by the exogenously given capital stock at the beginning of the period. The arrows in the flow-chart appended at the end of this volume indicate not only the logical cause-effect relationships in the system but also the steps taken to arrive at the solutions. The reader is therefore referred to the logical flows indicated by the arrows in reading the following explanations.

In the first block, when the capital stock is given at the beginning of each period, the capacity of production Q_j and the number of workers L_j will be determined for manufacturing and service sectors on the basis of the SFS production function. The process corresponds to the arrows numbered⑧⑨, ⑩,②,③, and④. These numbers also correspond to the numbers attached to equations listed at the end of this chapter.

When the total number of workers $\bar{L}$ is given exogenously, the agricultural labor force L_1 will be determined by ①. When the amount of public capital stock allocated for the agricultural sector K_{g1}, and the cultivated land A_1 are given in this situation, the amount of agricultural production X_1 will be determined through ⑪.

To obtain the equilibrium outputs for the four sectors, we will first give X_2*, X_3* and X_4* as the initial values. Given these initial amounts of output on the one hand, and the capacity of production on the other, hours of operation h_j will be determined in manufacturing and service sectors by arrows⑤, ⑥and⑦.

If the price p_1, hours of operation in Sector 1, and input coefficients of all the sectors a_{ij} are given exogenously, then by introducing the already determined values such as the number of workers, initial outputs in manufacturing and service sectors and the agricultural output into the system of simultaneous determination of wages and supply prices as indicated by arrows ⑫, ⑬, ⑭ , ⑮, ⑯ and ⑰, we can obtain supply prices p_2, p_3 and p_4, and wage levels w_1, w_{23} and w_4. Also, the number of employed persons E_{yj} for each of the four sectors will be determined through ⑱, ⑲ and ⑳.

This completes the overview of the logical flow in the first block, where we have obtained values of endogenous variables compatible with the initial levels of output X_j* for the fixed levels of capital stock given at the beginning of each period. The supply prices p_j, wages w_j, employment E_{yj} and the initial outputs X_j* ($j = 2, 3, 4$) will now be used in the second block.

In the second block, when the initial outputs X_j* ($j = 2, 3, 4$) and the agricultural output X_1, supply prices p_j and the indirect tax rates t_{Ij} are given, then the amounts of indirect tax will be determined through ㉒, ㉓, ㉔ and ㉕. The sectoral value added will be determined through the prices, outputs, and input coefficients as indicated by arrows ㉗, ㉘, ㉙ and ㉚. Compensation of employees E_{Ij} will be determined on the other hand from wages w_j, the number of employees E_{yj}, and hours of operation h_j through ㉛ , ㉜, ㉝ and ㉞. The amount of provisions for the consumption of fixed capital corresponding to the capital stock at the beginning of each period will be obtained through ㊱, ㊲ and ㊳.

When business consumption expenditure B_{cj} and current subsidies S_{cj} are given exogenously, the operating surplus B_{sj} for each sector will be obtained through ㊵, ㊶ and ㊷ by subtracting the exogenous elements of value added as above, indirect taxes, compensation employees and the provisions for the consumption of fixed capital from the total value added.

Further, the residual which would remain after subtracting the exogenous elements of stock variation adjustment A_{pj}, general government income from property and entrepreneurship Y_{Gj}, and interest on public debt and consumer's debt D_{cGj} from the operating surplus B_{sj} will now be classified into income from uncorporated enterprises through (43), (44) and (45), income from property through (47), (48) and (49) and income from private corporations through (51), (52) and (53). Personal income before taxes is determined by summing up compensation of employees, income from uncorporated enterprises, income from property and exogenous devidends payments through (55). Personal disposable income will be obtained by subtracting transfer payments, social insurance contributions, imputed service charges by persons and personal direct taxes from the personal income.

Personal disposable income will be subdivided into total private consumption expenditure E and personal savings S_p. The former will be used as the budget constraints on the simultaneous multi-item consumption function which will be used to determine the itemized personal consumption demands in the third block. The latter will be taken into account in determining the aggregate domestic saving through (154).

On the other hand, subtracting from income from private corporation such items as the exogenously determined income transfers dividends payment, imputed service charges by private corporation, and corporation income taxes and charges as shown by (62), (63), (64) and (65), we can obtain the retained earnings for the four sectors as indicated by (67), (68), (69) and (70). The retained earnings will be used on the one hand in the third block as a determinant of the cost of raising fund affecting the investment of firms, and on the other hand constitutes the aggregate domestic saving together with provisions for the consumption of fixed capital as indicated by (154). This completes the review of the logical flow of the second block.

In the third block, the final demand items will be determined on the basis of the prices of sectoral products p_j obtained in the first block and the incomes of institutional sectors obtained in the second block.

The sectoral prices p_j will be converted to prices by demand items η through the price converters. Of the prices of the final demand items, the prices of personal consumption items will determine, together with the aggregate personal consumption expenditure obtained in the second block, the personal consumption expenditure q_i per household for 5 major consumption items of households through the multi-item consumption functions. Note here that the habit formation variable H_i, a predetermined variable, and the average family size m, an exogenous variable, are also given in the determination of the consumption expenditure. The aggregate amounts of personal consumption expenditure for 5 major items will be obtained by incorporating, as indicated by arrows (90) to (94), the exogenously given total number of households M.

The demand for capital equipment consists of net investment and the investment to make up for the capital depreciation. The net investment ΔK_j for each sector will be determined when the predetermined variables such as the gross domestic production of the preceding period GDP_{-1}, the balance of debt D_j and the exogenous variables such as normal hours of operation h_j^*, interest rates of loans discounts of all banks i, and prime rate i_0, and further the prices of investment goods η_{Kp} obtained in the first block and the retained earnings obtained in the second block are given. The total value of depreciation on the other hand is obtained by (116). These two components are combined to make the gross investment for capital equipment I_{GT} as indicated by (117). Inventory increases INV_j for manufacturing and service sectors will be determined being constrained by the inventory stock $SINV_j$ through (143), (144) and (145).

Exports classified by six commodity categories will be obtained on the basis of the prices of export goods η_{EXi} and the exogenously determined world export W_i and world export prices P_i as indicated by the logical flows (118) to (123).

The final demand items obtained so far; personal consumption expenditure, investment for capital equipment, inventory increases, and export are now converted into the classification of four commodity-based sectors using the quantity converters. The final aggregate demand classified by the four sectors F_j will be obtained by giving in addition to the above demand items such items as business consumption expenditure, B_{Fcj}, personal housing investment I_{Hj}, gross fixed capital formation by government I_{Gj}, and general government consumption expenditure G_j which are also classified into the four sectors. This part is represented by the logical flows (146) to (149).

The amount imports for each sector by sectors are obtained through (131) to (134) on the basis of initial outputs X_j^* and import coefficients. This completes the brief explanation of the third block.

In the fourth block, the gross domestic product which is compatible with the final demand F_j obtained in the third block will be computed by giving the inverse matrix computed from the input coefficients and import coefficients as indicated by arrows (155) to (158). The difference between the output of Sector 1 obtained here and the output X_1 determined in the first block is regarded as inventory increases in Sector 1, whose direct and indirect repercussions on productions in other sectors also must be computed. Adding these derived outputs generated by the inventory increases in Sector 1 to the outputs of other respective sectors, we can obtain the amounts of domestic products, which are compatible with the final demand, classified by the four sectors. We distinguish these sectoral outputs $X_j^{\triangle}$ in the flow chart from the initial sectoral outputs X_j^*.

In order to determine the absolute levels of prices, on the other hand, we determine the currency demand of individual persons and firms, respectively, through (150) and (151) taking the interest rate as given exogenously. Combining these demands with the exogenously given government demand for currency, we can obtain the aggregate domestic demand for currency by (152). With the amount of money supply given in this way, the prices and wages obtained earlier will now be expressed in terms of absolute levels of money price. By arrow (153), aggregate domestic capital formation will be determined, which should be equivalent to the aggregate domestic saving obtained earlier by (154).

Thus far, we have followed the network of logical flows by which most of the endogenous variables are presumably determined.

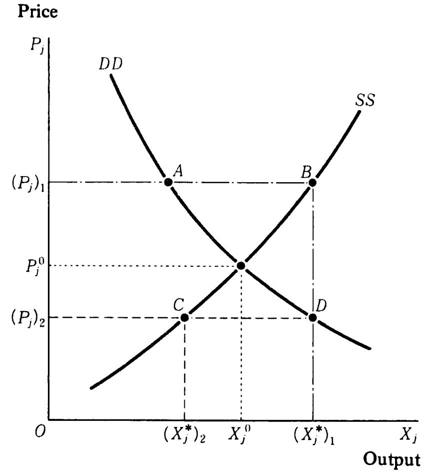
However, it should be born in mind that no assurance has been given so far that the demand will equal supply in each sector. In other words, it may very well happen that the initial output X_j^* given for each sector will deviate from the level output $X_j^{\triangle}$ which satisfies the demand in the same sector. If this is indeed the case, the gap between X_j^* and $X_j^{\triangle}$ would have to be suppressed. The process of suppressing the gap is carried out usually by kind of interative computation.

The gap between X_j^* and $X_j^{\triangle}$ may be explained as follows. The initial output X_j^* ($j = 2, 3, 4$) may be regarded as the amount of supply in each sector. Accordingly, the supply price corresponding to the initial amount of supply will be determined on the supply schedule of each respective sector.

On the other hand, the amount of demand $X_j^{\triangle}$ in that sector will be determined on the basis of the factor incomes associated to the supply price and the corresponding itemized price such as of consumption items, investment goods, exported goods etc. This situation may be illustrated by Figure 13.1.

In terms of Figure 13.1, if the initial output is given at $(X_j^*)_1$, then the supply price will be determined at the level $(p_j)_1$, and consequently, there will emerge the excess supply AB since the demand will be determined at A. In contrast, if the initial output is given at $(X_j^*)_2$, then the supply price will be $(p_j)_2$ and the excess demand CD will emerge accordingly. The aforementioned gap between X_j^* and $X_j^{\triangle}$ implies either excess demand or excess

Figure 13.1 AN ILLUSTRATION OF EXCESS DEMAND AND SUPPLY



supply as illustrated in this Figure, assuming that the demand and supply schedules do not shift in the process of adjustment. In the real world, however, prices and incomes change in response to changes in the initial values and, consequently, the demand and supply schedules themselves would not remain intact. Nevertheless, the interpretation on the nature of the gap between X_j* and $X_j^{\triangle}$, as given above, should be basically valid.

Figure 13.2 illustrates the process of convergence computation for the simple case of one commodity.

When the initial value X_1 is given, the corresponding suplly price p_1 will be determined at point A_2 on the supply schedule S_1S_1. For the given supply price p_1, the level of demand will be determined at the level X_1^D. Since $X_1 > X_1^D$ at the price level p_1, the existence of excess supply A_1A_2 is implied.

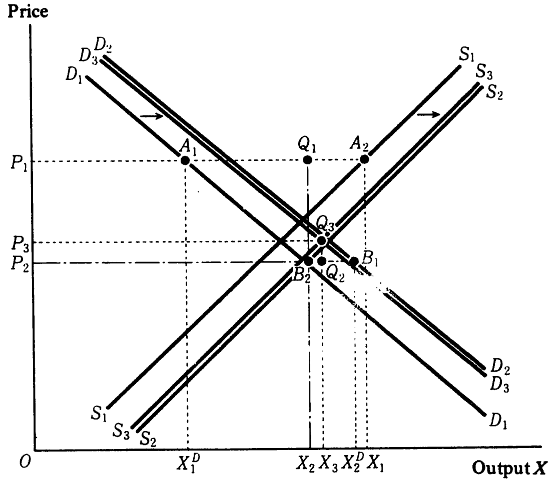
Therefore, we will give as the initial value in the second step the level X_2 which falls within the range A_1A_2. The new initial value X_2 is computed by

$$X_2 = X_1{}^D + \frac{3(X_1 - X_1{}^D)}{4} .$$

In response to this change in the initial value, the supply schedule S_1S_1 will shift to the positon S_2S_2, and the demand schedule shifts from D_1D_1 to D_2D_2. Consequently, excess demand B_1B_2 emerges at the price level p_2.

Then, as the third step, we give as the initial value the level X_3 which falls within the range B_1B_2. The value X_3 is computed by $X_3 = X_2 + \dfrac{(X_2{}^D - X_2)}{4}$.

Figure 13.2 AN ILLUSTRATION OF THE PROCESS OF CONVERGENCE COMPUTATION BY A SIMPLE EXAMPLE OF THE MARKET FOR ONE COMMODITY



Note: The process of convergence proceeds as follows.
When the initial level of output X_1 is given, the corresponding supply price P_1 will be determined at point A_2 on the supply schedule S_1 S_1. Then the amount of demand will be determined accordingly at the level X_1^D. This implies that there emerges the excess-supply A_1A_2. With the existence of this excess-supply at the end of the first step, then the amount of output in the second step will be given at an appropriate level X_2 within the range of the excess supply, which corresponds to a lower supply price P_2 on the supply schedule S_2S_2. The convergence computation proceeds in this way toward an equilibrium.

Then, the supply schedule S_2S_2 will shift to S_3S_3, the demand schedule D_2D_2 will shift to D_3D_3, and consequently the equilibrium point Q_3 will be obtained. The price p_3 and quantity X_3 which are compatible with the equilibrium point Q_3 are the equilibrium price and quantity in this commodity sector.

In our model, the determination of the equilibrium solutions through these processes has to be accomplished simultaneously in Sectors 2, 3 and 4. In doing so, we employed an approach by which we apply at each step of computation the method of interative convergence to the sector in which the gap between X_j^* and X_j^Δ is the largest while for other sectors the initial values adopted at the preceding step are maintained. It has been found that this method leads to convergence quicker than any other method. We have obtained the convergence within 10 to 15 steps in the interpolation test for each year using this method.

As we have repeatedly stressed, our model has peculiar properties in that it contains non-homogeneous production functions which explicitly allow for economies of scale and in that it allows for shifts in the preference field of consumers due to habit formation. And there are no *a priori* guarantees that our general equilibrium model, which heavily relies on these empirical findings in its specification, has stable and unique equilibrium solutions. Therefore, it is necessary, before examining the results of the interpolation tests, to give some proof of the stability and uniqueness of the equilibrium solutions obtained by the interative computation method explained above.

Since the system of our model is largely non-linear, it is difficult to verify the stability and uniqueness of the solutions by analytical methods. Therefore, we have taken instead the following procedure. We first provide exogenously alternative supply quantities ranging broadly around the equilibrium quantities. We then compute the amounts of excess demand of supply corresponding to these supply quantities by means of interpolation. For the purpose of finding the appropriate supply quantities which will minimize the amounts of excess demand or supply, we formulate the objective function $\phi = \sum_{j=2}^{4} (X_j^s - X_j^D)^2$.

If the equilibria are attained in all the sectors simultaneously, then the value ϕ of the objective function will be zero. By means of plotting the values of ϕ corresponding to a broad range of alternative output level for each sector, we may examine empirically whether the minimum value of the objective function is unique and stable.

Let us present the result of an experiment as explained above, taking the case of the equilibria, as an example, for 1965 for the three sectors with assigned values as; $X_2^* = 18231.8$, $X_3^* = 25918.5$, $X_4^* = 23070.8$. We have set, for each of Sector 2, 3 and 4, the range within which the equilibrium quantity will be sought as follows:

$$15747.3 < X_2^* = 18231.8 < 20525.0$$

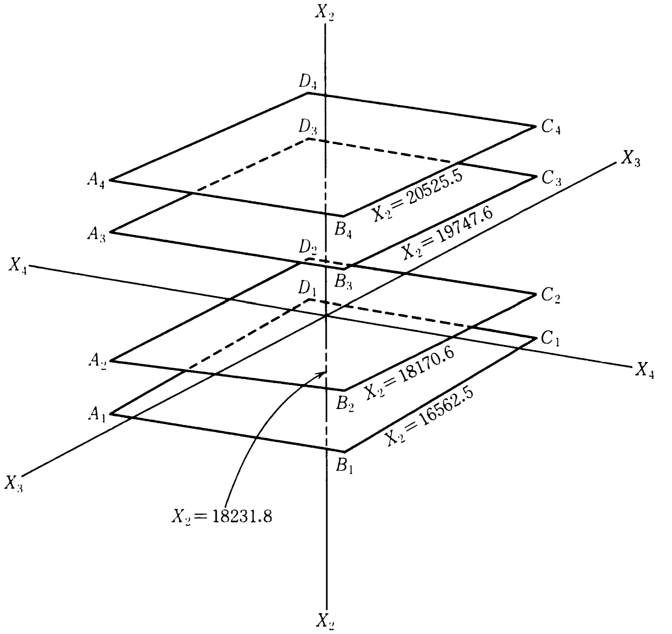
$$24350.8 < X_3^* = 25918.5 < 28145.6$$

$$22134.8 < X_4^* = 23070.8 < 27404.9$$

For the purpose of presenting the result of our experiment, we will use the procedure illustrated by Figure 13.3.

The three axes; X_2—X_2, X_3—X_3 and X_4—X_4, in the diagram correspond to the quantity scales of Sectors 2, 3 and 4, respectively. In this three dimensional space, we specify four tiers. of two-dimentional planes; $A_1\ B_1\ C_1\ D_1$, $A_2\ B_2\ C_2\ D_2$, $A_3\ B_3\ C_3\ D_3$ and $A_4\ B_4\ C_4\ D_4$, corres-

Figure 13.3 A DIAGRAMMATICAL EXPOSITION OF THE TEST OF THE STABILITY OF SOLUTIONS OF THE MODEL

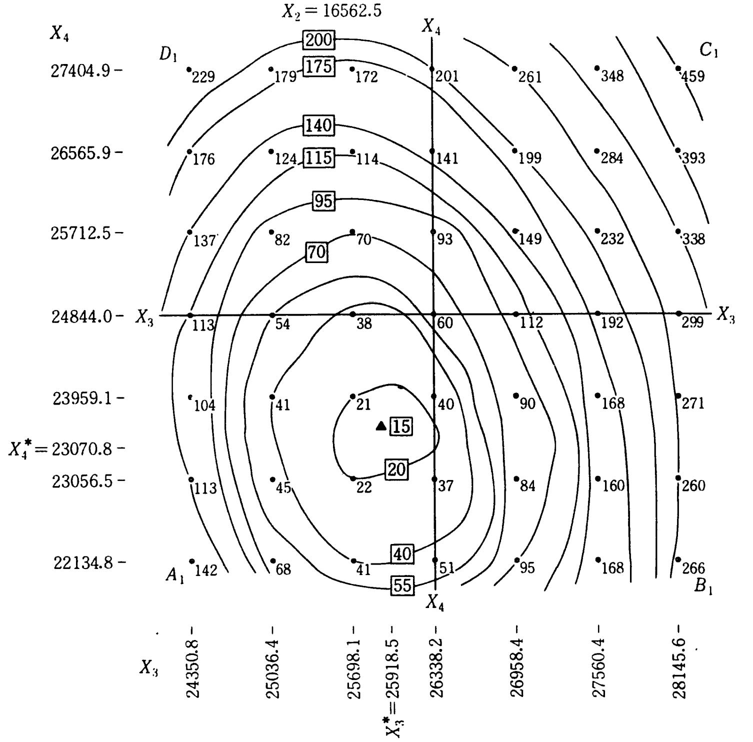


ponding respectively to the specific output level of Sector 2 X_2; 16562.5, 18170.6, 19747.6 and 20525.5. On each of the four planes, we will draw a map of contours of the objective function by plotting the values of the objective function within the range cited above corresponding to seven alternative level of output for Sector 3 and also seven alternative quantities for Sector 4. In other words, contours of the objective function will be drawn, for a certain fixed quantity in Sector 2, using 49 combinations of alternative quantities for Sectors 3 and 4. The isoquant maps drawn in this way on the four planes are shown by Figures 13.4(A) to (D).

Figures 13.4(A) presents isoquants of the objective function drawn for a fixed quantity $X_2=16562.5$ for Sector 2. The point marked by ▲ is the minimum value of the objective function on this plane. The numbers attached to plotted points in the diagram are the values of the objective function. According to this isoquant map, we can find the unique minimum value in the neighborhood of the equilibrium point at a given output level of X_2.

Similarly, isoquants drawn in Figures 13.4(B), (C) and (D) correspond to fixed values of X_2; 18170.6, 19747.6 and 20525.5, respectively. Like Figure 13.4(A), the unique minimum value will also be found for each of these

Figure 13.4 (A) THE ISOQUANT MAP OF THE OBJECTIVE FUNCTION ON PLANE $A_1\ B_1\ C_1\ D_1$

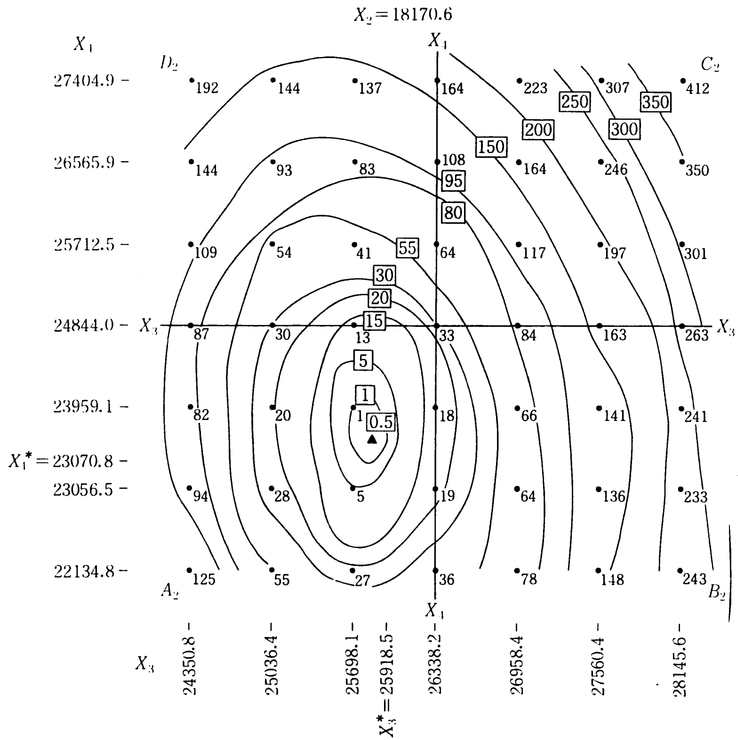


Note: The numbers attached to the 49 scatlered points on the plane represent the values of the objective function at those locations and the numbers in squares are the values of the objective function on the expressly drawn isoquants. The units of the values of the objective function are 1.0×10^6.

isoquant maps in the neighborhood of the equilibrium point at a given output level of X_2.

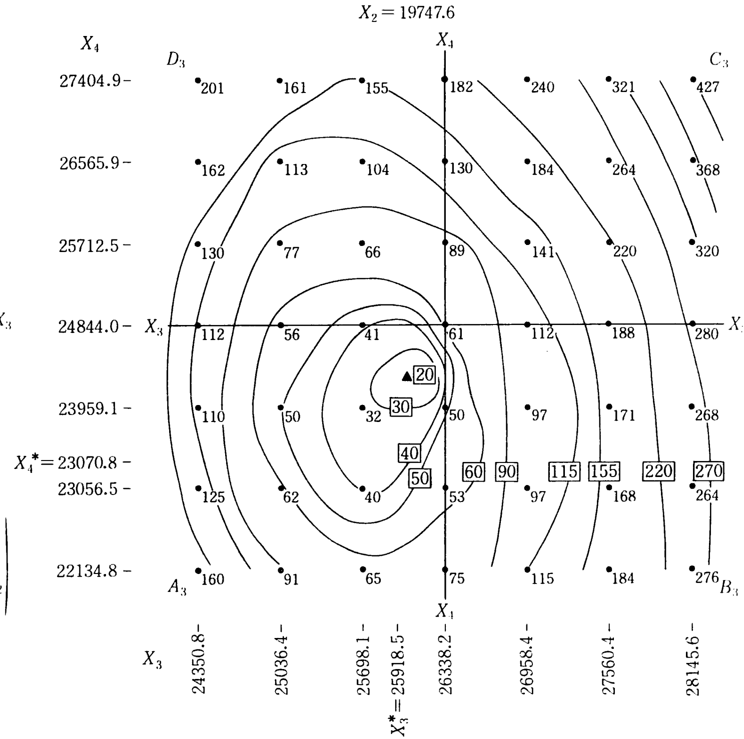
A comparison of the minimum values ▲ of the objective function assessed in the four maps will reveal that the value 0.5 attained in Figure 13.4(B) is the minimum, and that this minimum value can be uniquely identified within the range of X_2 for which the minimum value of the objective function has been sought. The value of X_2 for Figure 13.4(B) is 18170.6, which roughly corresponds to the equilibrium value $X_2^* = 18231.8$ which has been obtained through the iterative convergence method discussed earlier.

Figure 13.4 (B) THE ISOQUANT MAP OF THE OBJECTIVE FUNCTION ON PLANE $A_2 B_2 C_2 D_2$



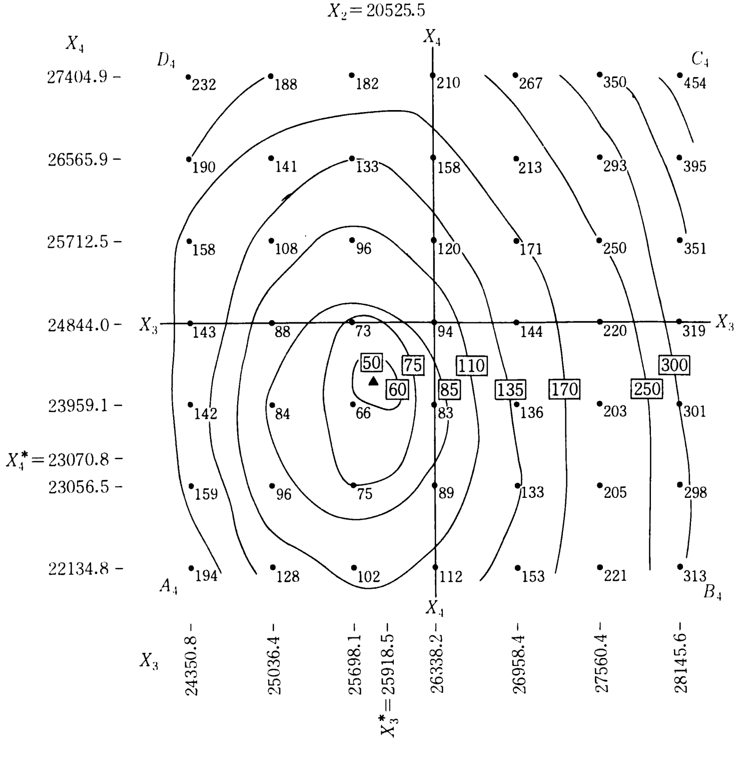
Note: See the footnote attached to figure 13.4 (A).

Figure 13.4 (C) THE ISOQUANT MAP OF THE OBJECTIVE FUNCTION ON PLANE $A_3 B_3 C_3 D_3$



Note: See the footnote attached to figure 13.4 (A).

Figure 13.4 (D) THE ISOQUANT MAP OF THE OBJECTIVE FUNCTION ON PLANE $A_4 B_4 C_4 D_4$



Note: See the footnote attached to figure 13.4 (A).

We have been able to ascertain, through the experiment explained so far, the existence, stability and uniqueness of the equilibrium values; X_2*, X_3*, and X_4* in the three sectors for 1965, which have been obtained through the method of iterative convergence. We have conducted similar experiments for other years within the period of observation and obtained the similar results. Therefore, it is safe to say the stability and uniqueness of the simultaneous solutions to the structural equations system of our model have been verified empirically for the period of observation.

13.3 The Interpolation Tests of the Model

Now let us make the interpolation test for the period of observation from 1955 to 1965. In both types of test, the observed values of an endogenous variable are compared with predicted values for that variable over the period of observation. In the total test, the endogenous variables are predicted for each time period solely from observed variables. In the final test, however, the endogenous variables are predicted from exogenous and lagged endogenous variables which are themselves predicted. The final test is therefore severer than the total test in the sense that prediction errors are allowed to accumulate. Table 13.1 presents the results of both tests for major variables. For each variable, rows OB, ES_1 and ES_2 represent the observed values, the results of the final test and the total test, respectively.

The results are more or less satisfactory except for a few variables such as hours of operation and inventory increases.

The function determining inventory increases has not been well developed as yet in our model, as we have noted in Chapter 12. For this we await improvements in the precision of data as well as the development of more autonomous structural equations.

The poor fit of hours of operation suggests an important methodological problem associated with estimation of this kind of model. This is the problem of the incompatibility between the direction of measuring errors in the case of statistical estimation of structural parameters and the causal orders by which to solve the equation system in purpose of interpolation tests or simulations.

Take the case of a simple regression model

$$y = \alpha x + \beta + u \quad , \tag{13.1}$$

for example for which the parameter α and β are estimated by the least squares method by measuring errors along the direction of the y-axis. The sum of squared errors obtained in this case is of course the minimum. However, it is difficult to avoid interpolation tests, in which variable x in the equilibrium quantity will be sought as follows:

$$x = \frac{1}{\hat{\alpha}} (y - \hat{\beta}) \, . \tag{13.2}$$

Since the errors measured along the direction of the x-axis may not be the minimum in this situation, the estimate of x is likely to contain greater errors. The effect of such errors can be quite large when the value of x is small. However, it is not easy to arrange from the beginning the derections of measuring errors of all variables to fit the causal orders implied in the equations used to compute the solution. This is especially true, in a system like ours in which many non-linear equations are contained. Appropriate methodological improvements are called for to resolve this problem.

Table 13.1 RESULTS OF FINAL AND TOTAL TESTS ON MAJOR ENDOGENOUS VARIABLES

Names of Variables		1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	$\bar{r}$
[Operation hours]													
Sector 2	OB	198.0	201.9	200.3	200.5	202.4	203.0	200.3	196.9	195.3	193.5	191.1	
h_2	ES_1	188.5	185.0	196.5	199.0	203.7	219.2	195.9	197.7	206.8	199.4	186.6	0.3251
	ES_2	188.5	201.9	204.0	200.5	202.4	217.6	196.1	200.1	195.3	191.8	185.0	0.7881
Sector 3	OB	172.1	178.9	177.7	200.5	205.5	209.2	205.1	199.1	198.3	197.5	193.1	
h_3	ES_1	110.1	151.6	171.0	188.8	234.8	283.6	203.3	185.9	181.2	220.1	180.6	0.8491
	ES_2	110.1	158.6	159.9	200.5	249.7	279.4	219.1	209.5	169.6	195.8	171.7	0.8894
Sector 4	OB	185.5	188.8	187.6	188.2	187.8	189.4	190.4	189.3	188.2	186.7	184.8	
h_4	ES_1	164.2	181.3	180.4	188.2	215.3	212.5	207.4	201.2	193.6	191.6	185.4	0.6597
	ES_2	164.2	177.7	177.1	196.6	198.9	209.9	193.2	192.8	183.0	183.5	173.3	0.7356
[Output deflator]													
Sector 2	OB	0.9453	0.9429	0.9590	0.9160	0.9300	0.9480	0.9700	0.9700	0.9940	0.9910	1.0000	
p_2	ES_1	0.9916	0.9660	0.9801	0.9029	0.8792	0.9049	0.9966	0.9878	1.0451	1.0113	1.0086	0.8435
	ES_2	0.9916	0.9935	0.9869	0.8981	0.8695	0.8969	0.9774	0.9735	1.0125	0.9838	1.0002	0.7138
Sector 3	OB	0.8619	0.9612	1.0180	0.9340	0.9410	0.9570	0.9740	0.9650	0.9720	0.9880	1.0000	
p_3	ES_1	0.8363	0.9657	1.0639	0.9338	0.9067	1.0164	0.9886	0.9448	1.0116	1.0387	0.9986	0.9093
	ES_2	0.8363	0.9770	1.0239	0.9245	0.9189	0.9913	1.0048	0.9659	0.9579	0.9722	0.9740	0.9112
Sector 4	OB	0.6395	0.6783	0.7210	0.7200	0.7580	0.7520	0.8010	0.8510	0.9070	0.9520	1.0000	
p_4	ES_1	0.6989	0.6904	0.7316	0.7014	0.7723	0.7482	0.8428	0.8935	1.0037	1.0131	1.0379	0.9773
	ES_2	0.6989	0.6753	0.7212	0.7117	0.7149	0.7302	0.7849	0.8393	0.9457	0.9707	0.9835	0.9690
[Wages]													
Marginal Value-added	OB	0.0765	0.0757	0.0855	0.0877	0.0949	0.1047	0.1177	0.1320	0.1521	0.1681	0.1930	
Productivity in Sector 1	ES_1	0.0777	0.0779	0.0862	0.0866	0.0904	0.1009	0.1149	0.1313	0.1557	0.1623	0.1922	0.9973
	ES_2	0.0777	0.0780	0.0863	0.0872	0.0907	0.1006	0.1154	0.1315	0.1590	0.1668	0.1958	0.9971
Sector 2.3	OB	0.00081	0.00089	0.00096	0.00096	0.00106	0.00120	0.00140	0.00162	0.00182	0.00207	0.00232	
w_{23}	ES_1	0.00092	0.00092	0.00100	0.00100	0.00103	0.00112	0.00130	0.00150	0.00188	0.00200	0.00237	0.4849
	ES_2	0.00090	0.00090	0.00100	0.00100	0.00100	0.00110	0.00130	0.00150	0.00190	0.00210	0.00240	0.4491
Sector 4	OB	0.00100	0.00104	0.00115	0.00122	0.00126	0.00139	0.00159	0.00187	0.00219	0.00245	0.00284	
w_4	ES_1	0.00109	0.00109	0.00118	0.00118	0.00122	0.00133	0.00155	0.00179	0.00225	0.00240	0.00285	0.9962
	ES_2	0.00109	0.00110	0.00120	0.00120	0.00120	0.00130	0.00160	0.00180	0.00230	0.00250	0.00290	0.8943

[Item deflator]													
Personal Consumption													
Food	OB	0.7160	0.7020	0.7250	0.7150	0.7170	0.7400	0.7800	0.8300	0.8970	0.9260	1.0000	
η_{c1}	ES_1	0.7522	0.7160	0.7369	0.7044	0.6948	0.7176	0.8031	0.8494	0.9488	0.9526	1.0147	0.9837
	ES_2	0.7522	0.7266	0.7382	0.7041	0.6780	0.7097	0.7800	0.8291	0.9167	0.9267	0.9959	0.9774
Clothing	OB	0.8260	0.8290	0.8310	0.8160	0.8080	0.8220	0.8530	0.8950	0.9390	0.9730	1.0030	
η_{c2}	ES_1	0.8750	0.8478	0.8478	0.8021	0.7777	0.7917	0.8816	0.9191	1.0027	1.0066	1.0217	0.9557
	ES_2	0.8750	0.8617	0.8495	0.8016	0.7570	0.7821	0.8537	0.8940	0.9632	0.9744	0.9975	0.9278
Fuel and Light	OB	0.8310	0.8410	0.9070	0.8910	0.8710	0.9130	0.9520	0.9760	0.9870	0.9910	1.0000	
η_{c3}	ES_1	0.8870	0.8535	0.9188	0.8737	0.8760	0.9047	0.9903	1.0125	1.0705	1.0401	1.0285	0.9494
	ES_2	0.8870	0.8440	0.9107	0.8816	0.8305	0.8885	0.9409	0.9675	1.0196	1.0025	0.9883	0.9798
Housing	OB	0.4830	0.5350	0.5760	0.6120	0.6580	0.7040	0.7640	0.8000	0.8590	0.9300	1.0000	
η_{c4}	ES_1	0.5196	0.5438	0.5878	0.5993	0.6597	0.7090	0.7957	0.8248	0.9336	0.9835	1.0263	0.9930
	ES_2	0.5196	0.5367	0.5781	0.6047	0.6245	0.6927	0.7596	0.7930	0.8826	0.9384	0.9826	0.9935
Miscellaneous	OB	0.6740	0.6890	0.6970	0.7020	0.7240	0.7480	0.7820	0.8410	0.8920	0.9290	0.9980	
η_{c5}	ES_1	0.7286	0.7012	0.7090	0.6855	0.7311	0.7446	0.8198	0.8774	0.9793	0.9840	1.0305	0.9804
	ES_2	0.7286	0.6907	0.6993	0.6934	0.6833	0.7274	0.7704	0.8314	0.9253	0.9429	0.9826	0.9725
Fixed Capital Formation (Private)	OB	0.7970	0.8860	0.9570	0.9300	0.9350	0.9570	0.9860	0.9860	0.9830	0.9910	1.0000	
η_{kp}	ES_1	0.7802	0.8911	0.9983	0.9282	0.9034	1.0110	1.0029	0.9703	1.0272	1.0415	1.0018	0.9622
	ES_2	0.7802	0.9009	0.9629	0.9202	0.9106	0.9865	1.0140	0.9862	0.9732	0.9778	0.9757	0.9751
[Taxes and charges]													
Personal Direct Taxes and Charges	OB	359.1	374.3	353.3	353.4	353.8	450.0	574.8	731.8	876.3	1072.5	1284.0	
	ES_1	353.9	364.6	350.4	347.2	361.4	469.1	562.5	716.4	889.0	1081.8	1300.7	0.9995
	ES_2	353.9	368.7	347.9	363.6	352.3	465.1	554.1	722.7	868.5	1062.9	1265.6	0.7992
Corporate Tax	OB	328.2	396.9	564.9	551.1	629.5	875.4	1131.2	1293.0	1385.9	1549.5	1668.4	
	ES_1	423.1	436.0	492.4	559.9	655.6	737.4	930.6	1158.8	1404.3	1591.9	2047.7	0.9601
	ES_2	423.1	447.1	484.8	606.2	637.3	728.6	930.2	1204.9	1354.2	1542.8	1964.8	0.9654
Indirect Tax	OB	788.6	839.2	939.6	993.4	1127.2	1295.9	1558.3	1662.0	1854.8	2045.3	2254.7	
	ES_1	817.1	846.9	923.3	969.3	1154.3	1304.9	1476.8	1605.6	1882.3	2016.4	2333.3	0.9963
	ES_2	817.1	845.8	915.7	1020.1	1125.9	1289.7	1484.2	1639.5	1848.1	2017.3	2281.9	0.9983
[Income]													
Disposable Income	OB	6375.4	6903.3	7788.1	8316.1	9386.7	10883.8	12726.9	14632.5	17116.9	19450.0	22298.5	
	ES_1	6283.6	6725.2	7725.1	8169.5	9588.2	11345.5	12455.7	14324.3	17365.5	19619.0	22588.6	0.9991
	ES_2	6283.6	6800.2	7668.3	8555.7	9346.0	11248.0	12270.0	14451.0	16963.0	19276.0	21978.0	0.9992
Compensation of Employees	OB	3352.8	3821.7	4385.1	4788.0	5332.5	6205.8	7368.1	8769.6	10312.4	11926.6	13829.8	
	ES_1	3362.1	3752.5	4298.2	4592.3	5530.4	6605.1	7104.9	8276.3	10511.5	12042.4	13831.2	0.9975
	ES_2	3362.1	3831.8	4272.4	4845.6	5445.9	6558.8	7059.8	8442.7	10321.0	11859.0	13539.0	0.9986
Income from Unincorporated Enterprises	OB	2801.8	2949.0	3086.7	3022.3	3285.2	3722.3	4353.6	4778.7	5543.8	6172.3	6785.3	
	ES_1	2660.4	2735.9	3074.6	3113.3	3379.5	3865.4	4375.8	4929.2	5568.2	6119.6	6886.7	0.9971
	ES_2	2660.4	2739.5	3053.1	3202.9	3276.6	3826.9	4285.5	4909.1	5401.8	5982.1	6652.8	0.9957

Table 13.1 (Continued)

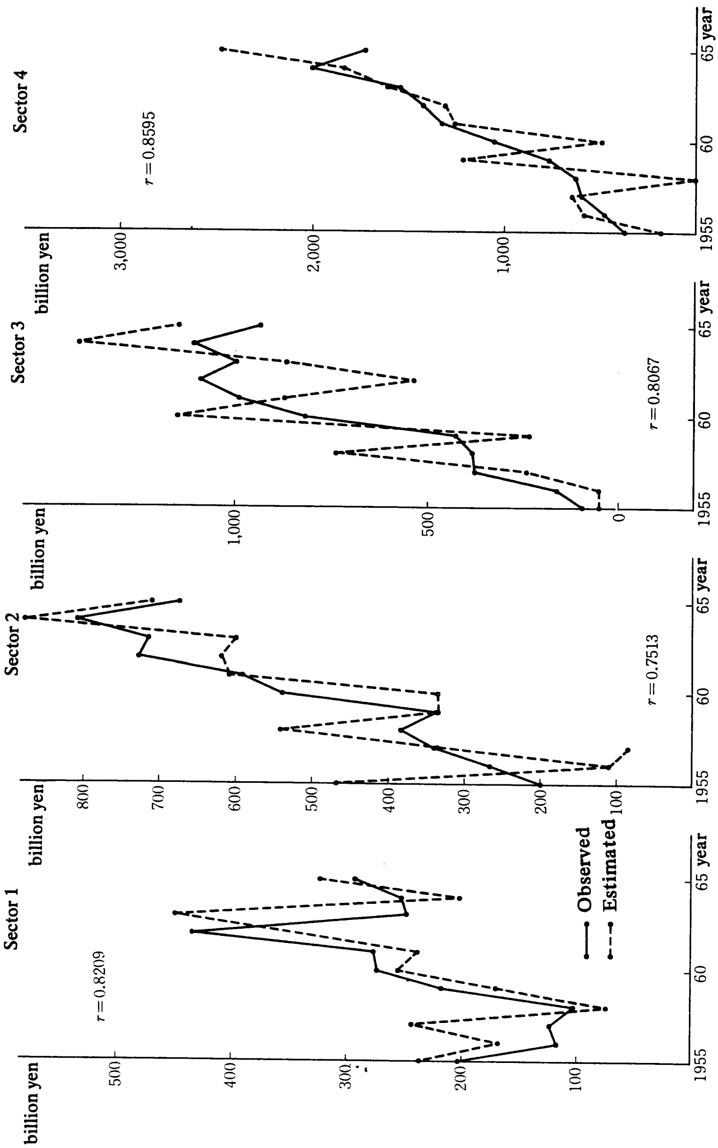
Names of Variables		1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	$\bar{r}$
Income from Property	OB	559.8	649.6	776.8	923.3	1143.0	1401.2	1726.1	2025.3	2374.3	2816.4	3284.4	
	ES_1	592.4	647.3	798.3	858.1	1064.8	1368.7	1653.1	2007.9	2414.6	2903.7	3403.9	0.9985
	ES_2	592.4	649.8	782.5	944.1	990.2	1347.1	1588.5	2012.4	2321.3	2835.5	3240.7	0.9977
Income from Private Corporation	OB	1182.7	1330.7	2001.5	1862.6	2077.3	3148.3	3791.2	4103.6	4379.6	5082.0	5061.2	
	ES_1	1414.7	1487.2	1781.5	1874.8	2126.7	2929.0	3198.5	3851.6	4283.6	5120.1	5762.5	0.9758
	ES_2	1414.7	1524.5	1755.8	2024.1	2080.2	2753.8	3207.8	3872.4	4218.2	4975.4	5549.5	0.9773
[Capital Stock] Sector 1	OB	5451.7	5629.2	5755.5	5856.5	6075.2	6350.5	6630.0	7066.7	7313.6	7567.7	7858.4	
	ES_1	5426.6	5670.6	5749.5	5926.4	6186.1	6426.9	6758.8	7209.2	7409.8	7616.3	7938.8	0.9987
	ES_2	5426.6	5673.5	5723.9	5932.3	6102.6	6341.9	6664.9	7053.0	7305.9	7567.7	7912.9	0.9991
Sector 2	OB	2908.4	3172.4	3511.1	3893.1	4235.5	4776.9	5366.9	6095.0	6808.8	7622.5	8333.4	
	ES_1	3179.7	3292.5	3373.7	3912.9	4242.5	4581.2	5185.9	5805.0	6402.9	7643.3	8314.3	0.9952
	ES_2	3179.7	3310.5	3561.7	4036.0	4316.6	4567.5	5197.5	6049.5	6799.9	7917.1	8448.8	0.9962
Sector 3	OB	2162.3	2323.0	2698.2	3075.2	3500.9	4320.9	5314.0	6413.8	7412.1	8571.1	9507.6	
	ES_1	2126.4	2179.3	2418.9	3153.1	3382.4	4524.8	5400.1	5936.1	6804.1	8433.6	9580.6	0.9954
	ES_2	2126.4	2191.2	2574.3	3329.0	3559.3	4433.5	5400.8	6317.6	7268.8	8592.8	9694.7	0.9987
Sector 4	OB	7684.4	8172.0	8772.6	9415.8	10178.1	11239.6	12553.4	13963.9	15513.2	17519.5	19257.0	
	ES_1	7498.0	8079.8	8736.9	8745.3	9962.6	10470.7	11714.5	13016.3	14621.5	16464.9	18933.0	0.9964
	ES_2	7498.0	8196.1	8443.6	9531.1	10613.3	10991.2	12108.4	13747.4	15653.9	17642.3	19654.7	0.9978
[Output] Sector 1	OB	4091.5	4046.4	4162.2	4246.9	4408.7	4467.9	4469.8	4554.3	4421.9	4727.6	4746.4	
	ES_1	3876.3	4076.4	4395.9	4387.9	4445.8	4637.5	4696.2	4739.8	4824.4	4780.6	4646.5	0.8283
	ES_2	3876.3	4078.7	4344.8	4328.3	4338.4	4536.9	4580.6	4578.7	4565.2	4532.6	4489.8	0.7852
Sector 2	OB	5523.3	6524.4	7409.1	7609.1	8468.8	10006.7	11513.5	12924.2	14879.7	16577.2	17769.4	
	ES_1	5547.2	6570.4	7186.3	7468.3	9033.4	10529.8	10499.3	12209.0	14426.4	15685.8	18231.8	0.9914
	ES_2	5547.2	6365.5	7090.6	7868.1	8934.1	10449.1	11029.6	12824.1	14565.4	16311.7	18041.8	0.9969
Sector 3	OB	5207.8	6674.6	7849.2	7849.9	9994.8	13249.9	16698.6	17937.3	19657.3	23256.0	24274.6	
	ES_1	5173.0	6095.7	6580.3	7618.5	10899.8	12681.3	14673.7	16837.6	18300.0	22796.0	25918.5	0.9889
	ES_2	5173.0	6320.0	6813.3	8718.2	10918.0	13040.3	14477.8	17448.0	19205.6	23601.1	25770.0	0.9896
Sector 4	OB	8659.3	9265.8	10048.1	10812.7	11656.4	13125.4	15652.9	17369.8	19548.1	21810.9	23241.0	
	ES_1	8099.3	8914.9	9711.2	10958.0	11980.5	13872.0	14479.1	16232.3	17942.9	20468.0	23070.8	0.9910
	ES_2	8099.3	9059.2	9727.3	11329.4	12421.8	14115.0	15043.6	17140.6	18804.5	21352.8	23770.6	0.9930
[Labor Input in terms of Man-Hours] Sector 1	OB	15360.0	15000.0	14670.0	14080.0	13480.0	13400.0	13030.0	12670.0	11940.0	11490.0	11130.0	
	ES_1	14189.9	14633.5	15259.2	14775.6	14379.4	14420.7	13908.7	13199.7	12515.6	11854.6	10884.9	0.8848
	ES_2	14189.9	14578.0	15114.0	14474.0	14044.0	14220.0	13575.0	12794.0	11784.0	11118.0	10424.0	0.9166

Sector 2	*OB*	5775.0	5878.0	6149.0	6290.0	6148.0	6252.0	6550.0	6839.0	6942.0	6953.0	7015.0	
	ES_1	5850.9	6033.9	6074.5	6103.1	6280.0	6378.6	6473.7	6630.2	6775.8	6905.0	7144.6	0.9502
	ES_2	5850.9	5931.1	6031.2	6150.2	6273.8	6376.6	6526.0	6674.1	6839.7	6987.2	7140.8	0.9674
Sector 3	*OB*	4195.0	4532.0	5051.0	5370.0	5722.0	6168.0	6680.0	7131.0	7368.0	7717.0	8055.0	
	ES_1	4661.6	4712.8	4761.6	4974.2	5558.4	5724.3	6466.4	6963.6	7245.6	7671.6	8393.5	0.9754
	ES_2	4661.6	4745.9	4890.6	5207.2	5500.5	5807.5	6342.7	6916.9	7484.0	7951.6	8450.5	0.9755
Sector 4	*OB*	15570.0	16300.0	16940.0	17240.0	18000.0	18546.0	18710.0	18920.0	19680.0	20380.0	21090.0	
	ES_1	16197.7	16329.9	16714.8	17127.1	17132.2	17842.5	18121.2	18766.5	19393.1	20108.9	20867.0	0.9743
	ES_2	16198.0	16455.0	16774.0	17149.0	17531.0	17962.0	18526.0	19175.0	19823.0	20483.0	21275.0	0.9797
[Inventory Increase] Sector 1	*OB*	423.0	327.9	325.6	117.0	145.6	123.4	205.7	76.8	139.4	161.3	118.1	
	ES_1	275.3	475.5	821.0	374.3	−127.9	19.2	719.9	421.2	602.4	326.1	−145.1	0.3545
	ES_2	275.3	540.3	737.4	42.1	−200.7	−64.3	443.7	70.3	310.8	−33.6	−264.2	0.6743
Sector 2	*OB*	141.5	174.3	268.2	99.8	170.8	213.1	596.6	162.9	437.8	611.2	420.7	
	ES_1	174.8	356.0	298.7	−153.5	324.1	507.4	95.9	24.8	708.9	318.0	601.7	0.3360
	ES_2	174.8	217.7	348.5	79.6	145.3	340.1	437.9	254.2	416.8	429.6	440.6	0.8678
Sector 3	*OB*	44.6	73.0	197.7	68.9	136.4	194.9	491.8	109.4	219.2	223.6	107.9	
	ES_1	57.2	109.0	118.6	−55.5	330.4	331.0	78.0	193.4	162.4	168.4	597.0	0.0819
	ES_2	57.2	126.6	230.7	−25.6	25.9	358.5	282.0	182.6	147.2	278.5	399.7	0.4420
Sector 4	*OB*	8.6	15.7	25.9	21.8	39.4	56.4	88.1	38.9	70.3	75.2	54.2	
	ES_1	10.5	−22.6	63.6	14.9	44.4	40.2	51.8	−8.2	105.6	60.9	105.8	0.6180
	ES_2	10.5	27.6	2.3	25.1	47.3	37.6	100.6	32.6	55.6	75.4	90.9	0.8467
Total of Nominal Imports	*OB*	890.6	1197.5	1245.4	1142.7	1393.7	1721.5	2215.1	2147.6	2614.7	3028.3	3166.4	
	ES_1	896.8	1072.2	1229.3	1307.1	1441.4	1650.9	1867.8	2092.5	2270.7	2692.6	3339.0	0.9925
	ES_2	896.8	1073.0	1230.4	1309.9	1421.0	1649.9	1846.5	2058.2	2279.1	2735.6	3318.9	0.9934
Total of Nominal Exports	*OB*	914.1	1123.0	1262.5	1259.2	1468.5	1698.8	1776.3	2051.1	2249.2	2507.9	3424.6	
	ES_1	896.8	1072.2	1229.3	1307.1	1441.4	1650.9	1867.8	2092.5	2270.7	2692.6	3339.0	0.9947
	ES_2	896.8	1073.0	1230.4	1309.9	1421.0	1649.9	1846.5	2058.2	2279.1	2735.6	3318.9	0.9929
Real GNP	*OB*	12858.7	13888.0	14996.0	15801.0	17257.7	19698.7	22765.7	24228.1	26785.3	30361.2	35111.7	
	ES_1	11485.3	12552.8	13717.0	15573.0	18057.0	20830.8	20933.1	23938.5	27423.1	31044.1	35658.6	0.9932
	ES_2	11485.3	12728.0	13766.0	16297.0	18318.0	21031.0	21417.0	24881.0	28074.0	31774.0	35815.0	0.9925
Nominal GNP	*OB*	9304.6	10549.5	12090.2	12116.0	14662.5	17644.5	20143.6	22983.4	27388.2	31454.6	35111.7	
	ES_1	9255.8	10319.5	11783.3	12503.5	15105.3	17918.9	19239.6	22251.8	27787.4	31646.9	36267.3	0.9984
	ES_2	9255.8	10467.0	11723.0	13249.0	14827.0	17752.0	19172.0	22673.0	27079.0	30933.0	35244.0	0.9990

Note: Notations are:
$\bar{r}$: correlation coefficient adjusted for the degree of freedom,
OB: actually observed values,
ES_1: estimates obtained from the total test,
ES_2: estimates obtained from the final test.

In our model which contains relatively few predetermined endogenous variables, the differences in the results between the total and final tests are presumably caused largely by the way in which the investment demand functions fit the observed data since these functions depend importantly on predetermined endogenous variables. Figure 13.5 compares the theoretically predicted values and the observed values of investment demand in the final

Figure 13.5 Estimates and Observed Values of Demand for Investment



Notes: (1) The dotted lines represent the estimates derived from the final test, and the solid lines represent the actual observations.
(2) The notation r in the diagrams denotes the correlation coefficient adjusted for the degree of freedom between estimates and observations.

test. The results that the theoretical values fluctuate more erratically than the actually observed values. This result suggests that our investment demand functions have failed to specify institutional elements which actually have an important effects on the investment demand.

Now let us construct input-output tables of the four sectors of our model on the basis of the theoretically predicted values derived from the final tests, and compare them with the data of the published Input-Output Tables reorganized into the same four sectors. This provides another test of the explanatory power of the entire model. Tables 13.2, 13.3 and 13.4 present the input-output tables constructed on the basis of the theoretically predicted values of the final tests for years 1955, 1960 and 1965, respectively. To see how well the theoretically derived values approximate the published values, we computed correlation coefficients between the theoretical values and the published values for each row or column, which is reported at the end of each respective row and column. For each year, the correlation coefficients are greater than 0.98, indicating that the fit between the theoretical values and published values in generally quite close.

Finally, the close fit of the theoretically predicted values of the gross domestic product to the actually observed values is shown by Figure 13.6.

In sum, the interpolation tests have yielded generally satisfactory results. Therefore, it is safe to conclude that the major objective of Part II of this volume, which is to express the structure of general interdependence of the Japanese economy for the period 1955 to 1965 by our four-sectoral model, has been reasonably fulfilled. The validity of this model as an analytical tool will be discussed further by policy simulations presented in the following Part III of this book.

Figure 13.6 ESTIMATES AND OBSERVED VALUES OF REAL AND NOMINAL GNP

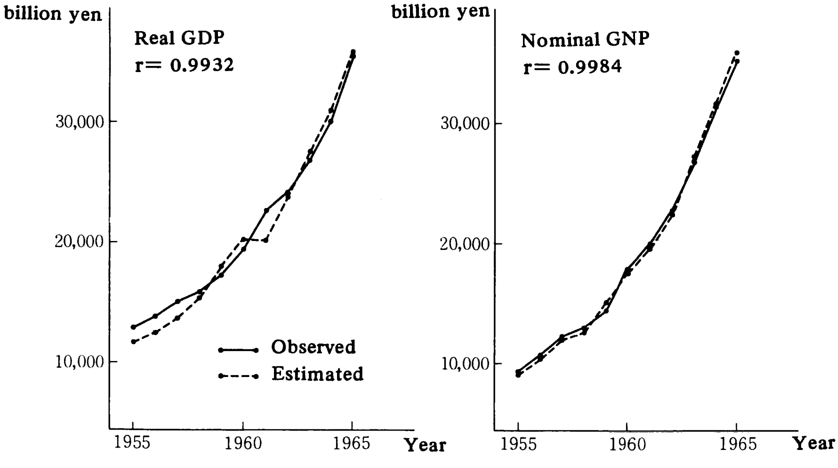


Table 13.2 ESTIMATED INPUT-OUTPUT TABLE OF 1955
(units: current prices in billions of yen)

Input \ Output	Intermediate Demand							Final Demand								
									Capital Formation							
	Agriculture Forestry and Fisheries	Light Manufacturing Industry	Heavy Manufacturing Industry	Commercial and Service Industry	Total Intermediate Demand	Business Consumption	Private Consumption	Government Expenditure	Private Fixed Capital Formation	Housing Investment	Government Fixed Capital Formation	Inventory Increases	Export	(Less) Import	Total Output	Correlation Coefficient
Intermediate Input																
1. Agriculture, Forestry and Fisheries	391.40	1623.40	86.26	10.47	2112.11	38.10	439.50	2.30	−2.69	0.00	0.00	169.39	31.93	−372.09	2385.10	0.9985
	0.1641	0.4758	0.0271	0.0021		0.0580		0.0021	−0.0037	0.00	0.00	0.8458	0.0603			
2. Light Manufacturing Industry	134.91	1039.03	747.87	299.56	2221.37	282.30	2642.49	43.00	45.31	0.00	0.90	173.34	342.23	−322.29	5500.41	0.9995
	0.0351	0.1889	0.1458	0.0373		0.2801		0.0247	0.0387	0.00	0.0013	0.2839	0.4010			
3. Heavy Manufacturing Industry	127.73	250.52	1797.64	181.54	2357.43	77.30	105.68	50.50	837.14	255.10	544.30	47.81	235.17	−182.84	4326.45	0.9989
	0.0394	0.0540	0.4155	0.0268		0.0842		0.0319	0.8477	0.7170	0.8279	0.0895	0.3267			
4. Commercial and Service Industry	147.92	642.77	561.46	491.32	1843.47	334.60	2261.86	798.10	41.43	0.00	15.50	7.31	287.46	25.64	5660.37	0.9993
	0.0546	0.1658	0.1553	0.0868		0.4909		0.6785	0.0502	0.00	0.0317	0.0172	0.4779			
Total Intermediate Input	801.96	3556.30	3193.23	982.88	8534.38	732.30	5449.52	893.90	921.19	255.10	560.70	397.85	896.79	−851.58	17872.33	0.9996
Value-Added																
1. Business Consumption Expenditure	−0.71	0.00	−85.73	430.75	344.30											
2. Compensation of Employees	170.23	874.53	439.55	1877.77	3362.08											
3. Income from Unincorporated Enterprises	1269.06	0.00	415.79	975.59	2660.44											
4. Income from Property	19.04	0.00	154.72	418.63	592.38											
5. Income from Private Corporations	7.31	0.00	941.23	466.17	1414.72											
6. General Government Income from Property and Enterpreneurship	3.08	0.00	19.61	57.98	80.67											
7. (Less) Interest on Public and Consumer Debt	−4.18	0.00	−85.73	−10.29	−100.20											
8. (Less) Stock Valuation Adjustment	0.06	0.00	−7.20	1.04	−6.10											
9. (Less) Imputed Service Charges	−4.08	0.00	−340.63	−133.40	−478.12											
10. Operating Surplus (10=3+4+...+9)	1290.28	0.00	1097.79	1775.71	4163.78											
11. Provisions for the Consumption of Fixed Capital	106.55	0.00	216.88	365.47	688.89											
12. Indirect Taxes	29.41	521.45	36.12	230.08	817.05											
13. (Less) Current Subsides	−12.61	1.97	0.13	−2.29	−12.85											
14. Income from Abroad	0.00	0.00	−25.30	0.00	−25.30											
15. Total Gross Value Added	1583.15	1944.11	1133.22	4677.49	9337.96											
Total Domestic Products	2385.10	5500.41	4326.45	5660.37	17872.33											
Correlation Coefficient	0.9995	0.9989		0.9988	0.9990											

Note: (1) The figures listed in the lower part within the cells in the block of intermediate transactions (upper left part of the table) represent input coefficients. The figures listed in the lower part within the cells in the block of final demand transactions (upper right part of the table) represent converter coefficients.

(2) Converter coefficients are not listed for the column of personal consumption expenditure for the reason that the relevant coefficients are estimated for disaggregated transactions of 5 major consumption items and cannot be expressed in the form of a single converter coefficient.

(3) The correlation coefficient listed at the right-hand end of each row represents the correlation between the estimates and the observed data (not listed in the table) in that row. The correlation coefficient at the bottom of each column, except the column of the total intermediates demand, represents the correlation between the estimates and the observed data in that column. The correlation coefficient at the bottom of the column for the total intermediate demand represents the correlation between all estimates in the table and the corresponding observed data.

Table 13.3 ESTIMATED INPUT-OUTPUT TABLE OF 1960
(units: current prices in billions of yen)

	Input \ Output	Intermediate Demand					Final Demand										
										Capital Formation							
		Agricul-ture Forestry and Fisheries	Light Manufac-turing Industry	Heavy Manufac-turing Industry	Commer-cial and Service Industry	Total Inter-mediate Demand	Business Consump-tion	Private Consump-tion	Govern-ment Expendi-ture	Private Fixed Capital Formation	Housing Invest-ment	Govern-ment Fixed Capital Formation	Inventory Increases	Export	(Less) Import	Total Output	Correla-tion Coeffi-cient
Intermediate Input	1. Agriculture, Forestry and Fisheries	491.26	2336.44	77.66	23.34	2928.70	38.00	603.29	0.90	12.08	0.00	0.00	12.89	82.43	−577.80	3121.06	0.9993
		0.1574	0.3297	0.0091	0.0025		0.0190		0.0006	0.0064	0.00	0.00	0.2208	0.0733			
	2. Light Manufacturing Industry	207.73	2100.06	1295.56	602.53	4205.88	1196.10	3711.35	42.30	44.16	0.00	2.70	459.17	602.40	−677.03	9528.41	0.9977
		0.0495	0.2204	0.1129	0.0480		0.4239		0.0213	0.0174	0.00	0.0021	0.3814	0.3984			
	3. Heavy Manufacturing Industry	221.53	805.88	5909.57	568.19	7505.17	178.10	470.93	61.30	2641.58	617.00	1141.10	336.37	584.05	−644.78	12888.91	0.9991
		0.0470	0.0753	0.4585	0.0403		0.0625		0.0306	0.9267	0.7952	0.8695	0.3489	0.3439			
	4. Commercial and Service Industry	111.72	1087.16	1246.67	996.33	3441.88	823.20	4272.13	1277.50	137.65	0.00	12.20	30.08	382.05	45.52	10378.45	0.9996
		0.0322	0.1380	0.1314	0.0960		0.3678		0.8113	0.0656	0.00	0.0118	0.1010	0.3056			
	Total Intermediate Input	1032.24	6329.54	8529.47	2190.40	18081.65	2235.40	9057.71	1382.00	2853.47	617.00	1156.00	838.51	1650.93	−1854.09	35916.84	0.9992
Value-Added	1. Business Consumption Expenditure	10.04	0.00	234.99	510.28	755.31											
	2. Compensation of Employees	237.56	1350.60	1705.42	3311.53	6605.11											
	3. Income from Unincorporated Enterprises	1536.06	0.00	908.04	1421.29	3865.39											
	4. Income from Property	42.43	0.00	513.67	812.57	1368.68											
	5. Income from Private Corporations	13.74	0.00	1875.36	878.91	2768.01											
	6. General Government Income from Property and Enterpreneurship	9.48	0.00	26.29	193.09	228.86											
	7. (Less) Interest on Public and Consumer Debt	−5.47	0.00	−64.17	−29.51	−99.16											
	8. (Less) Stock Valuation Adjustment	3.15	0.00	73.47	12.27	88.89											
	9. (Less) Imputed Service Charges	−7.19	0.00	−563.10	−223.12	−793.41											
	10. Operating Surplus (10=3+4+...+9)	1592.20	0.00	2769.57	3065.49	7427.27											
	11. Provisions for the Consumption of Fixed Capital	219.91	0.00	630.90	900.30	1751.11											
	12. Indirect Taxes	42.64	724.24	128.93	409.09	1304.90											
	13. (Less) Current Subsides	−13.53	−0.85	−1.58	−8.63	−24.59											
	14. Income from Abroad	0.00	0.00	16.10	0.00	16.10											
	15. Total Gross Value Added	2088.83	3198.88	4359.45	8188.06	17835.20											
	Total Domestic Products	3121.06	9528.41	12888.91	10378.45	35916.84											
	Correlation Coefficient	0.9997	0.9993		0.9992	0.9993											

Note: See the footnote attached to Table 13.2 for technical explanations of the content of the table.

Table 13.4 ESTIMATED INPUT-OUTPUT TABLE OF 1965
(units: current prices in billions of yen)

	Input \ Output	Intermediate Demand: Agriculture Forestry and Fisheries	Intermediate Demand: Light Manufacturing Industry	Intermediate Demand: Heavy Manufacturing Industry	Intermediate Demand: Commercial and Service Industry	Intermediate Demand: Total Intermediate Demand	Final Demand: Business Consumption	Final Demand: Private Consumption	Final Demand: Government Expenditure	Final Demand: Capital Formation: Private Fixed Capital Formation	Final Demand: Capital Formation: Housing Investment	Final Demand: Capital Formation: Government Fixed Capital Formation	Final Demand: Inventory Increases	Final Demand: Export	Final Demand: (Less) Import	Total Output	Correlation Coefficient
Intermediate Input	1. Agriculture, Forestry and Fisheries	543.64	3500.51	155.51	101.51	4301.17	120.90	1095.48	0.40	59.73	0.00	1.20	−145.06	69.94	−837.57	4646.47	0.9966
		0.1170	0.1920	0.0060	0.0044		0.0351		0.0001	0.0101	0.00	0.0004	0.1687	0.0211			
	2. Light Manufacturing Industry	470.99	4602.67	3610.13	1431.06	10114.85	1435.80	6509.96	104.90	160.46	0.00	68.80	606.87	1026.76	−1695.75	18388.61	0.9996
		0.1005	0.2503	0.1381	0.0615		0.4174		0.0357	0.0269	0.00	0.0240	0.6008	0.3071			
	3. Heavy Manufacturing Industry	301.14	1436.50	10190.04	1449.15	13376.83	105.70	1006.71	144.40	5245.61	1811.30	2725.70	596.14	1755.79	−933.59	25882.74	0.9974
		0.0649	0.0789	0.3937	0.0629		0.0307		0.0491	0.8882	1.0010	0.9500	0.1541	0.5304			
	4. Commercial and Service Industry	326.00	2060.67	3289.94	3141.58	8818.19	1777.80	9580.18	2699.30	459.13	0.00	73.40	109.82	486.49	40.60	23944.98	0.9995
		0.0676	0.1089	0.1223	0.1312		0.5168		0.9172	0.0748	0.00	0.0256	0.0774	0.1414			
	Total Intermediate Input	1641.77	11600.35	17245.62	6123.31	36611.05	3440.20	18192.32	2949.00	5924.93	1811.30	2869.10	1167.77	3338.99	−3426.32	72862.80	0.9988
Value-Added	1. Business Consumption Expenditure	−3.12	0.00	964.55	692.91	1654.34											
	2. Compensation of Employees	375.68	2724.38	3399.91	7331.21	13831.18											
	3. Income from Unincorporated Enterprises	2178.72	0.00	1707.02	3000.99	6886.73											
	4. Income from Property	98.74	0.00	1096.30	2208.83	3403.87											
	5. Income from Private Corporations	29.23	0.00	3391.53	2341.79	5762.55											
	6. General Government Income from Property and Enterpreneurship	6.82	0.00	101.72	150.29	258.83											
	7. (Less) Interest on Public and Consumer Debt	−11.45	0.00	133.89	−52.95	69.50											
	8. (Less) Stock Valuation Adjustment	−42.71	0.00	−53.17	−5.70	−101.58											
	9. (Less) Imputed Service Charges	−11.63	0.00	−1098.90	−528.59	−1639.12											
	10. Operating Surplus (10=3+4+...+9)	2247.72	0.00	5278.40	7114.66	14640.78											
	11. Provisions for the Consumption of Fixed Capital	341.92	0.00	1655.75	1926.58	3924.25											
	12. Indirect Taxes	68.46	1221.27	254.05	789.54	2332.32											
	13. (Less) Current Subsides	−25.96	−168.76	−2.76	−33.23	−230.71											
	14. Income from Abroad	0.00	0.00	98.60	0.00	98.60											
	15. Total Gross Value Added	3004.70	6788.26	8637.12	17821.67	36251.76											
	Total Domestic Products	4646.47	18388.61	25882.74	23944.98	72862.80											
	Correlation Coefficient	0.9999	0.9996		0.9991	0.9995											

Note: See the footnote attached to Table 13.2 for technical explanations of the content of the table.

[Estimated Structural Equations]

[Sector Classification]

Sector 1:	Agriculture, Forestry and Fisheries
Sector 2:	Light Manufacturing Industries
Sector 3:	Heavy Manufacturing Industries
Sector 4:	Commercial and Service Industries

[First Block]

§ Number of Workers

(1) $L_1=\bar{L}-\sum_{i=2}^{4} L_i$

(2) $\log L_2=7.1514541+0.19265695 \log K_2$ $r=0.9675$
$\quad(0.1409) \quad (0.0168)$

(3) $\log L_3=5.2481878+0.41891948 \log K_3$ $r=0.9712$
$\quad(0.2832) \quad (0.0342)$

(4) $\log L_4=6.9197492+0.31168829 \log K_4$ $r=0.9774$
$\quad(0.2087) \quad (0.0224)$

§ Operation Hours

(5) $h_2=(X_2/Q_2)^{\frac{1}{0.8173843}}$

(6) $h_3=(X_3/Q_3)^{\frac{1}{0.4318841}}$

(7) $h_4=(X_4/Q_4)^{\frac{1}{0.6541250}}$

§ Output Capacities

(8) $\log Q_2=-4.7953674+1.15550111 \log K_2$ $r=0.9959$
$\quad(0.2939) \quad (0.0350)$

(9) $\log Q_3=-1.0820038+0.99563189 \log K_3$ $r=0.9776$
$\quad(0.5908) \quad (0.0714)$

(10) $\log Q_4=-4.9290430+1.1905800 \log K_4$ $r=0.9957$
$\quad(0.3436) \quad (0.0370)$

§ Output Level of Sector 1

(11) $\log(X_1/L_1)=-8.3004598+0.3036 \log(A_1/L_1)+0.83086476 \log(K_1+K_{g1})$
$\quad(0.50113) \quad (0.0021) \quad (0.05602)$

§ Simultaneous Determination of Prices and Wages

(12) $\partial V_1/\partial L_1=(1-0.3036)(p_1-\sum_{i=1}^{4} p_i a_{i1})X_1/L_1=w_1 h_1$

(13) $w_{23}=0.0002327764+34.558113 w_1$ $r=0.9906$
$\quad(0.00005712) \quad (1.5938)$

(14) $w_4=0.0002537835+41.936762 w_1$ $r=0.9974$
$\quad(0.00003603) \quad (1.0053)$

(15) $$p_2=\frac{1}{-116428.63(a_{22}+t_{I2}-1)}\left\{(X_2+116428.63)\left(\frac{1}{0.8173843}L_2h_2w_2/X_2+\sum_{i=1\,(i\neq 2)}^{4}p_ia_{i2}\right)\right\}$$

(16) $$p_3=\frac{1}{-60920353.0(a_{33}+t_{I3}-1)}\left\{(X_3+60920353.0)\left(\frac{1}{0.4318841}L_3h_3w_3/X_3+\sum_{i=1\,(i\neq 3)}^{4}p_ia_{i3}\right)\right\}$$

(17) $$p_4=\frac{1}{-2305380.0(a_{44}+t_{I4}-1)}\left\{(X_4+2305380.0)\left(\frac{1}{0.654125}L_4h_4w_4/X_4+\sum_{i=1\,(i\neq 4)}^{4}p_ia_{i4}\right)\right\}$$

§ Number of Employees

(18) $E_{y1}=42.581300+0.08644268\,L_1$ $\qquad r=0.9999$
$\qquad(5.6667)\qquad(0.000424)$

(19) $E_{y2}=-56.703900+0.87011164\,L_2$ $\qquad r=0.9970$
$\qquad(145.1758)\qquad(0.02251)$

(20) $E_{y3}=-178.14270+0.96767723\,L_3$ $\qquad r=0.9999$
$\qquad(16.70690)\qquad(0.00264)$

(21) $E_{y4}=-1288.103+0.72751133\,L_4$ $\qquad r=0.9999$
$\qquad(86.3294)\qquad(0.004697)$

[Second Block]

§ Indirect Tax and Charges

(22) $T_{I1}=t_{I1}p_1X_1$

(23) $T_{I2}=t_{I2}p_2X_2$

(24) $T_{I3}=t_{I3}p_3X_3$

(25) $T_{I4}=t_{I4}p_4X_4$

(26) $T_I=\sum_{i=1}^{4}T_{Ii}$

§ Value Added

(27) $V_1=(p_1-\sum_{i=1}^{4}p_ia_{i1})X_1$

(28) $V_2=(p_2-\sum_{i=1}^{4}p_ia_{i2})X_2$

(29) $V_3=(p_3-\sum_{i=1}^{4}p_ia_{i3})X_3$

(30) $V_4=(p_4-\sum_{i=1}^{4}p_ia_{i4})X_4$

§ Compensation of Employees

(31) $E_{I1}=E_{y1}h_1w_1$

(32) $E_{I2}=E_{y2}h_2w_{23}$

(33) $E_{I3}=E_{y3}h_3w_{23}$

(34) $E_{I4}=E_{y4}h_4w_4$

(35) $E_I=\sum_{i=1}^{4}E_{Ii}$

§ Provisions for the Consumption of Fixed Capital

(36) $D_{e1}/\eta_{Kp}=-317.87463+0.086547698\,K_1$ $\quad r=0.9635$
$\qquad(50.490521)\quad(0.008018)$

(37) $D_{e23}/\eta_{Kp}=-303.98370+0.12170884(K_2+K_3)$ $\quad r=0.9966$
$\qquad(32.51584)\quad(0.003334)$

(38) $D_{e4}/\eta_{Kp}=-691.54620+0.15879786\,K_4$ $\quad r=0.9943$
$\qquad(65.8944)\quad(0.005690)$

(39) $D_e=D_{e1}+D_{e23}+D_{e4}$

§ Operating Surplus

(40) $B_{s1}=V_1-D_{e1}-B_{e1}-T_{I1}+S_{e1}-E_{I1}$

(41) $B_{s23}=(V_2+V_3)-D_{e23}-(B_{e2}+B_{e3})-(T_{I2}+T_{I3})+(S_{e2}+S_{e3})-(E_{I2}+E_{I3})$

(42) $B_{s4}=V_4-D_{e4}-B_{e4}-T_{I4}+S_{e4}-E_{I4}$

§ Income from Unincorporated Enterprises

(43) $U_{cI1}=67.9815+0.91253561(B_{s1}+A_{p1}-Y_{G1}+D_{cG1})$ $\quad r=0.9908$
$\qquad(72.1878)\quad(0.04151)$

(44) $U_{cI23}=B_{s23}+(A_{p2}+A_{p3})-(Y_{G2}+Y_{G3})+(D_{cG2}+D_{cG3})-P_{I23}-C_{I23}$

(45) $U_{cI4}=313.5087+0.35588237(B_{s4}+A_{p4}-Y_{G4}+D_{cG4})$ $\quad r=0.9899$
$\qquad(68.3632)\quad(0.01696)$

(46) $U_{cI}=U_{cI1}+U_{cI23}+U_{cI4}$

§ Income from Property

(47) $P_{I1}=-83.05872+0.078812881(B_{s1}+A_{p1}-Y_{G1}+D_{cG1})$ $\quad r=0.9707$
$\qquad(11.3025)\quad(0.006499)$

(48) $P_{I23}=-149.23602+0.20106021(B_{s23}+A_{p2}+A_{p3}-Y_{G2}-Y_{G3}+D_{cG2}+D_{cG4})$
$\qquad(41.3580)\quad(0.01091)$
$\quad r=0.9856$

(49) $P_{I4}=-166.5641+0.31455447(B_{s4}+A_{p4}-Y_{G4}+D_{cG4})$ $\quad r=0.9979$
$\qquad(27.2653)\quad(0.006764)$

(50) $P_I=P_{I1}+P_{I23}+P_{I4}$

§ Income from Private Corporations

(51) $C_{I1}=B_{s1}+A_{p1}-Y_{G1}+D_{cG1}-U_{cI1}-P_{I1}$

(52) $C_{I23}=150.26260+0.52321958(B_{s23}+A_{p2}+A_{p3}-Y_{G2}-Y_{G3}+D_{cG2}+D_{cG3})$
$\qquad(93.2235)\quad(0.02460)$
$\quad r=0.9880$

(53) $C_{I4}=-116.4137+0.32371964(B_{s4}+A_{p4}-Y_{G4}+D_{cG4})$ $\quad r=0.9922$
$\qquad(51.7530)\quad(0.01234)$

(54) $C_I=C_{I1}+C_{I23}+C_{I4}$

§ Personal Income and Saving

(55) $Y_p = E_I + U_{GI} + P_I + \sum_{i=1}^{4} D_{Vi}$

(56) $T_p = t_P Y_p$

(57) $Y_D = Y_p - T_p - TR_{pR} - TR_{pG} + TR_{Rp} + TR_{Gp} + TR_{cp} - TR_{sI} - I_{cp}$

(58) $E = 119.21050 + 0.4869900\, E_{t-1} + 0.4302\, Y_D$ $\quad r = 0.9997$
$\quad (145.5658)\ (0.08624)\ \ \ (0.1337)$

(59) $S_p = Y_D - E$

§ Retained Earnings of Private Corporations

(60) $C_{I2} = \{V_2/(V_2 + V_3)\} C_{I23}$

(61) $C_{I3} = \{V_3/(V_2 + V_3)\} C_{I23}$

(62) $T_{c1} = t_c C_{I1}$

(63) $T_{c2} = t_c C_{I2}$

(64) $T_{c3} = t_c C_{I3}$

(65) $T_{c4} = t_c C_{I4}$

(66) $T_c = \sum_{i=1}^{4} T_{ci}$

(67) $M_1 = C_{I1} - T_{c1} - TR_{cp1} - I_{cc1} - D_{V1}$

(68) $M_2 = C_{I2} - T_{c2} - TR_{cp2} - I_{cc2} - D_{V2}$

(69) $M_3 = C_{I3} - T_{c3} - TR_{cp3} - I_{cc3} - D_{V3}$

(70) $M_4 = C_{I4} - T_{c4} - TR_{cp4} - I_{cc4} - D_{V4}$

[Third Block]

§ Deflator of Final Demand Items

(71)～(83)

$$\begin{pmatrix} \eta_b \\ \eta_{c1} \\ \eta_{c2} \\ \eta_{c3} \\ \eta_{c4} \\ \eta_{c5} \\ \eta_{cp} \\ \eta_G \\ \eta_{Kp} \\ \eta_H \\ \eta_{KG} \\ \eta_{INV} \\ \eta_{EX} \end{pmatrix} = \begin{pmatrix} \text{Coefficients of} \\ \text{Price Converter} \\ B^T \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

(84) $\eta_{EX1}=0.72845037+0.609671\,p_1-0.137828\,p_2-0.1095\,p_3-0.0723530\,p_4$

(85) $\eta_{EX2}=-0.71985998-0.2276390\,p_1+2.0519999\,p_2+0.0072287\,p_3$
$-0.0996950\,p_4$

(86) $\eta_{EX3}=2.8499755+2.059051\,p_1-1.799260\,p_2+1.9019190\,p_3-4.1331910\,p_4$

(87) $\eta_{EX4}=-0.9063020-0.0199440\,p_1+0.428554\,p_2+2.8964970\,p_3$
$-1.4677420\,p_4$

(88) $\eta_{EX5}=2.4273587-2.277590\,p_1-1.682525\,p_2+0.254914\,p_3+2.332721\,p_4$

(89) $\eta_{EX6}=0.6776098+0.973714\,p_1-1.097138\,p_2+0.562961\,p_3-0.2041500\,p_4$

§ Personal Consumption Expenditures

(90)~(94)

$$\begin{pmatrix} \alpha_1^* & 0 & 0 & 0 & 0 & 1 \\ 0 & \alpha_2^* & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha_3^* & 0 & 0 & 1 \\ 0 & 0 & 0 & \alpha_4^* & 0 & 1 \\ 0 & 0 & 0 & 0 & \alpha_5^* & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_{c1}q_1 \\ \eta_{c2}q_2 \\ \eta_{c3}q_3 \\ \eta_{c4}q_4 \\ \eta_{c5}q_5 \\ -\lambda^* \end{pmatrix} = \begin{pmatrix} -\eta_{c1}a_1^* \\ -\eta_{c2}a_2^* \\ -\eta_{c3}a_3^* \\ -\eta_{c4}a_4^* \\ -\eta_{c5}a_5^* \\ E/M \end{pmatrix} \quad \begin{matrix} \alpha_i^*=1/\alpha_i \\ a_i^*=a_i/\alpha_i \\ (i=1\cdots5) \end{matrix}$$

$i=1$: Food
2 : Clothing
3 : Light and Fuel
4 : Housing
5 : Miscellaneous

where

$a_1=849356.81-223747.52m-0.0426120\,H_1,\quad \alpha_1=0.1978669$

$a_2=-135583.07+23020.71m-0.0064312\,H_2,\quad \alpha_2=0.0297410$

$a_3=59945.22-16083.89m-0.00841886\,H_3,\quad \alpha_3=0.010000$

$a_4=577876.99-124827.65m-0.0555299\,H_4,\quad \alpha_4=0.1935583$

$a_5=487519.75-116077.72m-0.0787848\,H_5,\quad \alpha_5=0.1479333$

(95) $H_1^t=\sum_{t=0}^{t-1}\eta_{C1}^t\,q_1^t \qquad H_1^1=0.0$

(96) $H_2^t=\sum\eta_{C2}^t q_2^t \qquad H_2^1=0.0$

(97) $H_3^t=\sum\eta_{C3}^t q_3^t \qquad H_3^1=0.0$

(98) $H_4^t=\sum\eta_{C4}^t q_4^t \qquad H_4^1=0.0$

(99) $H_5^t=\sum\eta_{C5}^t q_5^t \qquad H_5^1=0.0$

§ Demand for Investment Goods

(100) $p_{A1}=-0.10882425+1.2300317\,p_1+3.3858985(p_1-p_1^{t-1})$

(101) $\{p_{A1}(1-t_{I1}-a_{11})-\sum_{i\neq1}p_i a_{i1}\}e^{-8.3004598}\cdot0.8308676\,A^{0.3036}\,L_1^{1-0.3036}$
$(K_1^{t+1}+Kg)^{0.8308676-1}-K_1^{t+1}\eta_{Kp}^2\cdot0.98222970\times10^{-8}\cdot2.2125234\times$
$\{\eta_{Kp}(K_1^{t+1}-K_1^t)\}^{2.2125234-1}-\eta_{Kp}[0.98222970\times10^{-8}\{\eta_{Kp}(K_1^{t+1}$
$-K^t)^{2.2125234}\}+i+0.08655]=0$

(102) $YY_2=25419.738+2.2144873\,GDP^{t-1}-956.61477\,i_0+1501.5531\ (DUM2)$

(103) $-\dfrac{(1-a_{22}-t_{I2})YY_2\cdot e^{-4.795}\cdot 1.156\cdot K_2{}^{t+1\ 1.156-1}h_2{}^{*0.8176}(-21769.9)}{(e^{-4.795}K_2{}^{t+1\ 1.156}h_2{}^{*0.8173}+21769.9)^2}$

$-e^{-4.795}\cdot 1.156K_2{}^{t+1\ 1.156-1}h_2{}^{*0.8173}\sum_{i=1(i\neq 2)}^{4}p_i a_{i2}-e^{7.1514}\cdot 0.1926K_2{}^{t+1\ 0.1926-1}h_2{}^{*}w_{23}$

$-0.09314\eta_{Kp}\{\eta_{Kp}(K_2{}^{t+1}-K_2{}^{t})+D_2+2631.551\}^{0.5325985-1}\cdot M_2{}^{-0.53259}$

$\{(1-0.53259)\eta_{Kp}K_2{}^{t+1}+D_2+2631.551-\eta_{KP}K_2{}^{t}\}-(i+0.1217)\eta_{Kp}=0$

(104) $YY_3=2866.575+4.5797085\,GDP^{t-1}-2307.9288\,i_0+3121.8331\,(DUM_3)$

(105) $-\dfrac{(1-a_{33}+t_{I3})YY_3e^{-1.082}\cdot 0.9956K_3{}^{t+1\ 0.9956-1}h^{3*0.4319}(-14060.67)}{(e^{-1.082}\cdot K_3{}^{t+1\ 0.9956}h_3{}^{*0.4319}+14060.67)^2}$

$-e^{-1.082}\cdot 0.9956K_3{}^{t+1\ 0.9956-1}h_3{}^{*0.4319}\sum_{i=1\ (i\neq 3)}^{4}p_i a_{i3}-e^{5.2481}\cdot 0.4189K_3{}^{t+1\ 0.4189-1}h_3{}^{*}w_{23}$

$-0.3084\eta_{Kp}\{\eta_{Kp}\cdot(K_3{}^{t+1}-K_3{}^{t})+D_3+5575.125\}^{0.15198-1}M_3{}^{-0.151981}$

$\{(1-0.15198)\eta_{Kp}K_3{}^{t+1}+D_3+5575.125-\eta_{Kp}K_3{}^{t}\}+(i+0.1217)\eta_{Kp}=0$

(106) $YY_4=1543.1532+4.2895909\,GDP_{t-1}-2592.9616i_0-89.71262USINV_4$
$+3581.5803(DUM_4)$

(107) $-\dfrac{(1-a_{44}-t_{I4})YY_4e^{-4.929}1.1906K_4{}^{t+1\ 1.1906-1}h_4{}^{*0.6541}(-74968.75)}{(e^{-4.929}K_4{}^{t+1\ 1.1906}h_4{}^{*0.6541}+74968.75)^2}$

$-e^{-4.929}1.1906K_4{}^{t+1\ 1.1906}h_4{}^{*0.6541}\sum_{i=1(i\neq 4)}^{4}p_i a_{i4}-e^{6.9197}\cdot 0.3116K_4{}^{t+1\ 0.3116-1}h_4{}^{*}w_4$

$-0.09331\,\eta_{Kp}\{\eta_{Kp}(K_4{}^{t+1}-K_4{}^{t})+D\}^{1.6974-1}(M_4+5621.7)^{-1.6974}$

$\{(1+1.6974)\eta_{Kp}K_4{}^{t+1}+D-\eta_{Kp}K_4{}^{t}\}-(i+0.1588)\eta_{Kp}=0$

(108) $D_2=D_2{}^0+\sum_{t=1}^{t-1}\eta_{Kp}(K_2{}^{t+1}-K_2{}^{t})-\sum_{t=1}^{t-1}M_2{}^{t},\quad D_2{}^0=0$

(109) $D_3=D_3{}^0+\sum\eta_{Kp}(K_3{}^{t+1}-K_3{}^{t})-\sum M_3{}^{t},\quad D_3{}^0=0$

(110) $D_4=D_4{}^0+\sum\eta_{Kp}(K_4{}^{t+1}-K_4{}^{t})-\sum M_4{}^{t},\quad D_4{}^0=0$

(111) $I_1=K_1{}^{t+1}-K_1{}^{t}$

(112) $I_2=K_2{}^{t+1}-K_2{}^{t}$

(113) $I_3=K_3{}^{t+1}-K_3{}^{t}$

(114) $I_4=K_4{}^{t+1}-K_4{}^{t}$

(115) $I_{NT}=\sum_{i=1}^{4}I_i$

(116) $D_T=-485.751\ +\ 0.04460245\sum_{i=1}^{4}K_i$ $\qquad r=0.9122$
$\qquad (183.6130)\ (0.006680)$

(117) $I_{GT}=D_T+I_{NT}$

§ Demand for Exported Goods

(118) $\log(\eta_{EX1}E_{X1}/P_1)=-18.283927+2.2764686\log(W_1/P_1)$
$\qquad (5.8356)\qquad (0.5768)$

$-0.752038\log(\eta_{EX1}/P_1)+3.0283893\log(1/P_1)$ $\qquad r=0.9742$
$\qquad (1.03411)\qquad (0.9110)$

(119) $\eta_{EX2}E_{X2}/P_2 = -163.02734 + 0.07900781(W_2/P_2) + 149.35880(1/P_2)$
(61.8192) (0.005632) (76.6019)
$r=0.9879$

(120) $\log(\eta_{EX3}E_{X3}/P_3) = -12.164170 + 1.8738510 \log(W_3/P_3)$
(2.5213) (0.2793)

$-0.017772 \log(\eta_{EX}/P_3) - 1.615100 \log(1/P_3)$ $r=0.9898$
(0.8433) (2.2197)

(121) $\log(\eta_{EX4}E_{X4}/P_4) = -11.278056 + 1.8533378 \log(W_4/P_4)$
(1.3664) (0.1471)

$-2.175038 \log(\eta_{EX4}/P_4)$ $r=0.9940$
(0.21991)

(122) $\log(\eta_{EX5}E_{X5}/P_5) = -11.540847 + 1.7596699 \log(W_5/P_5)_{t-1}$
(3.1829) (0.3059)

$-0.551537 \log(\eta_{EX5}/P_5)$ $r=0.9826$
(0.4181)

(123) $\log(\eta_{EX6}E_{X6}/P_6) = 289.71353 + 0.048097215(W_6/P_6)$
(405.1822) (0.002821)

$-132.77320(\eta_{EX6}/P_6) - 308.548(1/P_6)$ $r=0.9975$
(129.3359) (344.9308)

(124)~(129) $EX_{Ni} = EX_i \cdot \eta_{EXi} \quad (i=1, \cdots, 6)$

(130) $$EX_T = \sum_{i=1}^{6} EX_{Ni}$$

§ Demand for Imported Goods

(131)~(134) $IM_i = m_i X_i \quad (i=1\cdots4)$

(135) $$IM_T = \sum_{i=1}^{4} IM_i p_i$$

§ Inventory Increases

(136) $SINV_2 = -751.12357 + 0.57685501K_2 + 1577.4271GW_2^{t-1}$ $r=0.9986$
(60.4058) (0.01159) (378.4357)

(137) $SINV_3 = -211.52590 + 0.26928396K_3 + 914.56080GW_3^{t-1}$ $r=0.9780$
(119.3521) (0.02081) (421.9863)

(138) $SINV_4 = -315.70109 + 0.045739371K_4 + 497.86770GW_4^{t-1}$ $r=0.9927$
(287.8357) (0.002222) (236.1142)

(139) $USINV_2 = 1577.4271GW_2^{t-1}$

(140) $USINV_3 = 914.56080GW_3^{t-1}$

(141) $USINV_4 = 497.86770GW_4^{t-1}$

(142) $INV_1 = X_1^D - X_1^S$

(143) $INV_2 = SINV_2^t - SINV_2^{t-1}$

(144) $INV_3 = SINV_3^t - SINV_3^{t-1}$

(145) $INV_4 = SINV_4^t - SINV_4^{t-1}$

§ Final Demand Vector

(146)~(149)

$$F=\begin{pmatrix}F_1\\F_2\\F_3\\F_4\end{pmatrix}=\begin{pmatrix}\text{Coefficients of}\\\text{Quantity Converter}\\B\\\end{pmatrix}\begin{pmatrix}q_1M\\q_2M\\q_3M\\q_4M\\q_5M\\I_{GT}\\E_X\end{pmatrix}+\begin{pmatrix}B_{FG1}/p_1\\B_{FG2}/p_2\\B_{FG3}/p_3\\B_{FG4}/p_4\end{pmatrix}+\begin{pmatrix}IH_1/p_1\\IH_2/p_2\\IH_3/p_3\\IH_4/p_4\end{pmatrix}+\begin{pmatrix}G_1/p_1\\G_2/p_2\\G_3/p_3\\G_4/p_4\end{pmatrix}$$

$$+\begin{pmatrix}C_{N1}/p_1\\C_{N2}/p_2\\C_{N3}/p_3\\C_{N4}/p_4\end{pmatrix}+\begin{pmatrix}I_{G1}/p_1\\I_{G2}/p_2\\I_{G3}/p_3\\I_{G4}/p_4\end{pmatrix}+\begin{pmatrix}INV_1\\INV_2\\INV_3\\INV_4\end{pmatrix}$$

[Forth Block]

§ Demand for Money

(150) $M_p=-41.0708+0.095081072Y_D$ $\quad r=0.9977$
$\qquad (25.10) \quad (0.002022)$

(151) $\log M_c=-9.5476+1.1115\log(\sum_{i=1}^{4}p_iX_i)-1.5087\log i$ $\quad r=0.9920$
$\qquad (2.417) \quad (0.1045) \qquad ((1.324)$

(152) $M^D=M_p+M_c+M_G=M^S$

§ Saving and Investment

(153) $I_T=I_H+I_G+I_{GT}+\sum INV_i$

(154) $S_T=S_p+\sum_{i=1}^{4}M_i+D_1+D_{23}+D_4+GRE$

§ Demand Level of Each Sector

(155)~(158) $X^D=[I-A+m]^{-1}F$

§ Revenue and Expenditure in Government Sector

(159) $GR=T_p+T_c+T_I+\sum_{i=1}^{4}Y_{Gi}+TR_{sI}+TR_{pG}+TR_{RG}+D_G$

(160) $GRE=GR-\sum_{i=1}^{4}G_i-\sum_{i=1}^{4}S_{ci}-TR_{Gp}-TR_{GR}$

§ Gross Domestic Product and its Growth Rate

(161) $NGNP=\sum_{i=1}^{4}V_i=\sum_{i=1}^{4}(F_i-I_{Mi})p_i$

(162) $RGNP=NGNP/PP$

(163) $PP=(\sum p_iX_i)/\sum X_i$

(164)~(166) $GW_i=X^t/X^t \quad (i=2,\cdots,4)$

Part III Policy Simulations and Evaluations of Japanese Economic Policies

Chapter 14
The Menu of Economic Policies in the Context of General Interdependence

14.1 The Measured Model and Economic Institutions

As we have stressed in Part I of this volume, our basic standpoint has been that "unlimited *laissez-faire*" and establishment of the "competitive market" do not necessarily mean the same thing. We maintain this basic position even when we evaluate the post-Keynesian free economic system. The competitive market is not something bestowed by nature. In order to realize the competitive market which Classical and Neo-Classical economists had envisaged, we need to build appropriate institutional and policy frameworks. The purpose of our enquiry is to find out, in this context, what would be the necessary and legitimate policy interventions.

Some institutions and policies might be unnecessary, unjustifiable and even harmful, as Adam Smith accused long ago. There are also types of governmental intervention which can be useful only when they are made to an appropriate degree, while they are useless or even counterproductive when they intervene too much or too little. Our motivation behind constructing the model in Part II has been to obtain clues by which to distinguish necessary policy interventions from unnecessary or even undesirable ones.

The impact that policies and institutions have on the economy depends upon how private economic decision making units react to them. In constructing our multi-sectoral model, we have tried, therefore, to describe the behavior of consumers and firms empirically using the framework of micro behavioral theory of economics. We have not taken for granted such *a priori* assumptions as the constancy of consumers' preference and the linear homogeneity of the production function which seems to be instrumental for the doctrine of *laissez-faire*. Our behavioral models of consumers and producers have been formulated ascertaining rigorously that the underlying theoretical consistency is supported by the observed data. Our model describes consumption behavior of households for each expenditure item and saving, as well as producers' behavior of commodity supply, employment, investment and financing. Through the interaction of these individual actions, equilibrium quantities and prices will be determined in the commodity, labor and money markets. We have not adopted a *priori* assumption of perfect competiton in describing these behaviors, but instead adopted a general model of imperfect competition, based not merely on the conventional theory but also on the theory of polypoly developed in Part I of this volume, which contains as its special case the case of perfect competition. By means of this formulation, we can assess the degree of imperfection of competition from the observed data of the market.

These theoretical components can be incorporated generally in constructing the multi-sectoral economic model which describes the allocation of resources and determination of prices among a plural number of sectors and commodities. For the moment, our model has been formulated on the basis of the four-sector and four-commodity classification scheme. The model is designed to reconstruct the basic mechanism behind the observed phenomena from which emerge the actual performances of consumers and producers in response to changes in extraneous conditions and their interactions. In other words, the behavioral patterns of economic decision making units and the market mechanism described in the model should be regarded as being separate and independent from the extraneous conditions given for each year during the period of observation. Therefore, by changing the values of the set of exogenous variables we can simulate the resultant changes in the performance of economic actors by our model.

Our eventual goal is to measure and evaluate the effect of institutions and policies upon actors and markets. However, the specification of our model needs to be elaborated further in order to identify the working of all the important institutions in terms of the model. For example, we were unable to incorporate labor supply functions as a theoretical component of the model. We instead have had to give the size of labor force exogenously each year. Consequently, the model is incapable of measuring directly the effects of the current Labor Standards Law and collective agreements upon the labor market. The effect of the Labor Standards Law is taken into account only indirectly by being implicity reflected in the determination of the size of labor force. There are a number of other legal and non-legal institutions whose effects are only implicit in the measured parameters of structural equations. Our present model is still incomplete, in this sense, for our eventual goal of evaluating the effects of many important institutions which affect the working of the economic system has not yet been attained.

It is nevertheless clearly possible to simulate, using the model constructed in Part II, the impacts of the explicitly specified policy variables upon the total endogenous system of the economy which take place through the interactive processes of the network of general interdependence. It should be noted, however, that the simple fact that the precision of interpolation by the total or final test is high, does not ensure that the precision of estimation of the endogenous variables in policy simulation is also high. This is because the values of endogenous variables, which correspond to the set of hypothetical values of exogenous policy variables, are determined by a kind of "extrapolation." The usefulness of the model as a policy simulator may be evaluated only after examining its empirical precision when applied to this kind of extrapolation and after identifying the maximum range of the values of exogenous variables which is permissible to maintain the precision of extrapolation.

There are of course innumerable cases of policy simulations which are worthy of trial. Let us select, however, only a few cases here and observe the final effects of the governmental policy action on the economy through interactions within the network of the general interdependence. This examination will serve not only as an examination of the impact of a policy but also an examination of the applicability of the policy simulation conducted by the present model.

14.2 Policy Instruments and Objectives Qualified by General Equilibrium Theory

In the real world, many economic problems such as inflation, pollution, social welfare, etc. emerge day after day, and remedial policies are being discussed constantly. Since many economic issues require prompt remedies, each issue tends to be hastily treated in isolation and without due consideration of its connections with other related issues. For example, the problem of inflation is discussed separately from the problem of employment or income distribution, and the problem of pollution is taken up separately from the problem of supply capacity of goods or the costs of production. It is not unusual that even a reduction in taxes and enrichment in social security programs are thought to be simultaneously attainable without limit.

Needless to say, however, it is meaningless to discuss each of these issues in isolation once we admit the fact that sub-systems of an economy are closely linked with each other. The trade-off relationship between policy targets has come to attract increasing attention. A typical example of such a trade-off is the relationship between the rate of economic growth and the stability of prices. More recently, with the international monetary system being shaken, tasks of economic policies have become quite complex: governments of many countries are said to have confronted the common problem of what has been called a "trilemma" which is the contradiction among the three policy objectives: full employment, the stability of prices and the balance of trade.

Although expressions such as "trade-off" and "trilemma" may still sound new, these are only natural problems in view of the general interdependence in the company. These problems have been known since the enlightening work of Quesney's *Tableau Economique*. Suppose there are N endogenous variables which may be grouped into M subsets corresponding to M policy objectives. However, it is obvious that the degrees of attainment of the M policy objectives are closely interrelated because of the general interdependence of the N endogenous variables. Therefore, the degree of attainment of each policy target can be evaluated meaningfully only in relation to the rest of the M targets. Suppose there is a particular set of values of G policy variables which determine unique values of N endogenous variables, then, in turn, they determine the degrees of attainment of M policy targets. Thus, it is implied that a set of values of G policy variables corresponds to a set of M policy targets. The results of various tests of our quantitative model developed in Part II of this volume suggests that we can obtain unique solutions of endogenous variable in the case of policy simulations so long as the values of policy variables do not deviate too much from their observed values. From this, we conclude that a numerical correspondence between a certain degree of attainment of policy target and each set of values of policy variables can be ascertained by our model.

It is of course meaningful to examine the partial effects of a change in the value of one of the G policy variables under the Marshallian assumption of *ceteris paribus*. It is also possible to estimate the effects using the estimated model. In such a case, however, it is not only the value of one of the M policy objectives of our particular interest that changes but also those of all M targets. Whether one likes it or not, all of the rest of M targets are affected unavoidably. This is the natural consequence generated from the general interdependence built into the economy.

When politicians or journalists discuss policies, they tend to discuss the partial effect of an individual policy independently. In other words, each policy is discussed assuming implicitly that other things are held constant. If, however, a variety of policies are implemented more or less simultaneously in reality, each of the policy targets would eventually be affected by all of these policies, and the result would be quite different from what would be expected from the partial derivatives.

Since the sum of partial equilibrium analyses cannot substitute for a general equilibrium analysis, it is quite reckless to implement policies on the basis of the former type of reasoning.

14.3 The Menu for Alternative Policies

In contemporary welfare economics, attention has been concentrated on pure theoretical analysis of production in competitive markets, while the aspect of distribution has been dealt with relatively less intensively. In other words, the major issue in New Welfare Economics has been the determination of the criteria of efficient allocation of resources assuming that a social concensus has been reached a desirable pattern of distribution, at best. In this respect, however, we have already demonstrated theoretically in Part I of this book that the patterns of distribution and bargaining positions of bargainers in the market are closely related with each other. In other words, our theoretical analysis suggests that an analysis of market competition which takes the patterns of distribution as given is not meaningful.

Let us digress for a moment and see how policy problems are dealt with in the real world. There are hardly any policy issues there which are unrelated to the question of distribution. In the real world, it is the patterns of distribution themselves that are the central focus of policy debates. The question of distribution in this situation includes many specific problems: income distribution between individual persons and juridical persons, income distribution of persons by different traits, and income distribution between persons with identical traits. Moreover, the question of distribution includes such problems as changes in the distribution of real purchasing power among economic actors, changes in the income distribution among industries or relative changes in the book values of assets which all take place due to changes in absolute and relative prices in the economy. The question of distribution relates still further to such problems as the different effects of external economies and diseconomies upon different persons triggered, for example, by intensified environmental pollution or appropriations of private land for public use etc.

To conclude that one distribution is better than the other, we need to have a social concensus to support the evaluation. The attainability of a social concensus through democratic means has been qualified traditionally by pure theorists as the question of existence of social welfare functions. It is well known, however, that K. J. Arrow has revealed through his rigorous analysis that the existence of such a social welfare function is not always warranted.[1] This problem has remained an important topic for pure theorists and many analyses have been made. It is important to note in this respect that the assumption of independence of the utility of one individual from that or others is maintained even in the cases of analyses with relaxed assumptions. We have ascertained empirically, however, that the constancy of preference functions cannot be taken for granted even in the case of such a limited aspect of preference as consumers' preference, as we reported in

Chapter 10. In an individual's utility at one point in time is indeed dependent on his utility at a different point in time, as found by our analysis, it would also be difficult to deny on empirical grounds the notion that utilities of different persons are interdependent which has been put forth initially by Dusenberry.[2] In view of this, it seems also not easy to prove, using purely abstract models, the existence of social welfare functions as such. Thus, we have to depend on actual choices of alternative economic policy targets made by the public.

We have said earlier, however, that the actual selection of policy objectives is in many cases dependent on intuitive partial analysis-type predictions and that the judgement based on such predictions is different from the understanding of the true network of causal relationships in the economy. A good example in point in Japanese economy is the determination of the official price of rice. When the price of rice is determined by interventions of public authorities, it is usually unclear whether the public pricing policy aims at strengthening a national self-sufficiency in rice or at promoting an equitable income distribution between urban and rural areas. Even in the case in which an increase in the consumers' price of rice is discussed in connection with the problem of inflation, effects of the increase on prices in general are only poorly understood. Consumers in urban areas generally think that it is only the consumers' price that affects their home economies. Therefore, they are concerned only with the consumer's price and not with the producers' price. They even encourage an increase in the producers' price for the naive reason that an increase in income of farm-households should be welcomed.

However, it is obvious from the viewpoint of the general interdependence of an economy that it is not only through an increase in the consumers' rice price that an increase in the producers' price affects prices in general. An increase in the producers' price directly affects the rural-urban labor force migration by shifting the marginal value added productivity curve in agricultural sector upward, and consequently gives rise to an increase in urban wages and to a shortage of labor force in manufacturing and service sectors. An increase in income from rice production will also increase the demand of farm-households for investment and consumption. This would aggravate the rate of inflation when the economy is suffering from an over-all demand-pull inflation. From the viewpoint of general equilibrium analysis, an increase in the producers' price of rice, i.e. the purchasing price by the government, would affect prices in general through three major routes: (1) a concomitant increase in the consumers' price, (2) an increase in labor costs in general, and (3) intensification of demand-pull effect for inflation. Even if the consumers' price were kept unchanged, it may well happen that urban consumers have to bear even heavier indirect burdens than the consumers' price were increased directly, depending upon what measures are taken in fiscal policies to alleviate deficits in the staple food account of the government.

If urban consumers agree to an increase in the producers' price, understanding these points fully, that agreement may be interpreted as a concrete expression of a social welfare function. However, past experiences suggest the contrary.

In other words, although in principle the public chooses appropriate policies to attain their objectives under the name of people's sovereignty, in reality what has been attained often deviates far from what was initially intended since the knowledge on which the judgements are made is deficient. The actual choice of policies in this situation is no longer regarded as a reasonable expression of people's objectives. Consequently, we cannot deduce a social welfare function as such on the basis of the people's actual choices of policies.

Needless to say, there is no question about the righteousness of individual judgements such as a greater income is better than a less or, stable prices are better than unstable ones, and pollution is bad etc. However, these judgements which reflect directly people's needs should be distinguished from the judgements by which to choose appropriate policies for the purpose of satisfying the above stated people's needs. This problem may be illustrated, for example, by the case of the land tax which was adopted sometime ago in Japan with the intention of stabilizing prices of land. Although this tax has been levied formally on owners of land, it has resulted eventually in increasing the burden of purchasers in the form of increased prices since it was levied without limiting the private property right which would have been necessary to realize the spirit of the tax. The eventual effects of economic policies often turn out to be quite different from what was originally intended because of modifications by complex reactions of economic actors and their interactions.

Moreover, in the case where policies adopted are based on intuitions, it is always possible that a policy generates even an opposite effect from what was intended due to interactive or offsetting effects between endogenous variables of an economy. For the purpose of enabling the public to make accurate policy choices, therefore, we need to prepare a menu which exhibits the correspondence between alternative feasible sets of policies and the probable effects of these policy sets which are specified on the basis of empirical economic analysis of general interdependence.

It is quite unreasonable to expect that each member of the society understands completely the structure of general interdependence of an economy. It is the job of professional economists to make alternative menues each of which presents the correspondence between a set of quantitative policy variables and a set of degrees of objective attainments for the purpose of helping individual persons' policy judgements. Only when such a supplementary instrument is used effectively can people's actual policy choices be regarded as a reasonable expression of a social welfare function. It is for this purpose, namely, the making of policy menues, that policy simulations on the basis of an measured general equilibrium-type model is required.

Notes to Chapter 14

1) Arrow(1963).
2) Duesenberry(1958).

Chapter 15
Effects of Public Expenditures on Prices

The purpose of this chapter is to make alternative policy simulations on the basis of our model, and to examine the differences in policy effects due to alternative methods of financing the government budget.

First, let us examine short-run policy effects. We will simulate policy effects for the following three alternative cases for 1960 and 1965:

Case I:	The increased government expenditure for government fixed capital formation is financed entirely by government bonds.
Case II:	Part of the increased government expenditure for government fixed capital formation is financed by increasing the rate of corporation income tax by 10 percent and the rest is financed by government bonds.
Case III:	Part of the increased government expenditure for government fixed capital formation is financed by increasing the rate of indirect taxes by 10 percent and the rest is financed by government bonds.

It is assumed that all of the increased government expenditure for fixed capital formation is spent in Sector 3 (heavy manufacturing industries) of our model.

All exogenous variables other than the policy variables specified in this simulation are assigned observed values. Results of this simulation, therefore, will be comparable with the results of the total tests reported earlier.

Before examining the simulation results, let us mention two points of qualifications. The first point is the fact that the description of the money market is considerably simplified in our model. In particular, money supply is assumed to be made automatically to meet the demand for money for a given rate of interest. This implies that we have assumed implicitly that the Bank of Japan supplies an additional amount of currency just as much as the government issues bonds through open market operations, buying up foreign exchange and loans of the Bank of Japan. This also implies to assume that the issuance of government bonds is not likely to oppress private monetary transactions by altering the allocation of funds directly.

The second point relates to the difficulty of including the legal rates of corporation income tax and indirect taxes explicitly in our model. We have used instead the ex-post average tax rate as a proxy for the legal tax rate.

Now let us look at actual developments of related variables in the first half of the 1960s. The ratio of government fixed capital formation to the nominal *GNP* was 7.5 percent in 1960 and 9 percent in 1965.
The amounts of government securities issued in 1960 and 1965 were 72 billion yen and 313 billion yen respectively. If we consider the amount of redemption, the balance of government securities was 446 billion yen in 1960 and 688 billion yen 1965. On the other hand, the amount of treasury bills reached 679 billion yen in 1960 and 718 billion yen in 1965, respectively.

The average rate of corporate income tax was 35 percent in 1960 and 44 percent in 1965. The indirect tax rates for the four sectors are:

	1960	1965
Sector 1	0.01366	0.01473
Sector 2	0.07601	0.06641
Sector 3	0.01000	0.00982
Sector 4	0.03942	0.03297

In terms of average rates, the rate of corporation income tax increased from 1960 to 1965, while the rates of indirect taxes declined from 1960 to 1965. With this factual overview in mind, let us examine economic effects of an increase in government expenditure for capital formation for alternative cases of financing methods.

The Table 15.1 summarizes the effects of an increase in government capital formation for the two years. To facilitate comparison of effects on different endogenous variables we have computed impact multipliers as explained in the footnote to the table.

Table 15.1 EFFECTS OF A 10 PERCENT INCREASE IN GOVERNMENT CAPITAL FORMATION ON VARIOUS ENDOGENOUS VARIABLES

	Case I		Case II		Case III	
	1960	1965	1960	1965	1960	1965
Real Output						
Sector 2	0. 0617	0. 1266	0. 0713	0. 1248	△0. 0381	0. 0909
Sector 3	0. 6712	0. 7650	0. 5580	0. 6685	0. 4362	0. 6700
Sector 4	0. 1103	0. 1372	0. 0904	0. 1193	△0. 0586	0. 0843
Nominal GNP	0. 5849	0. 7408	0. 4848	0. 6714	0. 4108	0. 6869
Real GNP	0. 2732	0. 5741	0. 2345	0. 4829	0. 0662	0. 5813
Nominal Exports	△0. 0052	△0. 0342	△0. 0035	△0. 0244	△0. 0012	△0. 0274
Nominal Imports	0. 0681	0. 0697	0. 0610	0. 0621	0. 0580	0. 0661
Personal Consumption Investment	0. 1572	0. 1928	0. 1303	0. 1749	0. 0391	0. 1469
	△0. 4781	△0. 2897	△0. 5281	△0. 3782	△0. 5618	△0. 3436
Disposable Income	0. 3246	0. 3959	0. 2682	0. 3593	0. 1267	0. 3016
Compensation of Employees	0. 2279	0. 2869	0. 1887	0. 2595	0. 0768	0. 2276
Retained Earnings	0. 0642	0. 0128	△0. 0679	△0. 0310	0. 0199	0. 0428

Note: The effects presented in this table are expressed in terms of the impact coefficient, which is defined as

$$\text{Impact Coefficient} = \frac{\text{Simulated changes in the value of an endogenous variable}}{\text{Hypothetical changes in the amount of government capital formation}}$$

Mark Δ stands for minus sign.

The simulation result reveals that the impact multiplier on *GNP* is less than 1 for both 1960 and 1965. This implies that an increase in government capital formation depresses the effective demand in the private sector. the impact multiplier on real *GNP* turns out to be even smaller. This suggests that government demand competes with the private demand and thereby tends to increase prices.

The impact multipliers on nominal *GNP* are greater in 1965 than in *1960* for all three cases. This is probably due to an increase in the output capacities achieved in the process of "heavy industrialization" during the early 1960s. The impact multiplier for the case of financing by issuing bonds was greater for both years. The impact multiplier for the case of financing through increasing the rate of corporation income tax turned out to be greater in 1960 when the actual rate of corporation income tax was low, while the multiplier for the case of financing through increasing the rates of indirect taxes turned out to be relatively greater in 1965. This result seems to suggest that the suppressive effect of an additional government demand on private demand is greater the higher the actual tax rate.

Let us now examine these effects in terms of effects on real production by different production sectors. Let us estimate impact multipliers on production in the case of increasing the final demand in Sector 3 by 1 percent using the inverse matrices of input coefficients of the four sectors for both years.

	1960	1965
Sector 2	0.2549	0.2941
Sector 3	1.7511	1.6259
Sector 4	0.2984	0.2712

In this case, the likely effects of price changes are totally ignored since the input-output tables are used as "open models." According to this simplistic estimation, the impact multiplier for Sector 3 is greater in 1960 rather than in 1965. However, once we introduce the effects of price changes we will see that the impact multipliers will be greater for 1965 for all cases.

In all cases of simulation an increase in government demand has been found to have an oppressive effect on private demand. Theoretically speaking, a depressive effect on aggregate demand should be most conspicuous in the case of an increase in the rate of corporation income tax since aggregate demand would be oppressed in this case both by increases in prices of investment goods and by a decrease in retained earnings of private corporations. According to our simulation result, however, an inflationary effect due to an increase in the rate of indirect tax is also conspicuous. Especially in 1960 when the actual rate of indirect tax was high, the depressive effect on private investment for capital formation turned out to be even greater in Case III than in Case II.

On the other hand, the effect of increasing the aggregate personal disposable income and compensation of employees was greatest in the case of financing by bonds, which is followed by the case of financing by increasing the rate of corporation income tax.

Table 15.2 summarizes the simulation results on price changes. Each figure in Table 15.2 indicates the simulated price elasticity with respect to a one percent change in government fixed capital formation.

Wholesale prices, composed primarily of investment goods, tend to increase most greatly in the case of financing by bonds. This finding may be interpreted as the result of an intensified demand-pull effect due to the additional government demand which has had a smaller oppressive effect on private demand than the other two types of financing.

Impacts on consumers' prices have been generally moderate according to

Table 15.2 IMPACT MULTIPLIERS IN ALTERNATIVE CASES OF FINANCING GOVERNMENT CAPITAL FORMATION

Year	1960			1965		
Cases	I	II	III	I	II	III
Output Deflator						
Sector 2	0. 0067	0. 0055	0. 0224	0. 0105	0. 0096	0. 0191
Sector 3	0. 0491	0. 0382	0. 0362	0. 0596	0. 0525	0. 0544
Sector 4	0. 0041	0. 0028	0. 0053	0. 0064	0. 0055	0. 0072
Consumer Price Index	0. 0060	0. 0056	0. 0136	0. 0107	0. 0095	0. 0140
Wholesale Price Index	0. 0450	0. 0359	0. 0346	0. 0538	0. 0469	0. 0506
Deflator of Consumption Goods						
Food	0. 0042	0. 0041	0. 0155	0. 0077	0. 0072	0. 0133
Clothing	0. 0051	0. 0049	0. 0187	0. 0096	0. 0083	0. 0151
Fuel & Light	0. 0045	0. 0033	0. 0074	0. 0077	0. 0069	0. 0104
Housing	0. 0130	0. 0114	0. 0140	0. 0195	0. 0168	0. 0195
Miscellaneous	0. 0055	0. 0051	0. 0085	0. 0098	0. 0087	0. 0123
Deflator of Housing Inv.	0. 0469	0. 0381	0. 0362	0. 0647	0. 0524	0. 0560
Deflator of Export Goods	0. 0196	0. 0161	0. 0222	0. 0378	0. 0313	0. 0364
GNP Deflator	0. 0220	0. 0185	0. 0228	0. 0250	0. 0232	0. 0273

Notes: (1) The impact multiplier is defined as

$$\eta_x = \frac{\Delta X}{\Delta G_I} \cdot \frac{G_I}{X} ,$$

where G_I and ΔG_I denote, respectively, government fixed capital formation and its change, and X and ΔX denote, respectively, an endogenous variable and its change.
(2) Case I: An increase in government fixed capital formation is financed entirely by issurance of long-term government bonds,
Case II: An increase in government fixed capital formation is financed by an increase in the rate of corporation taxes.
Case III: An increase in government fixed capital formation is financed by an increase in the rate of indirect taxes.

our simulation results. Among various cases, however, a considerable inflationary effect on consumers' goods has been found in the case of financing by increasing the rate of indirect tax (Case III).

This is because an increase in the tax rate has a direct effect in shifting the supply schedule upward. The pattern of changes in prices of investment goods for housing resembles the pattern of price changes of investment goods in general since the former partially competes with the latter in the markets. Table 15.2 also reveals that increases in export prices triggered by increases in domestic prices have only a minor effect in reducing the quantity of exports.

We have been examining so far short-run policy effects of additional government demand for capital formation while assuming the output capacities remain intact. Let us now examine various effects of government and private investments which have long-run implications.

Government investment is likely to induce inflation when the level of capacity utilization is high so long as it competes with private investment demand. Our simulation result indicates that the multiplier effect of an increase in government fixed capital formation on real *GNP* is merely 0.4 to 0.7 because of its oppressive effect on private investment for capital equipment.

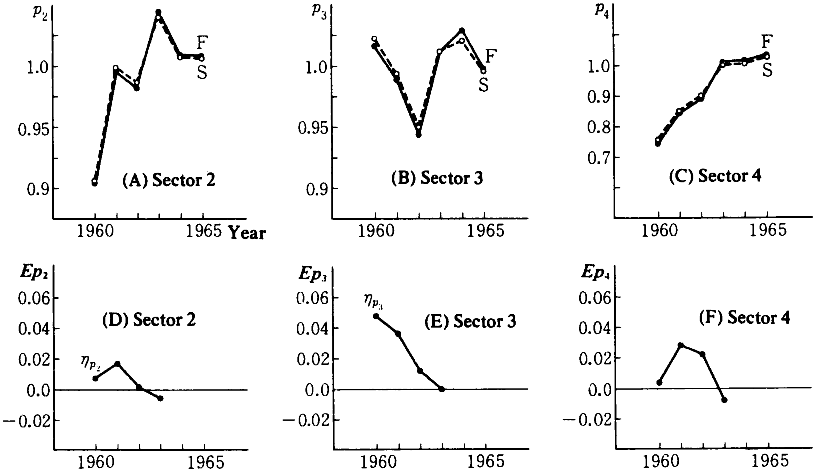
A decrease in private investment for capital equipment will consequently limit the output capacities in private production sectors. However, an increase in government fixed capital formation will have a positive effect on the other hand in increasing the supply capacities in private sectors by enriching the infra-structure of the economy. To the extent that this is true, part of the depressive effect of government investment demand on the output capacity in private sectors can be offset. In order to examine this kind of long-run effect, let us attempt simulations of long-run effects of Case I for 1960.

We will examine long-run impacts of a hypothetical once-and-for-all increase of 10 percent in government fixed capital formation in 1960. It is assumed that this additional increase in government fixed capital formation is made entirely within Sector 3 (heavy manufacturing industries). It is also assumed that this additional government expenditure is financed entirely by issuing government bonds while all other exogenous variables, including the rates of corporation income tax and indirect taxes, are assigned observed values. Therefore, the simulation is made in the form of a final test.

Table 15.3 presents the simulation results of economic effects of a 10 percent increase in government fixed capital formation in 1960 in Sector 3. Figures 15.1 and 15.2 illustrate on the basis of the simulation results the impacts upon major endogenous variables.

Figure 15.1 exhibits movements in output deflators and price elasticities with respect to increases in government fixed capital formation for Sectors 2, 3 and 4. The price of output of Sector 1 is given as an exogenous variable, as we have explained earlier. The lines indicated by F represent the results of the final test and the lines indicated by S express the simulation results. Since the result of the final test represents approximately the actual observations, the time-series movements in deflators simulated here may be said to conform quite well with the movements of actual observations. In Panels (D), (E) and (F), movements of price elasticities are presented in order to show directions of change more explicitly. The movements in price elasticities suggest that in all sectors the impact of the hypothetical once-and-for-all exogenous change in government fixed capital formation in 1960 decays within three to four

Figure 15.1 ESTIMATED CHANGES IN IMPACT MULTIPLIERS ON PRICES (OUTPUT DEFLATORS) DUE TO A 10 PERCENT INCREASE IN GOVERNMENT CAPITAL FORMATION IN 1955



Note: In Panels (A), (B) and (C), the vertical axes measure the price index p_j ($j=2$, 3 and 4) standardized by setting the 1965 value as 1.0. The solid lines F represent the estimates derived from the final test and the dotted lines S represent the results of the simulation made by a hypothetical increase of 10 percent government capital formation.

In Panels (D), (E) and (F), the vertical axes measure the impact multipliers, denoted as $E_{\hat{p}j}$ ($j=2$, 3 and 4), which are calculated by

$$E_{pj} = [\frac{\Delta p_j}{\Delta G_I} \cdot \frac{G_I}{p_j}] ,$$

where p_j and G_I represent, respectively, the price index and government capital formation.

periods. In Sector 3, the price elasticity increases in the first period due probably to the competitive relationship between government and private demand. However, the rate of increase diminishes gradually and returns to the original level within three periods. In contrast, thė Sectors 2 and 4 where the proportion of consumers' goods is large, prices tend to increase rather sizeably after the first period since the decline in private capital formation in the initial period constrains subsequent expansions in output capacities.

These trends in sectoral deflators are reflected also in consumers' prices, wholesale prices and prices of investment goods for housing. Consumers' prices, which consist largely of outputs of Sectors 2 and 4, increase with a considerable lag. In contrast, wholesale prices and prices of investment goods for housing, which are dominated by output of Sector 3, increase

Table 15.3 EFFECTS OF A 10 PERCENT INCREASE IN GOVERNMENT

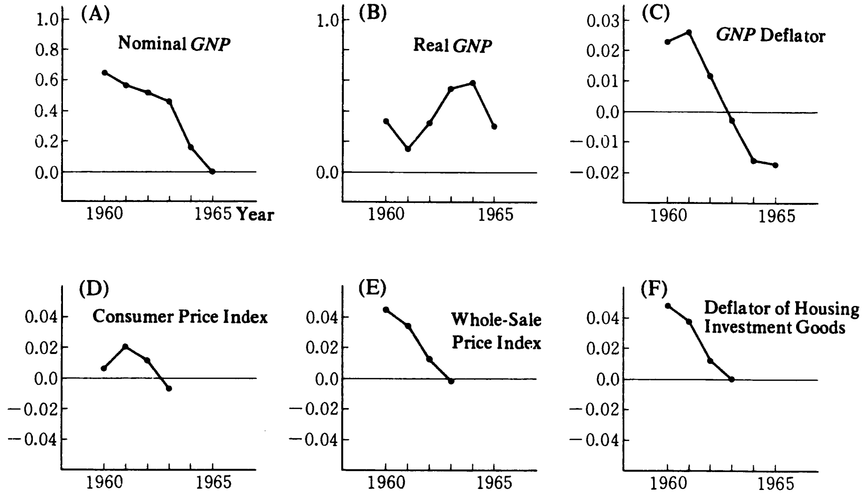
		1960	1961	1962	1963	1964	1965
Output Deflator							
Sector 2	FN.	0.9048	0.9966	0.9878	1.0451	1.0113	1.0086
	S.	0.9055	0.9983	0.9880	1.0445	1.0102	1.0073
	η	0.0077	0.0171	0.0020	△0.0057	△0.0109	△0.0129
Sector 3	FN.	1.0164	0.9886	0.9448	1.0116	1.0387	0.9986
	S.	1.0213	0.9923	0.9459	1.0116	1.0370	0.9960
	η	0.0483	0.0374	0.0117	0.0	△0.0164	△0.0261
Sector 4	FN.	0.7482	0.8428	0.8935	1.0037	1.0131	1.0379
	S.	0.7485	0.8452	0.8954	1.0030	1.0106	1.0354
	η	0.0040	0.0285	0.0213	△0.0071	△0.0247	△0.0241
Nominal GNP	FN.	17918.9	19239.6	22251.8	27787.4	31646.9	36267.3
	S.	17992.6	19305.5	22311.7	27840.9	31664.2	36230.3
	η	0.6466	0.5780	0.5252	0.4697	0.1517	△0.3240
Real GNP	FN.	20814.1	20946.8	23998.9	27572.3	31191.5	35773.6
	S.	20851.4	20963.8	24035.0	27633.8	31257.9	35807.8
	η	0.3320	0.1516	0.3215	0.5466	0.5907	0.3014
GNP Deflator							
	FN.	0.8609	0.9185	0.9272	1.0078	1.0146	1.0138
	S.	0.8629	0.9209	0.9283	1.0075	1.0130	1.0118
	η	0.0233	0.0261	0.0118	△0.0030	△0.0158	△0.0177
Deflator of Personal Consumption							
Food	FN.	0.7176	0.8031	0.8494	0.9488	0.9526	1.0147
	S.	0.7180	0.8045	0.8500	0.9483	0.9514	1.0133
	η	0.0056	0.0174	0.0071	△0.0053	△0.0126	△0.0138
Clothing	FN.	0.7917	0.8816	0.9191	1.0027	1.0066	1.0217
	S.	0.7923	0.8834	0.9198	1.0021	1.0051	1.0199
	η	0.0076	0.0204	0.0076	△0.0060	△0.0149	△0.0176
Fuel and Light	FN.	0.9047	0.9903	1.0125	1.0705	1.0401	1.0285
	S.	0.9051	0.9925	1.0140	1.0698	1.0381	1.0263
	η	0.0044	0.0222	0.0148	△0.0065	△0.0183	△0.0214
Housing	FN.	0.7090	0.7957	0.8248	0.9336	0.9835	1.0263
	S.	0.7101	0.7980	0.8262	0.9330	0.9814	1.0238
	η	0.0155	0.0289	0.0169	△0.0064	△0.0213	△0.0244
Miscellaneous	FN.	0.7446	0.8198	0.8774	0.9793	0.9840	1.0305
	S.	0.7451	0.8220	0.8791	0.9787	0.9818	1.0281
	η	0.0067	0.0269	0.0193	△0.0061	△0.0224	△0.0233
Consumer Price Index	FN.	0.7371	0.8221	0.8662	0.9647	0.9756	1.0222
	S.	0.7376	0.8239	0.8673	0.9641	0.9739	1.0203
	η	0.0067	0.0219	0.0127	△0.0062	△0.0174	△0.0186

Note: The notation F stands for the estimates obtained from the final tests, S the estimates obtained from the simulation and η the impact multiplier. For the definition of the impact multiplier, see the footnote attached to Table 15.2.

CAPITAL FORMATION ON VARIOUS ENDOGENOUS VARIABLES

		1960	1961	1962	1963	1964	1965
Price of Export Goods	FN.	0. 9880	0. 9931	0. 9612	1. 0285	1. 0357	1. 0073
	S.	0. 9901	0. 9957	0. 9622	1. 0282	1. 0342	1. 0051
	η	0. 0213	0. 0262	0. 0104	△0. 0029	△0. 0145	△0. 0218
Price of Capital Goods	FN.	1. 0110	1. 0029	0. 9703	1. 0272	1. 0415	1. 0018
	S.	1. 0156	1. 0064	0. 9715	1. 0271	1. 0398	0. 9993
	η	0. 0455	0. 0349	0. 0124	△0. 0009	△0. 0016	△0. 0239
Investment	FN.	0. 8082	0. 8901	0. 8959	0. 9825	1. 0283	0. 9996
	S.	0. 8122	0. 8935	0. 8969	0. 9825	1. 0266	0. 9970
	η	0. 0495	0. 0382	0. 0112	0. 0	△0. 0165	△0. 0260
Personal Consumption	FN.	9057. 71	10088. 01	11557. 20	13764. 75	15790. 77	18192. 32
	S.	9076. 91	10100. 41	11582. 25	13778. 12	15791. 19	18181. 51
	η	0. 1683	0. 1087	0. 2195	0. 1172	0. 0037	△0. 0949
Invertment	FN.	2835. 47	3740. 53	3607. 84	4325. 63	6254. 90	5924. 93
	S.	2786. 44	3798. 20	3656. 23	4369. 10	6288. 77	5909. 78
	η	△0. 4297	0. 5054	0. 4241	0. 3810	0. 2968	△0. 1328
Exports	FN.	1650. 93	1867. 84	2092. 52	2270. 71	2692. 60	3338. 99
	S.	1650. 55	1867. 83	2092. 89	2270. 47	2692. 85	3341. 37
	η	△0. 0033	△0. 0001	0. 0034	△0. 0021	0. 0022	0. 0209
Imports	FN.	1854. 09	2209. 92	2195. 44	2784. 16	3252. 50	3426. 32
	S.	1862. 50	2217. 79	2199. 98	2788. 80	3253. 84	3423. 10
	η	0. 0737	0. 0689	0. 0398	0. 0407	0. 0117	△0. 0282
Real Output Sector 2	FN.	10529. 8	10499. 3	12209. 0	14426. 4	15685. 8	18231. 8
	S.	10540. 1	10493. 1	12218. 4	14461. 5	15714. 2	18261. 2
	η	0. 0917	△0. 0552	0. 0836	0. 3121	0. 2526	0. 2615
Sector 3	FN.	12681. 3	14673. 7	16837. 6	18299. 9	22795. 9	25918. 5
	S.	12758. 8	14697. 7	16904. 2	18379. 2	22862. 5	25947. 6
	η	0. 6893	0. 2135	0. 5924	0. 7053	0. 5924	0. 2588
Sector 4	FN.	13871. 9	14479. 1	16232. 3	17942. 9	20467. 9	23070. 8
	S.	13886. 3	14442. 9	16276. 9	17980. 1	20521. 9	23122. 8
	η	0. 1281	△0. 3220	0. 3967	0. 3309	0. 4803	0. 4625
Disposable Income	FN.	11345. 6	12455. 7	14324. 3	17365. 5	19619. 1	22588. 6
	S.	11385. 0	12481. 1	14375. 8	17392. 9	19619. 9	22566. 4
	η	0. 3453	0. 2226	0. 4515	0. 2401	0. 0070	△0. 1945
Retained Earnings	FN.	1087. 03	1257. 10	1274. 35	1384. 99	1919. 78	1829. 27
	S.	1094. 84	1262. 46	1281. 40	1389. 48	1920. 47	1826. 17
	η	0. 0684	0. 0469	0. 0618	0. 0394	0. 0060	△0. 0271
Compensation of Emplyees	FN.	6605. 1	7104. 9	8276. 3	10511. 5	12042. 4	13831. 2
	S.	6632. 8	7119. 0	8307. 2	10531. 0	12046. 9	13820. 5
	η	0. 2428	0. 1236	0. 2708	0. 1709	0. 0394	△0. 0938

Figure 15.2 ESTIMATED CHANGES IN IMPACT MULTIPLIERS ON VRIOUS ENDOGENOUS VARIABLES DUE TO A 10 PERCENT INCREASE IN GOVERNMENT CAPITAL FORMATION IN 1955



Note: The vertical axis of each panel measures the value of impact multiplier on each relevant endogenous variable. For the derivation of the impact multipliers, see the footnote attached to Figure 15.1

immediately reflecting the effect of competition between the government and private demand. Figure 15.2 depicts these different patterns of price changes.

The result presented in Panel (A) of Figure 15.2, indicates that the once-and-for-all increase in government capital formation will have a positive effect on nominal *GNP* for five periods but its effect will become only minor after the third period. Panel (B) presents the impact on real *GNP* which takes into account changes in prices. The impact multiplier in this case changes irregularly within the range of 0.2 to 0.6. Particularly in the first period, the impact on real *GNP* will be depressed due to increases in consumers' prices.

Thus far, we have summarized briefly our simulation results for hypothetical increases in government expenditures for capital formation. The results suggest that the eventual economic effects of policy actions are highly complex. To achieve certain policy targets, it is suggested therefore that the likely effects of alternative policies need to be investigated rigorously on the basis of empirical analysis of general interdependence. To make such an analysis more elaborate and reliable, many improvements have yet to be made, however. One such problem, suggested in our simulation, is the probable competitive relationship between government and private demand. This problem deserves further and more intensive investigations using models with finer sectoral classifications.

Chapter 16
A Simulation Analysis of Tax Shifts

Changes in the rate of corporation income tax or of indirect tax cannot be discussed separately from their implications of tax shifts. It seems, however, that the problem of tax shift has been discussed within a limited scope of partial equilibrium analysis.[1]

The problem of tax shift is by itself an important and intriguing issue from the viewpoint of an equitable distribution of tax burdens. It is certainly important to realize, for example, that an increase in corporation taxes which is seemingly an increased burden on the firm may eventually be shifted to consumers through increases in the commodity prices depending upon particular conditions in the market.

If the increase in the rate of indirect tax is shifted to commodity prices, then relative prices would change. An increase in the price of a certain commodity would also affect the demand for it. This demand also depends upon the level of income of purchasers. Therefore, it is not easy to investigate who eventually would bear the burden of the increased indirect tax when the question is viewed in the context of general interdependence within the economy.

Our focus is on the eventual effect of a tax which is generated through the general interdependence within an economy and, unlike the usual way of dealing with the issue of tax shift, not on whether or not a producer shifts tax burdens to someone else in an attempt to escape from them. Indeed, it should be difficult to determine whether the producer did escape from the tax burdens unless the net effect of his tax shift through the working of interdependent system of the economy is ascertained.[2]

We shall attempt in this chapter to demonstrate this point more concretely using specific examples of the Japanese economy. In doing so, we shall again apply the method of simultation analysis of alternative tax policies using the estimated results of our model. Simulations will be made for 1955, 1960 and 1965. All exogenous variables except the rates of corporation income tax and indirect tax are assigned observed values for each year of simulation. The observed rates of corporation income tax and indirect taxes for the selected years of simulation are as follows:

The rates of corporation income tax:
1955; 0.417, 1960; 0.350, 1965; 0.444.

The rates of indirect taxes:

	Sector 1	Sector 2	Sector 3	Sector 4
1955	0.01233	0.09480	0.00835	0.04060
1960	0.01366	0.07601	0.01001	0.03942
1965	0.01473	0.06641	0.00982	0.03297

These figures are average tax rates and not exactly the same as the effective legal rates.

Let us first compute the convergence solutions for major endogenous variables for the hypothetical cases in which the rates of corporation income tax and indirect taxes are raised by 10 percent from the observed rates for 1955, 1960 and 1965. We will then compare the thus obtained results with the results of the total tests.

Table 16.1 presents a summary of results of simulations and total tests for major endogenous variables. Notations T represent the result of the total test, $S(1)$ the simulation result of the case of a 10 percent increase in the rate of corporation income tax, and $S(2)$ the simulation result of the case of a 10 percent increase in the rate of indirect taxes. It is assumed in these simulations that the increased government revenue due to increases in tax rates is not used immediately for government expenditures. The level of government expenditure is given exogenously for all cases of simulations.

Let us first examine the simulation results of the case, denoted by $S(1)$, in which the rate of corporation income tax is raised. Even though short-run supply schedules of different sectors are assumed to be unchanged, retained earnings of private corporations will be affected. The retained earnings decrease by 10 percent in 1955, by 7 percent in 1960 and by 12 percent in 1965. The decline in retained earnings of private corporations will naturally depress the demand for investment goods. In Sector 2, the demand for investment goods decreases by 14 to 15 percent each year. In Sector 3, the decline is particularly conspicuous in 1955 while there are only insignificant

Table 16.1 SIMULATED VALUES OF ENDOGENOUS VARIABLES FOR ALTERNATIVE TAX RATE

		1955	1960	1965
Output Deflator				
Sector 2	T	0.9916	0.8969	1.0002
	S(1)	0.9912	0.8963	0.9989
	S(2)	1.0030	0.9049	1.0078
Sector 3	T	0.8363	0.9913	0.9740
	S(1)	0.8327	0.9877	0.9672
	S(2)	0.8286	0.9864	0.9704
Sector 4	T	0.6989	0.7302	0.9835
	S(1)	0.6986	0.7298	0.9824
	S(2)	0.6983	0.7300	0.9833
Nominal Wage		(yen/man-hour)		
Sector 1	T	19.9	25.6	62.9
	S(1)	19.9	25.6	63.0
	S(2)	19.9	25.6	63.0
Sector 2, 3				
	T	75.0	91.7	200.0
	S(1)	75.0	91.7	200.0
	S(2)	75.0	91.7	200.0
Sector 4	T	91.7	108.3	241.7
	S(1)	91.7	108.3	241.7
	S(2)	91.7	108.3	241.7

		1955	1960	1965
Deflator of Personal Consumption				
: Food	T	0.7522	0.7097	0.9959
	S(1)	0.7519	0.7093	0.9949
	S(2)	0.7578	0.7136	1.0004
: Clothing	T	0.8750	0.7821	0.9975
	S(1)	0.8747	0.7816	0.9962
	S(2)	0.8823	0.7875	1.0024
: Fuel and Light	T	0.8870	0.8885	0.9883
	S(1)	0.8867	0.8880	0.9872
	S(2)	0.8876	0.8894	0.9901
: Housing	T	0.5196	0.6927	0.9826
	S(1)	0.5192	0.6917	0.9802
	S(2)	0.5192	0.6921	0.9823
: Miscellaneous				
	T	0.7286	0.7274	0.9826
	S(1)	0.7282	0.7268	0.9811
	S(2)	0.7286	0.7274	0.9830
Deflator of Investment Goods	T	0.7802	0.9865	0.9757
	S(1)	0.7771	0.9831	0.9695
	S(2)	0.7740	0.9820	0.9727
Deflator of Exported Goods	T	1.0419	0.9707	0.9839
	S(1)	1.0405	0.9691	0.9798
	S(2)	1.0437	0.9722	0.9843

		1955	1960	1965
Compensation of Employees	T	3362.1	6558.8	13539.0
	S(1)	3351.5	6534.6	13440.0
	S(2)	3293.6	6450.4	13345.0
Income from Unincorporated Enterprises	T	2660.4	3826.9	6652.8
	S(1)	2656.7	3819.2	6620.9
	S(2)	2631.1	3779.2	6564.9
Personal Consumption Expenditure	T	592.4	1347.1	3240.7
	S(1)	589.0	1340.3	3213.7
	S(2)	570.0	1310.9	3174.4
Disposable Income	T	6283.6	11248.0	21978.0
	S(1)	6268.0	11213.0	21839.0
	S(2)	6174.7	11072.0	21669.0
Retained Earnings	T	435.4	1073.9	1752.5
	S(1)	390.7	994.9	1537.6
	S(2)	419.7	1043.7	1709.0
Total of Nominal Export	T	896.9	1649.9	3319.9
	S(1)	898.3	1650.3	3333.0
	S(2)	902.6	1651.5	3318.3

	1955	1960	1965
Personal Direct Taxes and Charges T	353.9	465.1	1265.6
S(1)	353.0	463.6	1257.5
S(2)	347.8	457.8	1247.7
Corporation Income Taxes			
T	423.1	728.6	1964.8
S(1)	462.6	796.7	2136.0
S(2)	408.9	709.0	1918.6
Indirect Taxes			
T	817.1	1289.7	2281.9
S(1)	815.6	1286.3	2270.2
S(2)	895.5	1412.2	2500.0
Income from Property			
: Food T	191229	234220	248548
S(1)	191050	233957	247905
S(2)	189177	231368	245610
: Clothing T	41020	62744	80537
S(1)	41004	62711	80446
S(2)	40750	62330	80082
: Fuel and Light T	13293	15869	21054
S(1)	13287	15859	21023
S(2)	13220	15781	20932
: Housing T	48885	87053	118160
S(1)	48737	86905	117810
S(2)	46589	85123	116210

	1955	1960	1965
Miscellaneous			
T	65951	117057	170920
S(1)	65863	116889	170493
S(2)	64672	115560	169322
Net Investment			
Sector 1 T	175.8	266.6	345.2
S(1)	200.9	266.0	345.2
S(2)	200.9	266.6	345.2
Sector 2 T	469.7	331.9	826.3
S(1)	405.5	277.9	697.7
S(2)	425.3	291.4	768.7
Sector 3 T	54.6	932.6	1123.7
S(1)	41.3	922.2	1106.1
S(2)	32.4	917.2	1100.3
Sector 4 T	193.1	813.1	2135.2
S(1)	223.1	828.5	2106.1
S(2)	175.1	797.7	2109.9
Output			
Sector 1 T	3876.3	4536.9	4489.8
S(1)	3876.3	4536.9	4489.8
S(2)	3876.3	4536.9	4489.8
Sector 2 T	5547.2	10449.1	18041.8
S(1)	5541.8	10434.0	17995.0
S(2)	5496.3	10365.0	17916.0
Sector 3 T	5173.0	13040.3	25770.0
S(1)	5135.3	12982.0	25522.0
S(2)	5048.6	12906.0	25529.0

	1955	1960	1965
Sector 4 T	8099.3	14115.0	23770.6
S(1)	8090.2	14096.0	23706.0
S(2)	8012.5	13965.0	23574.0
Total of Nominal Import T	851.6	1832.1	3340.0
S(1)	849.6	1825.3	3320.9
S(2)	848.0	1822.7	3330.1
Nominal GNP			
T	9337.9	17678.7	35368.3
S(1)	9311.3	17616.8	35112.5
S(2)	9253.6	17539.3	35113.6
Real GNP			
T	11858.1	20892.3	35891.6
S(1)	11840.9	20857.8	35748.2
S(2)	11743.7	20721.8	35616.9
GNP Deflator			
T	0.7875	0.8462	0.9853
S(1)	0.7864	0.8446	0.9822
S(2)	0.7869	0.8464	0.9859

Note: The results of three alternative cases presented in this table are:
T: estimates obtained from the total tests, S(1): estimates obtained from the simulation assuming that the rate of corporation income taxes has been raised by 10 percent, and S(2): estimates obtained from the simulation assuming that the rate of indirect taxes has been raised by 10 percent.

declines for 1960 and 1965. In Sector 1, on the other hand, the demand for investment goods increased by 14 percent in 1955 and remained unchanged for the other years. In Sector 4, the demand increased by 15 percent in 1955, 2 percent in 1960 and decreased by more than one percent in 1965.

These results imply that although an increase in the rate of corporation income tax would generally reduce retained earnings proportionately, its effects on the demand for investment goods are not uniform. A reduction in the demand for investment goods will have a depressive effect on prices of investment goods. The decline in prices of investment goods would stimulate the demand for investment goods, which would partially offset the depressive effect of the decreased retained earnings on the demand for investment goods. The major reasons why the demand for investment goods varies from year to year or between different sectors seem to be in the differences in demand elasticities with respect to retained earnings of private corporation and also to prices of investment goods under different conditions. When all sectors are integrated, the composite price of investment goods does not vary much from year to year.

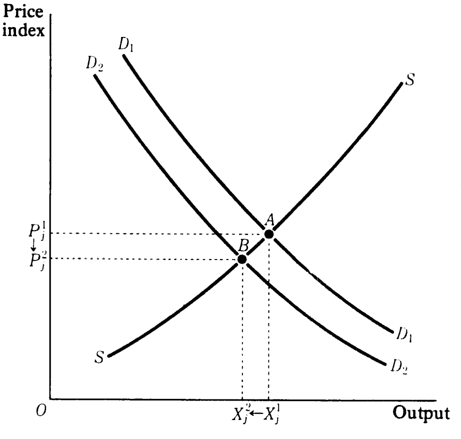
Changes in demand-supply balance in the market of investment goods affect compensation of employees, income from unincorporated enterprises, and income from property through alterations in income distributions. The 10 percent increase in the rate of corporation income tax thus reduces personal disposable income by 0.2 to 0.6 percent. The decline in personal disposable income, which is the budget constraint on consumption, will then decrease personal consumption expenditures by 0.1 to 0.3 percent for all major items.

These effects on markets of investment goods and consumers' goods will eventually be reflected in the demand-supply balances of commodity sectors. The change in the demand-supply balance for each sector may be explained by the illustration in Figure 16.1.

Let us evaluate the overall effect of an increase in the rate of corporation income tax using *GNP* as an indicator. Nominal *GNP* is reduced by 0.3 to 0.7 percent, while the *GNP* deflator declines by 0.1 to 0.3 percent, which together indicate that real *GNP* is reduced by 0.2 to 0.4 percent. It is implied therefore that the phenomenon of a shift of the increased rate of corporation income tax to increased prices has not been ascertained from the result of this simulation unless the increased tax revenue is purposefully used for increasing government expenditures.

Next, let us examine simulation results of case $S(2)$ in which the rate of indirect taxes has been raised by 10 percent for all sectors. Table 16.2

Figure 16.1 AN ILLUSTRATION OF A SHIFT OF THE DEMAND-SUPPLY EQUILIBRIUM POINT IN A CERTAIN SECTOR IN RESPONSE TO AN INCREASE IN THE RATE OF CORPORATION INCOME TAX: CASE 1



Note: This diagram illustrates the case in which the increased general government revenue is not expended.

presents partial elasticities of supply prices reduced from supply equations with respect to rates of indirect taxes for different sectors.

The partial elasticities reported in Table 16.2 indicate that an increase in the rate of indirect taxes generally tends to increase supply prices. The elasticity is greatest in Sector 2 where it ranges between 0.1 and 0.13, and followed next by Sector 4 and Sector 3.

It should be added quickly, however, that the eventual effects of an increase in the tax rate after taking the general interdependence of an economy into account may well be different from this result.

Let us look at the eventual demand-supply balances of Case $S(2)$ for different sectors. In Sector 2, equilibrium price has increased by 0.8 to 1.1 percent, while the equilibrium quantity has decreased by 0.7 to 0.9 percent. On the other hand, in Sector 3, the equilibrium price declined by 0.4 to 1.0 percent and the equilibrium quantity also decreased by 0.9 to 2.5 percent. Similarly, in Sector 4, the equilibrium price declined by 0.1 percent and the equilibrium quantity decreased by 0.8 to 1.2 percent.

Although the previous partial equilibrium analysis indicated that an increase in the rate of indirect tax has a tax shift effect in the form of raising supply prices for all sectors, this result suggests that the effects of tax shift

Table 16.2 ELASTICITIES TO INDIRECT TAX RATE

	1955	1960	1965
Sector 2	t_{I2}=0. 09480	0. 07601	0. 06641
	a_{22}=0. 18890	0. 22040	0. 25030
	η=0. 13230	0. 10800	0. 09719
Sector 3	t_{I3}=0. 00835	0. 01001	0. 00982
	a_{33}=0. 41550	0. 45850	0. 39370
	η=0. 01449	0. 01883	0. 01645
Sector4	t_{I4}=0. 04060	0. 03942	0. 03297
	a_{44}=0. 08680	0. 09600	0. 13120
	η=0. 04652	0. 04559	0. 03945

Note: Notations are:

t_{Ij} : the rate of indirect tax for the j-th sector (j=2, 3 and 4),

a_{jj} : input-coefficient representing the input to its own sector (j=2, 3 and 4), and

η : elasticity computed on the basis of specification of supply function of each sector as follows;

$$\eta = \frac{\partial \log P_j}{\partial \log t_{Ij}} = \frac{\partial P_j}{\partial t_{Ij}} \cdot \frac{t_{Ij}}{P_j} = \frac{t_{Ij}}{a_{jj} \times t_{Ij} - 1}$$

are not so readily discernible. Indeed the result reveals that supply prices tended to decrease except in Sector 2.

When examined using *GNP* as a comprehensive indicator of the impact, nominal *GNP* is found to have declined by 0.7 to 0.9 percent. Since the *GNP* deflator has not changed appreciably, this means that real *GNP* has also decreased similarly.

These results suggest that when the increased tax revenues due to an increase in tax rates are not used for government expenditures, the total size of economic activities shrink and that the increase in prices due to a tax shift will be hardly observable.

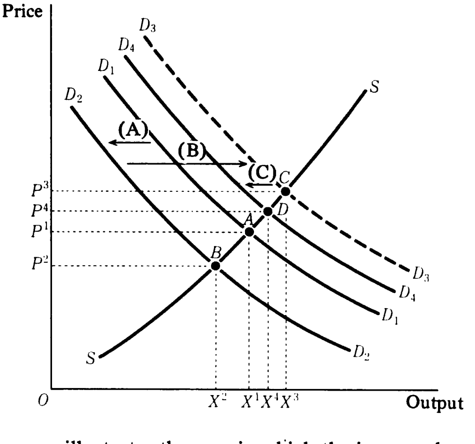
Thus far, we have only considered effects of alternative tax revenue policies on endogenous variable. Needless to say, the expenditure side is unseparable from the revenue side when we evaluate the entire system of tax policies. Implications of tax policies can be quite different depending on whether the increased revenue is spent for general government consumption expenditures or for government fixed capital formation. The changes of the composition of government expenditures will affect directly not only the structure of aggregate demand, but also importantly the allocation of resources within the economy.

It is with this interest that we examine in what follows the effects on resource allocation by two cases of simulations: (1) the increased government revenue, obtained by a 10 percent increase in the rates of corporation income tax and indirect taxes, is expended entirely as an additional general government consumption expenditure, and (2) the same increased government revenue is expended entirely for the purpose of government fixed capital formation. The simulation experiment is made only for the case of 1965. The simulation results will be compared with the results of the total tests. Table 16.3 presents both of these results side by side.

General government consumption expenditure tends to be concentrated more in Sector 4 than in other sectors, while government fixed capital formation is concentrated more in Sector 3 relative to other sectors. Therefore, the former type of government demand tends to compete with private demand in the area of consumers' goods and the latter type in the area of investment goods.

Figure 16.2 illustrates simplistically the simulation result of the case in which the rate of corporation income tax is raised and the increased government revenue is expended entirely as an additional government expenditure. Let us assume for simplicity that the supply schedule is fixed at the position S—S. So long as we can ignore the effect of changes in raw material prices upon the supply schedule in the short-run, this assumption is not unreasonable since changes in the rate of corporation income tax would not shift the position of the supply schedule.

Figure 16.2 AN ILLUSTRATION OF A SHIFT OF THE DEMAND-SUPPLY EQUILIBRIUM POINT IN A CERTAIN SECTOR IN RESPONSE TO AN INCREASE IN THE RATE OF CORPORATION INCOME TAX: CASE 2



Note: This diagram illustrates the case in which the increased general government revenue is expended.

Table 16.3 SIMULATED VALUES OF ENDOGENOUS VARIABLES FOR

	10 percent increase in the corporation income tax rate		10 percent increase in the indirect tax rate	
	I	II	I	II
Output Deflator				
Sector 2 1.0002	1.0130 (101.3)	1.0094 (100.9)	1.0221 (102.2)	1.0183 (101.8)
Sector 3 0.9740	0.9798 (100.6)	1.0224 (105.0)	0.9835 (100.9)	1.0256 (105.3)
Sector 4 0.9835	1.0345 (105.2)	0.9835 (100.5)	1.0363 (105.4)	0.9886 (100.7)
Disposable Income 21978.0	24247.0 (110.3)	22908.0 (104.2)	24095.0 (109.6)	22758.0 (103.5)
Deflator of Fixed Capital Formation 0.9757	0.9850 (100.9)	1.0193 (104.5)	0.9887 (101.3)	1.0225 (104.8)
Deflator of Personal Consumption				
Food 0.9959	1.0165 (102.1)	1.0027 (100.7)	1.0224 (102.7)	1.0084 (101.3)
Clothing 0.9975	1.0234 (102.6)	1.0053 (100.8)	1.0300 (103.3)	1.0117 (101.4)
Fuel and Light 0.9883	1.0271 (103.9)	0.9947 (100.6)	1.0367 (104.8)	0.9981 (100.9)
Housing 0.9826	1.0199 (103.8)	0.9983 (101.6)	1.0228 (104.1)	1.0009 (101.9)
Miscellaneous 0.9826	1.0270 (104.5)	0.9907 (100.8)	1.0295 (104.8)	0.9931 (101.1)
Demand for Investment Goods				
Sector 1 345.2	327.0 (94.7)	295.2 (85.8)	327.0 (94.7)	290.7 (84.2)
Sector 2 826.3	639.3 (77.4)	466.9 (56.5)	702.1 (84.9)	542.4 (65.6)
Sector 3 1123.7	954.1 (84.9)	989.2 (88.0)	948.3 (84.4)	983.4 (87.5)
Sector 4 2135.2	2159.7 (101.1)	1471.5 (68.9)	2186.0 (102.4)	1480.9 (69.4)

Note: Column I represents the values of endogeneous variables simulated on the assumption that an increase in government revenue due to a 10 percent increase in the rate of either corporation income tax or indirect tax is expended for the purpose of general government consumption expenditure.

Column II represents the values of endogeneous variables simulated on the assumption that an increased government revenue due to a 10 percent increase in the tax rate is expended for the purpose of government fixed capital formation.

ALTERNATIVE CASES OF GOVERNMENT EXPENDITURES

		10 percent increase in the corporation income tax rate		10 percent increase in the indirect tax rate	
		I	II	I	II
Total Output					
Sector 2		18476. 9	18373. 3	18391. 0	18283. 0
	18041. 8	(102. 4)	(101. 8)	(101. 9)	(101. 3)
Sector 3		25673. 7	27546. 0	25645. 0	27547. 0
	25770. 0	(99. 4)	(106. 9)	(99. 5)	(106. 9)
Sector 4		26815. 9	24087. 6	26720. 0	23994. 0
	23770. 6	(112. 8)	(101. 3)	(112. 4)	(100. 9)
Retained Earnings		1739. 6	1686. 1	1937. 1	1876. 9
	1752. 5	(99. 3)	(96. 2)	(110. 5)	(107. 1)
Deflator of Exported Goods		0. 9981	1. 0131	1. 0032	1. 0178
	0. 9839	(101. 4)	(102. 9)	(101. 9)	(103. 4)
Total of Personal Consumption Expenditure		257785. 0	252893. 0	255521. 0	250662. 0
Food	248548. 0	(103. 7)	(101. 7)	(102. 8)	(100. 8)
Clothing		81754. 0	81156. 0	81394. 0	80800. 0
	80537. 0	(101. 5)	(100. 8)	(101. 1)	(100. 3)
Fuel and Light		21328. 0	21279. 0	21239. 0	21190. 0
	21054. 0	(101. 3)	(101. 1)	(100. 9)	(100. 6)
Housing		123755. 0	120595. 0	122166. 0	119035. 0
	118160. 0	(104. 7)	(102. 1)	(103. 4)	(100. 7)
Miscellaneous		174077. 0	173981. 0	172918. 0	172809. 0
	170920. 0	(101. 8)	(101. 8)	(101. 2)	(101. 1)
Nominal GNP		38703. 9	37105. 3	38835. 4	37143. 0
	35368. 3	(109. 4)	(104. 9)	(109. 8)	(105. 1)
Real GNP		38453. 9	36850. 4	38350. 0	36744. 5
	35891. 6	(107. 1)	(102. 7)	(106. 8)	(102. 4)
GNP Deflator		1. 0086	1. 0069	1. 0127	1. 0108
	0. 9853	(102. 4)	(102. 2)	(102. 8)	(102. 6)

An increase in the rate of corporation income tax, in this situation, will shift the demand curve from D_1—D_1 to D_2—D_2, for a moment, as indicated by arrow (A) in the diagram. If the increased government revenue were not expended, then the equilibrium would have been reached at point B. However, if the increased government revenue is expended in the form of an additional government fixed capital formation, then the demand curve would shift rightward as indicated by arrow (B). Consequently, the equilibrium point would shift to position C and the price would rise to p_3 accordingly. However, the increased government demand would probably compete with the private demand here and the demand schedule would therefore be pulled back somewhat along the direction of arrow (C).

The equilibrium point will be reached eventually at D through this kind of adjustment process. The extent of shifts of the demand curve as indicated by arrows (A), (B) and (C) would depend largely upon the shapes of demand curves in respective sectors. It can be said, nevertheless, that the shift indicated by arrow (B) is large in Sectors 2 and 4 when the additional government demand takes the form of an increased general government consumption expenditure, and is large in Sector 3 when the additional government demand takes the form of an additional government fixed capital formation.

According to the results reporeted in Table 16.3, in the case in which the increased government revenue is spent for general government consumption expenditure, the equilibrium prices would increase by 1.3 percent in Sector 2 and by 5.2 percent in Sector 4, and the equilibrium output would increase by 2.4 percent in Sector 2 and by 12.8 percent in Sector 4. In contrast, in the case in which the increased government revenue is used for government fixed capital formation the equilibrium price would increase by 5.0 percent in Sector 3 and the equilibrium output would increase by 6.9 percent in the same sector.

In the former case, prices of consumers' goods, which consist largely of outputs of Sectors 2 and 4, increase appreciably. However, since personal disposable income will at the same time increase substantially, the increase in personal consumption expenditures will range from 1.3 to 4.7 percent. On the other hand, in the latter case, an increase in government fixed capital formation pushes up prices of investment goods by as much as 4.5 percent. This increase in prices, coupled with a decrease in retained earnings of private corporations due to an increase in the rate of corporation income tax, will depress private demand for investment goods considerably.

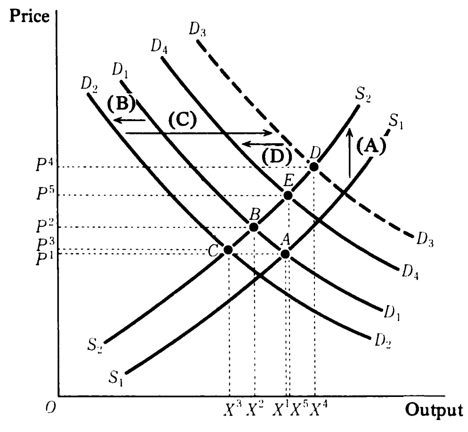
On the other hand, the effects of an increase in indirect tax rates on equilibrium prices and quantities do not differ appreciably from the simulation results of the case of an increase in the rate of corporation income tax. Note, however, that an increase in the rate of indirect tax will shift the supply schedule upward even in the short-run as indicated by Figure 16.3.

Due to an increase in the rate of indirect tax, the supply curve will shift upward from S_1—S_1 to S_2—S_2 as indicated by arrow (A). The demand curve, on the other hand, will shift in turn from D_1—D_1, D_2—D_2, D_3—D_3 and to D_4—D_4 as indicated by arrows (B), (C) and (D). The equilibrium point therefore will be reached eventually at E.

In the case of an increase in the rate of indirect tax, personal disposable income will not increase much. Therefore, the competitive relationship between government consumption expenditure and private consumption demand will be intensified in this case. On the other hand, an increase in the rate of indirect tax will not directly reduce retained earnings of private corporations. Indeed, the increase in retained earnings of private corporations ranged in this case from 7.1 to 10.5 percent. Therefore, the competitive relationship between government fixed capital formation and private demand for investment goods will be mitigated accordingly in this case compared to the case of an increase in the rate of corporation income tax.

Evaluating the simulation results in terms of a comprehensive indicator of *GNP,* both nominal and real *GNP* tend to increase because of the demand inducement effect of an additional government expenditure. Nominal *GNP* increases more greatly in the case of an increase in the rate of indirect taxes than in the case of an increase in the rate of corporation income tax regar-

Figure 16.3 AN ILLUSTRATION OF A SHIFT OF THE DEMAND-SUPPLY EQUILIBRIUM POINT IN A CERTAIN SECTOR IN RESPONSE TO AN INCREASE IN THE RATE OF INDIRECT TAX



dless of whether the increased government revenue is spent for consumption expenditure or for capital formation. However, the *GNP* deflator also increases more greatly in the case of an increase in the rate of indirect taxes, and consequently, real *GNP* increases more greatly in the case of an increase in the rate of corporation income tax.

The inflationary effect as measured by the *GNP* deflator turned out to fall in the range of 2.0 to 3.0 percent in both cases of an increase in the rate of corporation income tax and of indirect taxes. The result is quite contrary to the simulation result reported earlier in Table 16.1. Although not undebatable, this phenomenon may be interpreted as an effect of tax shift onto prices.

Following Kilpatrick (1965), let us define the rate of shift of corporation income tax as

$$S_h = \frac{(1-\rho_1)C_{I1} - (1-\rho_1)C_{I0}}{(1-\rho_0)C_{I0} - (1-\rho_1)C_{I0}},$$

where S_h represents the rate of shift of corporation income tax, C_{It} $(t = 0, 1)$ corporation income before and after the change in the tax rate, and $\rho_t (t = 0, 1)$ the tax rate before and after the revision.[3] Table 16.4 presents the rates of tax shifts S_h computed on the basis of simulation results of the case of an increase in the rate of corporation income tax reported earlier.

Table 16.4 The Rate of a Shift of Corporation Income Tax

		1955	1960	1965
[Observed Data]	Income from Private Corporation after Taxes	435.4	1073.9	1752.5
	Rate of Corporation Income Taxes	0.417	0.350	0.444
[Partial Analysis]	Income from Private Corporation after Taxes	404.2	1016.1	1612.5
	Rate of Tax Shift	0.0	0.0	0.0
[Experiment I]	Income from Private Corporation after Taxes	390.7	994.7	1537.6
	Rate of Tax Shift	−0.4327	−0.3702	−0.5350
[Experiment II]	Income from Private Corporation after Taxes	426.4	1056.0	1739.6
	Rate of Tax Shift	0.7115	0.6903	0.9078
[Experiment III]	Income from Private Corporation after Taxes	421.6	1031.8	1686.1
	Rate of Tax Shift	0.5576	0.2716	0.5257

Note: The meaning of row headings "Partial Analysis," "Experiment I," and "Experiment II" is explained in the text.

The computed rates of tax shifts in Table 16.4 indicate that in the case of Experiment I in which the increased government revenue due to increased corporation tax is not expended, the rate of tax shift has been in the range of negative 40 to 50 percent for years 1955, 1960 and 1965. In the case of Experiment II where the increased government revenue is expended in the form of an additional general government consumption expenditure, the rate of tax shift turned out to be in the range of positive 70 to 90 percent. In the case of Experiment III where the increased government revenue is used in the form of an additional government fixed capital formation, the rate of tax shift has varied in the range of positive 30 to 50 percent.

Notes to Chapter 16

1) The notion of "tax shift" has been the traditional concept in economics ever since the days of Cournot(1959). The word "shift" has usually been used in the sense of "evasion" from tax burdens. Although we deal with the problem area which has often been analyzed with interest of "tax shifts", in our analysis here, we are interested particularly in eventual changes in market balances which would take place through the working of general interdependence of an economy in response to changes in tax rates.
2) Krzyzaniak and Musgrave(1963), Gordon(1967) and Orkland(1972).
3) Kilpatrick(1965).

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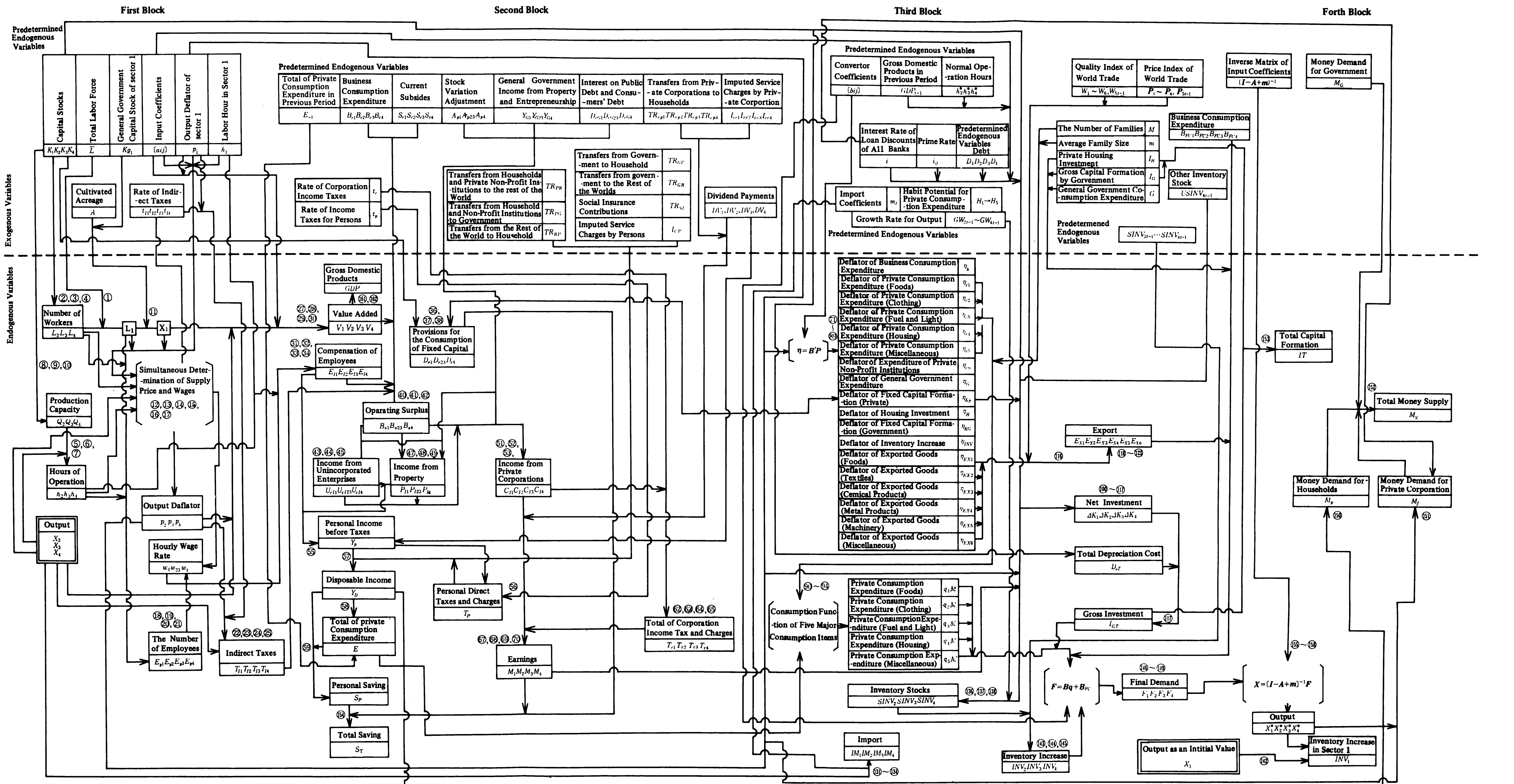
KEO Multi-sectoral Econometric Model Flow Chart



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